General Certificate of Education
Advanced Level Examination
January 2010

Mathematics MPC3

Unit Pure Core 3

Friday 15 January 2010 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables
- an insert for use in Question 2 (enclosed).

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- Fill in the boxes at the top of the insert.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A curve has equation $y = e^{-4x}(x^2 + 2x - 2)$.

(a) Show that $\dfrac{dy}{dx} = 2e^{-4x}(5 - 3x - 2x^2)$. *(3 marks)*

(b) Find the exact values of the coordinates of the stationary points of the curve. *(5 marks)*

2 [**Figure 1**, printed on the insert, is provided for use in this question.]

(a) (i) Sketch the graph of $y = \sin^{-1} x$, where y is in radians. State the coordinates of the end points of the graph. *(3 marks)*

(ii) By drawing a suitable straight line on your sketch, show that the equation

$$\sin^{-1} x = \tfrac{1}{4}x + 1$$

has only one solution. *(2 marks)*

(b) The root of the equation $\sin^{-1} x = \frac{1}{4}x + 1$ is α. Show that $0.5 < \alpha < 1$. *(2 marks)*

(c) The equation $\sin^{-1} x = \frac{1}{4}x + 1$ can be rewritten as $x = \sin\left(\frac{1}{4}x + 1\right)$.

(i) Use the iteration $x_{n+1} = \sin\left(\frac{1}{4}x_n + 1\right)$ with $x_1 = 0.5$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

(ii) The sketch on **Figure 1** shows parts of the graphs of $y = \sin\left(\frac{1}{4}x + 1\right)$ and $y = x$, and the position of x_1.

On **Figure 1**, draw a cobweb or staircase diagram to show how convergence takes place, indicating the positions of x_2 and x_3 on the x-axis. *(2 marks)*

3 (a) Solve the equation

$$\operatorname{cosec} x = 3$$

giving all values of x in radians to two decimal places, in the interval $0 \leqslant x \leqslant 2\pi$.
(2 marks)

(b) By using a suitable trigonometric identity, solve the equation

$$\cot^2 x = 11 - \operatorname{cosec} x$$

giving all values of x in radians to two decimal places, in the interval $0 \leqslant x \leqslant 2\pi$.
(6 marks)

4 (a) Sketch the graph of $y = |8 - 2x|$. *(2 marks)*

(b) Solve the equation $|8 - 2x| = 4$. *(2 marks)*

(c) Solve the inequality $|8 - 2x| > 4$. *(2 marks)*

5 (a) Use the mid-ordinate rule with four strips to find an estimate for $\int_0^{12} \ln(x^2 + 5)\,dx$, giving your answer to three significant figures. *(4 marks)*

(b) A curve has equation $y = \ln(x^2 + 5)$.

(i) Show that this equation can be rewritten as $x^2 = e^y - 5$. *(1 mark)*

(ii) The region bounded by the curve, the lines $y = 5$ and $y = 10$ and the y-axis is rotated through 360° about the y-axis. Find the exact value of the volume of the solid generated.
(4 marks)

(c) The graph with equation $y = \ln(x^2 + 5)$ is stretched with scale factor 4 parallel to the x-axis, and then translated through $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ to give the graph with equation $y = f(x)$. Write down an expression for $f(x)$. *(3 marks)*

Turn over for the next question

6 The functions f and g are defined with their respective domains by

$$\mathrm{f}(x) = \mathrm{e}^{2x} - 3, \quad \text{for all real values of } x$$

$$\mathrm{g}(x) = \frac{1}{3x+4}, \quad \text{for real values of } x, \ x \neq -\frac{4}{3}$$

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 0$. *(2 marks)*

(c) (i) Find an expression for $\mathrm{gf}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{gf}(x) = 1$, giving your answer in an exact form. *(3 marks)*

7 It is given that $y = \tan 4x$.

(a) By writing $\tan 4x$ as $\dfrac{\sin 4x}{\cos 4x}$, use the quotient rule to show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = p(1 + \tan^2 4x)$, where p is a number to be determined. *(3 marks)*

(b) Show that $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = qy(1 + y^2)$, where q is a number to be determined. *(5 marks)*

8 (a) Using integration by parts, find $\displaystyle\int x \sin(2x - 1)\,\mathrm{d}x$. *(5 marks)*

(b) Use the substitution $u = 2x - 1$ to find $\displaystyle\int \frac{x^2}{2x - 1}\,\mathrm{d}x$, giving your answer in terms of x. *(6 marks)*

END OF QUESTIONS