



# Cambridge International AS & A Level

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## MATHEMATICS

9709/21

Paper 2 Pure Mathematics 2

October/November 2021

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

### INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

### INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.



1 Find the exact value of  $\int_{-1}^2 (4e^{2x} - 2e^{-x}) dx$ . [4]

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2 (a) Sketch, on the same diagram, the graphs of  $y = 3x$  and  $y = |x - 3|$ . [2]

(b) Find the coordinates of the point where the two graphs intersect. [3]

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(c) Deduce the solution of the inequality  $3x < |x - 3|$ . [1]

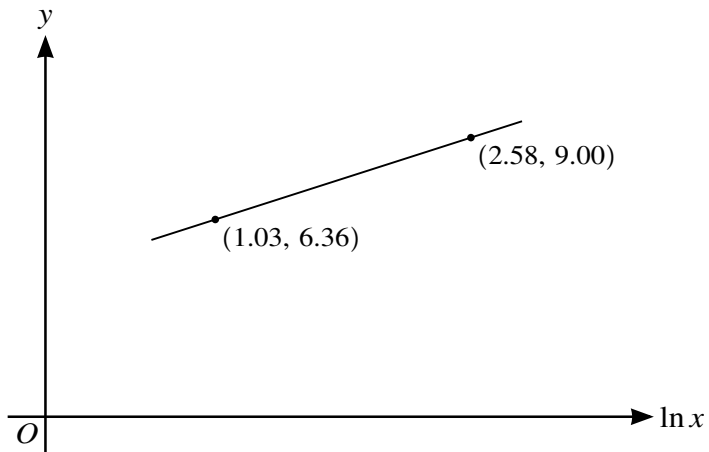
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The variables  $x$  and  $y$  satisfy the equation  $a^y = kx$ , where  $a$  and  $k$  are constants. The graph of  $y$  against  $\ln x$  is a straight line passing through the points  $(1.03, 6.36)$  and  $(2.58, 9.00)$ , as shown in the diagram.

Find the values of  $a$  and  $k$ , giving each value correct to 2 significant figures. [5]

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4 The curve with equation  $y = xe^{2x} + 5e^{-x}$  has a minimum point  $M$ .

(a) Show that the  $x$ -coordinate of  $M$  satisfies the equation  $x = \frac{1}{3} \ln 5 - \frac{1}{3} \ln(1 + 2x)$ . [5]

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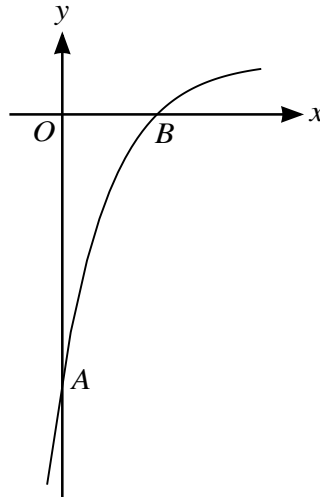
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The diagram shows the curve with parametric equations

$$x = \ln(2t + 3), \quad y = \frac{2t - 3}{2t + 3}.$$

The curve crosses the y-axis at the point  $A$  and the x-axis at the point  $B$ .

- (a) Show that  $\frac{dy}{dx} = \frac{6}{2t + 3}$ . [4]

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(b) Find the gradient of the curve at  $A$ . [2]

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(c) Find the gradient of the curve at  $B$ . [2]

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6 The polynomials  $f(x)$  and  $g(x)$  are defined by

$$f(x) = 4x^3 + ax^2 + 8x + 15 \quad \text{and} \quad g(x) = x^2 + bx + 18,$$

where  $a$  and  $b$  are constants.

(a) Given that  $(x + 3)$  is a factor of  $f(x)$ , find the value of  $a$ . [2]

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(b) Given that the remainder is 40 when  $g(x)$  is divided by  $(x - 2)$ , find the value of  $b$ . [2]

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(c) When  $a$  and  $b$  have these values, factorise  $f(x) - g(x)$  completely. [3]

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(d) Hence solve the equation  $f(\operatorname{cosec} \theta) - g(\operatorname{cosec} \theta) = 0$  for  $0 < \theta < 2\pi$ . [3]

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7 (a) By first expanding  $\cos(2\theta + \theta)$ , show that  $\cos 3\theta \equiv 4\cos^3 \theta - 3\cos \theta$ . [3]

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(b) Find the exact value of  $2\cos^3(\frac{5}{18}\pi) - \frac{3}{2}\cos(\frac{5}{18}\pi)$ . [2]

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(c) Find  $\int (12 \cos^3 x - 4 \cos^3 3x) dx$ . [4]

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**Additional Page**

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