Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8450647717

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

CAMBRIDGE International Examinations

[Turn over

1 Solve the equation $\ln(1 + 2^x) = 2$, giving your answer correct to 3 decimal places. [3]

2 Solve the inequality $|x - 4| < 2|3x + 1|$. [4]

3 **(i)** By sketching suitable graphs, show that the equation $e^{-\frac{1}{2}x} = 4 - x^2$ has one positive root and one negative root. [2]

(ii) Verify by calculation that the negative root lies between -1 and -1.5. [2]

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(iii) Use the iterative formula $x_{n+1} = -\sqrt{(4 - e^{-\frac{1}{2}x_n})}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 **(i)** Express $8\cos\theta - 15\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8\cos 2x - 15\sin 2x = 4,$$

for $0° < x < 180°$. [4]

5 The curve with equation $y = e^{-ax} \tan x$, where a is a positive constant, has only one point in the interval $0 < x < \frac{1}{2}\pi$ at which the tangent is parallel to the x-axis. Find the value of a and state the exact value of the x-coordinate of this point. [7]

6 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$. The plane p has equation $3x + y - 5z = 20$.

(i) Show that the line l lies in the plane p. [3]

(ii) A second plane is parallel to l, perpendicular to p and contains the point with position vecto
$3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [5]

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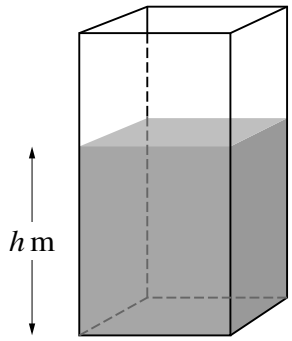

A water tank has vertical sides and a horizontal rectangular base, as shown in the diagram. The area of the base is $2\,\text{m}^2$. At time $t = 0$ the tank is empty and water begins to flow into it at a rate of $1\,\text{m}^3$ per hour. At the same time water begins to flow out from the base at a rate of $0.2\sqrt{h}\,\text{m}^3$ per hour, where h m is the depth of water in the tank at time t hours.

(i) Form a differential equation satisfied by h and t, and show that the time T hours taken for the depth of water to reach 4 m is given by

$$T = \int_0^4 \frac{10}{5 - \sqrt{h}}\,\mathrm{d}h. \qquad [3]$$

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(ii) Using the substitution $u = 5 - \sqrt{h}$, find the value of T. [6]

8 Throughout this question the use of a calculator is not permitted.

The polynomial $z^4 + 3z^2 + 6z + 10$ is denoted by $p(z)$. The complex number $-1 + i$ is denoted by u.

(i) Showing all your working, verify that u is a root of the equation $p(z) = 0$. [3]

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(ii) Find the other three roots of the equation $p(z) = 0$. [7]

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9 Let $f(x) = \dfrac{x(6-x)}{(2+x)(4+x^2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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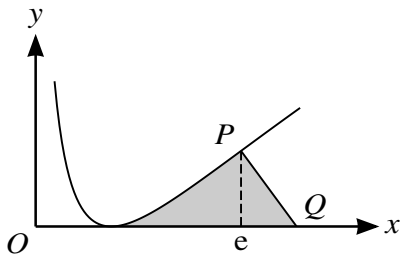

The diagram shows the curve $y = (\ln x)^2$. The x-coordinate of the point P is equal to e, and the normal to the curve at P meets the x-axis at Q.

(i) Find the x-coordinate of Q. [4]

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(ii) Show that $\int \ln x \, dx = x \ln x - x + c$, where c is a constant. [1]

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(iii) Using integration by parts, or otherwise, find the exact value of the area of the shaded region between the curve, the x-axis and the normal PQ. [5]

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