UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2012 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/32   Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2012 question papers for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level syllabuses and some Ordinary Level syllabuses.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

• When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

• The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

• Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

• Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

• For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF   Any Equivalent Form (of answer is equally acceptable)

AG   Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD   Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO   Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO   Correct Working Only – often written by a 'fortuitous' answer

ISW   Ignore Subsequent Working

MR   Misread

PA   Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS   See Other Solution (the candidate makes a better attempt at the same question)

SR   Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1   A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1   This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1   *EITHER*: Use law of the logarithm of a power or quotient and remove logarithms   M1

         Obtain a 3-term quadratic equation $x^2 - x - 3 = 0$, or equivalent   A1

         Solve 3-term quadratic obtaining 1 or 2 roots   M1

         Obtain answer 2.30 only   A1

    OR1:   Use an appropriate iterative formula, e.g. $x_{n+1} = \exp\left(\frac{1}{2}\ln(3x_n + 4)\right) - 1$ correctly at

         least once   M1

         Obtain answer 2.30   A1

         Show sufficient iterations to at least 3 d.p. to justify 2.30 to 2 d.p., or show there is a

         sign change in the interval (2.295, 2.305)   A1

         Show there is no other root   A1

    OR2:   Use calculated values to obtain at least one interval containing the root   M1

         Obtain answer 2.30   A1

         Show sufficient calculations to justify 2.30 to 3 s.f., e.g. show it lies in (2.295, 2.305)   A1

         Show there is no other root   A1   [4]

2   **(i)**   Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}bh$, form an equation an a and θ M1

         Obtain given answer   A1   [2]

  (ii)   Use the iterative formula correctly at least once   M1

         Obtain answer $\theta = 1.32$ A1

         Show sufficient iterations to 4 d.p. to justify 1.32 to 2 d.p., or show there is a sign change

         in the interval (1.315, 1.325)   A1   [3]

3 *EITHER*: State a correct unsimplified term in x or x^2 of $(1-x)^{\frac{1}{2}}$ or $(1+x)^{-\frac{1}{2}}$ B1

State correct unsimplified expansion of $(1-x)^{\frac{1}{2}}$ up to the term in x^2 B1

State correct unsimplified expansion of $(1+x)^{-\frac{1}{2}}$ up to the term in x^2 B1

Obtain sufficient terms of the product of the expansions of $(1-x)^{\frac{1}{2}}$ and $(1+x)^{-\frac{1}{2}}$ M1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1

*OR*1: State that the given expression equals $(1-x)(1-x^2)^{-\frac{1}{2}}$ and state that the first term of

the expansion of $(1-x^2)^{-\frac{1}{2}}$ is 1   B1

State correct unsimplified term in x^2 of $(1-x^2)^{-\frac{1}{2}}$ B1

State correct unsimplified expansion of $(1-x^2)^{-\frac{1}{2}}$ up to the term in x^2 B1
Obtain sufficient terms of the product of $(1-x)$ and the expansion   M1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1

*OR*2: State correct unsimplified expansion of $(1+x)^{\frac{1}{2}}$ up to the term in x^2 B1
Multiply expansion by $(1-x)$ and obtain $1-2x+2x^2$ B1
Carry out correct method to obtain one non-constant term of the expansion of

$\left(1-2x+2x^2\right)^{\frac{1}{2}}$ M1
Obtain a correct unsimplified expansion with sufficient terms   A1

Obtain final answer $1-x+\frac{1}{2}x^2$ A1   [5]

[Treat $(1+x)^{-1}(1-x^2)^{\frac{1}{2}}$ by the *EITHER* scheme.]

[Symbolic coefficients, e.g. $\binom{\frac{1}{2}}{2}$, are not sufficient for the B marks.]

4 Use trig formulae to express equation in terms of $\cos\theta$ and $\sin\theta$ M1
Use Pythagoras to obtain an equation in $\sin\theta$ M1
Obtain 3-term quadratic $2\sin^2\theta-2\sin\theta-1=0$, or equivalent   A1
Solve a 3-term quadratic and obtain a value of θ M1
Obtain answer, e.g. 201.5°   A1
Obtain second answer, e.g. 338.5°, and no others in the given interval   A1   [6]
[Ignore answers outside the given interval. Treat answers in radians (3.52, 5.91) as a misread
and deduct A1 from the marks for the angles.]

5 Separate variables correctly and attempt integration of both sides   B1
Obtain term $-e^{-y}$, or equivalent   B1

Obtain term $\frac{1}{2}e^{2x}$, or equivalent   B1

Evaluate a constant, or use limits $x=0$, $y=0$ in a solution containing terms ae^{-y} and be^{2x} M1

Obtain correct solution in any form, e.g. $-e^{-y}=\frac{1}{2}e^{2x}-\frac{3}{2}$ A1

Rearrange and obtain $y=\ln(2/(3-e^{2x}))$, or equivalent   A1   [6]

6 (i) State derivative in any correct form, e.g. $3\cos x - 12\cos^2 x \sin x$ B1 + B1

Equate derivative to zero and solve for $\sin 2x$, or $\sin x$ or $\cos x$ M1

Obtain answer $x = \dfrac{1}{12}\pi$ A1

Obtain answer $x = \dfrac{5}{12}\pi$ A1

Obtain answer $x = \dfrac{1}{2}\pi$ and no others in the given interval   A1✓ [6]

(ii) Carry out a method for determining the nature of the relevant stationary point   M1

Obtain a maximum at $\dfrac{1}{12}\pi$ correctly   A1 [2]

[Treat answers in degrees as a misread and deduct A1 from the marks for the angles.]

7 (i) *EITHER*: Multiply numerator and denominator by 1 + 3i, or equivalent   M1

Simplify numerator to –5 + 5i, or denominator to 10, or equivalent   A1

Obtain final answer $-\dfrac{1}{2} + \dfrac{1}{2}\mathrm{i}$, or equivalent   A1

OR: Obtain two equations in x and y, and solve for x or for y M1

Obtain $x = -\dfrac{1}{2}$ or $y = \dfrac{1}{2}$, or equivalent   A1

Obtain final answer $-\dfrac{1}{2} + \dfrac{1}{2}\mathrm{i}$, or equivalent   A1 [3]

(ii) Show B and C in relatively correct positions in an Argand diagram   B1

Show u in a relatively correct position   B1✓ [2]

(iii) Substitute exact arguments in the LHS arg(1 + 2i) – arg(1 – 3i) = arg u, or equivalent   M1

Obtain and use $\arg u = \dfrac{3}{4}\pi$ A1

Obtain the given result correctly   A1 [3]

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8 (i) State or imply $2u\,\mathrm{d}u = -\mathrm{d}x$, or equivalent   B1

Substitute for x and $\mathrm{d}x$ throughout   M1

Obtain integrand $\dfrac{-10u}{6-u^2+u}$, or equivalent   A1

Show correct working to justify the change in limits and obtain the given answer correctly   A1   [4]

(ii) State or imply the form of fractions $\dfrac{A}{3-u}+\dfrac{B}{2+u}$ and use a relevant method to find A or B M1

Obtain $A = 6$ and $B = -4$ A1

Integrate and obtain $-6\ln(3-u)-4\ln(2+u)$, or equivalent   A1√ + A1√

Substitute limits correctly in an integral of the form $a\ln(3-u)+b\ln(2+u)$ M1

Obtain the given answer correctly having shown sufficient working   A1   [6]

[The f.t. is on A and B.]

9 (i) Use correct product rule   M1

Obtain derivative in any correct form, e.g. $\dfrac{\ln x}{2\sqrt{x}}+\dfrac{\sqrt{x}}{x}$ A1

Carry out a complete method to form an equation of the tangent at $x = 1$ M1

Obtain answer $y = x - 1$ A1   [4]

(ii) State or imply that the indefinite integral for the volume is $\pi\displaystyle\int x(\ln x)^2\,\mathrm{d}x$ B1

Integrate by parts and reach $ax^2(\ln x)^2 + b\displaystyle\int x^2\cdot\frac{\ln x}{x}\,\mathrm{d}x$ M1*

Obtain $\dfrac{1}{2}x^2(\ln x)^2 - \displaystyle\int x\ln x\,\mathrm{d}x$, or unsimplified equivalent   A1

Attempt second integration by parts reaching $cx^2\ln x + d\displaystyle\int x^2\cdot\frac{1}{x}\,\mathrm{d}x$ M1(dep*)

Complete the integration correctly, obtaining $\dfrac{1}{2}x^2(\ln x)^2 - \dfrac{1}{2}x^2\ln x + \dfrac{1}{4}x^2$ A1

Substitute limits $x = 1$ and $x = e$, having integrated twice   M1(dep*)

Obtain answer $\dfrac{1}{4}\pi(e^2 - 1)$, or exact equivalent   A1   [7]

[If π omitted, or 2π or $\pi/2$ used, give B0 and then follow through.]

[Integration using parts $x\ln x$ and $\ln x$ is also viable.]

10 (i) *EITHER*: Substitute coordinates of a general point of l in given equation of plane m M1

Obtain equation in λ in any correct form   A1

Verify that the equation is not satisfied for any value of λ A1

*OR*1: Substitute for **r** in the vector equation of plane m and expand scalar product   M1

Obtain equation in λ in any correct form   A1

Verify that the equation is not satisfied for any value of λ A1

*OR*2: Expand scalar product of a normal to m and a direction vector of l M1

Verify scalar product is zero   A1

Verify that one point of l does not lie in the plane   A1

*OR*3: Use correct method to find perpendicular distance of a general point of l from m M1

Obtain a correct unsimplified expression in terms of λ A1

Show that the perpendicular distance is 4/3, or equivalent, for all λ A1

*OR*4: Use correct method to find the perpendicular distance of a particular point of l from m M1

Obtain answer 4/3, or equivalent   A1

Show that the perpendicular distance of a second point is also 4/3, or equivalent   A1   [3]

(ii) *EITHER*: Express general point of l in component form, e.g. $(1 + 2\lambda, 1 + \lambda, -1 + 2\lambda)$ B1

Substitute in given equation of n and solve for λ M1

Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ from $\lambda = 2$ A1

OR: State or imply plane n has vector equation $\mathbf{r}.(2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 7$, or equivalent   B1

Substitute for **r**, expand scalar product and solve for λ M1

Obtain position vector $5\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ from $\lambda = 2$ A1   [3]

(iii) Form an equation in λ by equating perpendicular distances of a general point of l from m and n M1*

Obtain a correct modular or non-modular equation in λ in any form   A1

Solve for λ and obtain a point, e.g. $7\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ from $\lambda = 3$ A1

Obtain a second point, e.g. $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ from $\lambda = 1$ A1

Use a correct method to find the distance between the two points   M1(dep*)

Obtain answer 6   A1   [6]

[The f.t. is on the components of l.]