UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/51**

Paper 5 Mechanics 2 **(M2)** **May/June 2012**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

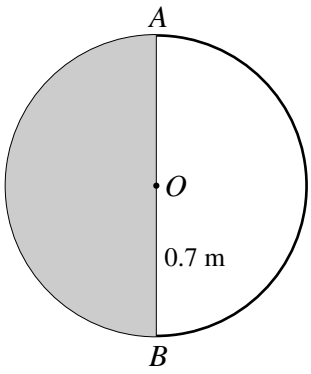
Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\text{m}\,\text{s}^{-2}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

1 The end A of a rod AB of length 1.2 m is freely pivoted at a fixed point. The rod rotates about A in a vertical plane. Calculate the angular speed of the rod at an instant when B has speed 0.6 m s^{-1}. [2]

2



The diagram shows a circular object formed from a uniform semicircular lamina of weight 11 N and a uniform semicircular arc of weight 9 N. The lamina and the arc both have centre O and radius 0.7 m and are joined at the ends of their common diameter AB.

(i) Show that the distance of the centre of mass of the object from O is 0.0371 m, correct to 3 significant figures. [3]

The object hangs in equilibrium, freely suspended at A.

(ii) Find the angle between AB and the vertical and state whether the lowest point of the object is on the lamina or on the arc. [3]

3

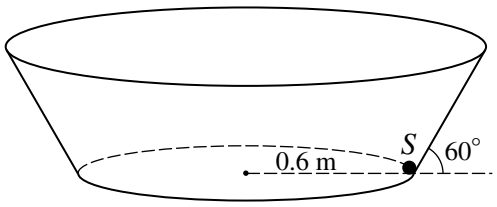


Fig. 1

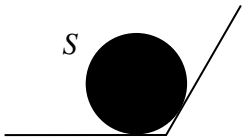


Fig. 2

A small sphere S of mass m kg is moving inside a smooth hollow bowl whose axis is vertical and whose sloping side is inclined at 60° to the horizontal. S moves with constant speed in a horizontal circle of radius 0.6 m (see Fig. 1). S is in contact with both the plane base and the sloping side of the bowl (see Fig. 2).

(i) Given that the magnitudes of the forces exerted on S by the base and sloping side of the bowl are equal, calculate the speed of S. [4]

(ii) Given instead that S is on the point of losing contact with one of the surfaces, find the angular speed of S. [3]

4 A light elastic string has natural length 2.4 m and modulus of elasticity 21 N. A particle P of mass m kg is attached to the mid-point of the string. The ends of the string are attached to fixed points A and B which are 2.4 m apart at the same horizontal level. P is projected vertically upwards with velocity 12 m s^{-1} from the mid-point of AB. In the subsequent motion P is at instantaneous rest at a point 1.6 m above AB.

(i) Find m. [4]

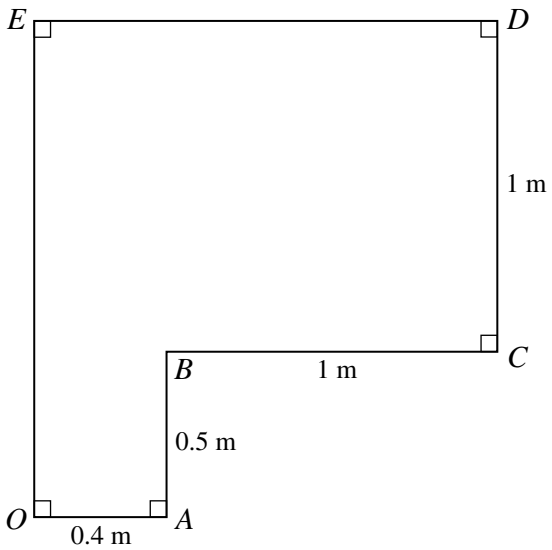
(ii) Calculate the acceleration of P when it first passes through a point 0.5 m below AB. [4]

5 A particle P of mass 0.4 kg is released from rest at the top of a smooth plane inclined at 30° to the horizontal. The motion of P down the slope is opposed by a force of magnitude $0.6x$ N, where x m is the distance P has travelled down the slope. P comes to rest before reaching the foot of the slope. Calculate

(i) the greatest speed of P during its motion, [7]

(ii) the distance travelled by P during its motion. [2]

6



The diagram shows the cross-section $OABCDE$ through the centre of mass of a uniform prism. The interior angles of the cross-section at O, A, C, D and E are all right angles. OA = 0.4 m, AB = 0.5 m and BC = CD = 1 m.

(i) Calculate the distance of the centre of mass of the prism from OE. [3]

The weight of the prism is 120 N. A force of magnitude F N acting along DE holds the prism in equilibrium when OA rests on a rough horizontal surface.

(ii) Find the set of possible values of F. [6]

[Question 7 is printed on the next page.]

[Turn over

7 A small ball B is projected with speed $15\,\text{m s}^{-1}$ at an angle of 41° above the horizontal from a point O which is 1.6 m above horizontal ground. At time t s after projection the horizontal and vertically upward displacements of B from O are x m and y m respectively.

(i) Express x and y in terms of t and hence show that the equation of the trajectory of B is

$$y = 0.869x - 0.0390x^2,$$

where the coefficients are correct to 3 significant figures. [4]

A vertical fence is 1.5 m from O and perpendicular to the plane in which B moves. B just passes over the fence and subsequently strikes the ground at the point A.

(ii) Calculate the height of the fence, and the distance from the fence to A. [5]