UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

**MATHEMATICS** **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

**1 hour 45 minutes**

*9836805872*

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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UNIVERSITY *of* CAMBRIDGE
International Examinations

**1** Solve the inequality $|4x + 3| > |x|$. [4]

**2** It is given that $\ln(y + 1) - \ln y = 1 + 3\ln x$. Express $y$ in terms of $x$, in a form not involving logarithms. [4]

**3** Solve the equation $\tan 2x = 5\cot x$, for $0° < x < 180°$. [5]

**4** **(i)** Express $(\sqrt{3})\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of $R$ and $\alpha$. [3]

**(ii)** Hence show that

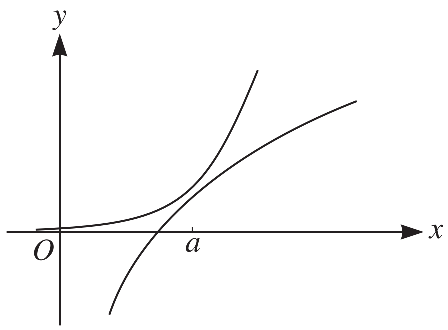
$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{\left((\sqrt{3})\cos x + \sin x\right)^2}\,dx = \frac{1}{4}\sqrt{3}.$$ [4]

**5** The polynomial $8x^3 + ax^2 + bx + 3$, where $a$ and $b$ are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x - 1)$ the remainder is 1.

**(i)** Find the values of $a$ and $b$. [5]

**(ii)** When $a$ and $b$ have these values, find the remainder when $p(x)$ is divided by $2x^2 - 1$. [3]

**6**




The diagram shows the curves $y = e^{2x-3}$ and $y = 2\ln x$. When $x = a$ the tangents to the curves are parallel.

**(i)** Show that $a$ satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [3]

**(ii)** Verify by calculation that this equation has a root between 1 and 2. [2]

**(iii)** Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate $a$ correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

**7** The complex number $z$ is defined by $z = a + ib$, where $a$ and $b$ are real. The complex conjugate of $z$ is denoted by $z^*$.

**(i)** Show that $|z|^2 = zz^*$ and that $(z - k\mathrm{i})^* = z^* + k\mathrm{i}$, where $k$ is real. [2]

In an Argand diagram a set of points representing complex numbers $z$ is defined by the equation $|z - 10\mathrm{i}| = 2|z - 4\mathrm{i}|$.

**(ii)** Show, by squaring both sides, that

$$zz^* - 2\mathrm{i}z^* + 2\mathrm{i}z - 12 = 0.$$

Hence show that $|z - 2\mathrm{i}| = 4$. [5]

**(iii)** Describe the set of points geometrically. [1]

**8** The variables $x$ and $t$ satisfy the differential equation

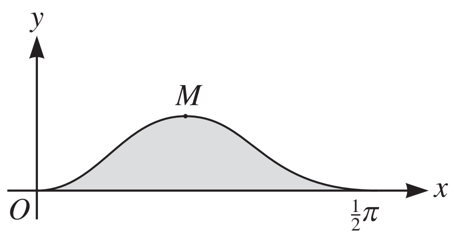
$$t\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k - x^3}{2x^2},$$

for $t > 0$, where $k$ is a constant. When $t = 1$, $x = 1$ and when $t = 4$, $x = 2$.

**(i)** Solve the differential equation, finding the value of $k$ and obtaining an expression for $x$ in terms of $t$. [9]

**(ii)** State what happens to the value of $x$ as $t$ becomes large. [1]

**9**




The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point $M$.

**(i)** Find the $x$-coordinate of $M$. [6]

**(ii)** Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the $x$-axis. [4]

**10** The line $l$ has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where $a$ is a constant. The plane $p$ has equation $x + 2y + 2z = 6$. Find the value or values of $a$ in each of the following cases.

**(i)** The line $l$ is parallel to the plane $p$. [2]

**(ii)** The line $l$ intersects the line passing through the points with position vectors $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$. [4]

**(iii)** The acute angle between the line $l$ and the plane $p$ is $\tan^{-1} 2$. [5]

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