

General Certificate of Education
January 2008
Advanced Level Examination



MATHEMATICS
Unit Pure Core 3

MPC3

Thursday 17 January 2008 1.30 pm to 3.00 pm

For this paper you must have:

- an 8-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Find $\frac{dy}{dx}$ when:

(i) $y = (2x^2 - 5x + 1)^{20}$; (2 marks)

(ii) $y = x \cos x$. (2 marks)

(b) Given that

$$y = \frac{x^3}{x-2}$$

show that

$$\frac{dy}{dx} = \frac{kx^2(x-3)}{(x-2)^2}$$

where k is a positive integer. (3 marks)

2 (a) Solve the equation $\cot x = 2$, giving all values of x in the interval $0 \leq x \leq 2\pi$ in radians to two decimal places. (2 marks)

(b) Show that the equation $\operatorname{cosec}^2 x = \frac{3 \cot x + 4}{2}$ can be written as

$$2 \cot^2 x - 3 \cot x - 2 = 0$$
 (2 marks)

(c) Solve the equation $\operatorname{cosec}^2 x = \frac{3 \cot x + 4}{2}$, giving all values of x in the interval $0 \leq x \leq 2\pi$ in radians to two decimal places. (4 marks)

3 The equation

$$x + (1 + 3x)^{\frac{1}{4}} = 0$$

has a single root, α .

(a) Show that α lies between -0.33 and -0.32 . (2 marks)

(b) Show that the equation $x + (1 + 3x)^{\frac{1}{4}} = 0$ can be rearranged into the form

$$x = \frac{1}{3}(x^4 - 1) \quad (2 \text{ marks})$$

(c) Use the iteration $x_{n+1} = \frac{(x_n^4 - 1)}{3}$ with $x_1 = -0.3$ to find x_4 , giving your answer to three significant figures. (3 marks)

4 The functions f and g are defined with their respective domains by

$$f(x) = x^3, \quad \text{for all real values of } x$$

$$g(x) = \frac{1}{x-3}, \quad \text{for real values of } x, x \neq 3$$

(a) State the range of f . (1 mark)

(b) (i) Find $fg(x)$. (1 mark)

(ii) Solve the equation $fg(x) = 64$. (3 marks)

(c) (i) The inverse of g is g^{-1} . Find $g^{-1}(x)$. (3 marks)

(ii) State the range of g^{-1} . (1 mark)

5 (a) (i) Given that $y = 2x^2 - 8x + 3$, find $\frac{dy}{dx}$. (1 mark)

(ii) Hence, or otherwise, find

$$\int_4^6 \frac{x-2}{2x^2-8x+3} dx$$

giving your answer in the form $k \ln 3$, where k is a rational number. (4 marks)

(b) Use the substitution $u = 3x - 1$ to find $\int x\sqrt{3x-1} dx$, giving your answer in terms of x . (4 marks)

Turn over for the next question

Turn over ►

- 6 (a) Sketch the curve with equation $y = \operatorname{cosec} x$ for $0 < x < \pi$. (2 marks)
- (b) Use the mid-ordinate rule with four strips to find an estimate for $\int_{0.1}^{0.5} \operatorname{cosec} x \, dx$, giving your answer to three significant figures. (4 marks)
- 7 (a) Describe a sequence of **two** geometrical transformations that maps the graph of $y = x^2$ onto the graph of $y = 4x^2 - 5$. (4 marks)
- (b) Sketch the graph of $y = |4x^2 - 5|$, indicating the coordinates of the point where the curve crosses the y -axis. (3 marks)
- (c) (i) Solve the equation $|4x^2 - 5| = 4$. (3 marks)
- (ii) Hence, or otherwise, solve the inequality $|4x^2 - 5| \geq 4$. (2 marks)
- 8 (a) Given that $e^{-2x} = 3$, find the exact value of x . (2 marks)
- (b) Use integration by parts to find $\int x e^{-2x} \, dx$. (4 marks)
- (c) A curve has equation $y = e^{-2x} + 6x$.
- (i) Find the exact values of the coordinates of the stationary point of the curve. (4 marks)
- (ii) Determine the nature of the stationary point. (2 marks)
- (iii) The region R is bounded by the curve $y = e^{-2x} + 6x$, the x -axis and the lines $x = 0$ and $x = 1$.
- Find the volume of the solid formed when R is rotated through 2π radians about the x -axis, giving your answer to three significant figures. (5 marks)

END OF QUESTIONS