Cambridge Pre-U

Cambridge Assessment International Education
Cambridge Pre-U Certificate

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MATHEMATICS (PRINCIPAL) **9794/01**

Paper 1 Pure Mathematics 1 **May/June 2019**

2 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This syllabus is regulated for use in England, Wales and Northern Ireland as a Cambridge International Level 3 Pre-U Certificate.

This document consists of **3** printed pages and **1** blank page.

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1 **(a)** Express $x^2 + 6x + 4$ in the form $(x + a)^2 + b$ where a and b are integers. [1]

(b) State the coordinates of the turning point of the curve $y = x^2 + 6x + 4$.

Hence sketch the curve. [2]

2 The terms of an arithmetic progression are u_1, u_2, u_3, … and the sum of the first n terms of the progression is S_n.

The values of u_1 and u_2 are 100 and 97.5 respectively.

(a) Find the largest value of n such that $u_n > 0$. [3]

(b) Hence find the largest positive value of S_n. [2]

3 In triangle ABC, $AB = 7$ cm, $BC = 12$ cm and angle $ABC = 120°$.

(a) Find the area of triangle ABC. [2]

(b) Find AC. [2]

4 **(a)** Given that $\mathrm{f}(x) = \left(\frac{1}{2}x - 1\right)$ and $\mathrm{gf}(x) = \left(\frac{1}{2}x - 1\right)^4 - 2$, write down $\mathrm{g}(x)$. [1]

(b) For $y = \left(\frac{1}{2}x - 1\right)^4 - 2$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Hence find the tangent to the curve $y = \left(\frac{1}{2}x - 1\right)^4 - 2$ at the point $(4, -1)$. [3]

5 Show that the exact value of $\int_0^1 x\mathrm{e}^{-x}\,\mathrm{d}x$ is $\frac{\mathrm{e} - 2}{\mathrm{e}}$. [5]

6 It is given that 2 and $3 + \mathrm{i}$ are roots of the equation $z^3 + az^2 + bz - 20 = 0$ where a and b are real numbers.

(a) Write down the third root. [1]

(b) Find a and b. [4]

7 Let $\mathrm{f}(x) = \frac{1}{x}$. Use differentiation from first principles to find an expression for $\mathrm{f}'(x)$. [5]

8 Solve the equation $\log_3(x^2 - 3x - 10) = \frac{1}{2}\log_3 9 + \log_3(x + 2)$. [5]

9 In the binomial expansion of $(1 + 2x)^p$, the coefficient of x is twice the coefficient of x^3. The coefficient of x^2 is negative.

(a) Find p. [7]

(b) State the range of values of x for which the expansion of $(1 + 2x)^p$ is valid. [1]

10 Two straight lines have equations

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2} \quad \text{and} \quad \frac{x}{4} = \frac{y-1}{-1} = \frac{z-1.5}{1}.$$

(a) Show that the lines intersect and find the coordinates of their point of intersection. [6]

(b) Find the acute angle between the lines. [3]

11 (a) Prove that $\tan 15° = 2 - \sqrt{3}$. [5]

(b) Solve the equation $\cos(x + \frac{1}{6}\pi)\cos(x - \frac{1}{6}\pi) = \cos 2x$ for $0 \leqslant x \leqslant \pi$. [8]

12 Biologists decided to stock a lake with salmon. The lake contained no salmon before it was stocked. The biologists suggested that the rate at which the population of the salmon stock would grow could be modelled by the differential equation

$$12\,000\frac{\mathrm{d}P}{\mathrm{d}t} = kP(12\,000 - P)$$

where P is the number of salmon in the lake, k is a constant to be determined and t is time measured in years.

The biologists initially stocked the lake with 500 salmon and estimated that the number of salmon had reached 2000 after 3 years.

(a) Solve the differential equation to show that

$$P = \frac{12\,000}{1 + 23\mathrm{e}^{-kt}}$$

where $k = 0.509$ correct to 3 significant figures. [12]

(b) State the maximum number of salmon which the lake is expected to support and justify your answer. [2]

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