June 2019 A Level Mathematics

Bronze, Silver, Gold Graduated Difficulty Papers

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Key Information

We have created these papers by taking the Summer 2019 A Level Maths exam series, putting the questions from the Pure and Applied papers in the order of difficulty that the students found when they sat the papers. These have been used to created three levels of paper: Bronze, Silver and Gold. Each contains a mix of Pure and Applied Questions. Bronze can be used to build confidence and Gold can be used to extend your more able students that need more concentrated experience of harder questions.

Since the Gold paper contains the marks the students found hardest, and took the longest to answer, we have kept it below 100 marks. Likewise, as the Foundation paper contains the questions that students find easiest and quickest, to answer, it contains slightly more than 100 marks.

On the next page you will find the performance data for these questions, at each grade, in case you need to replace questions to ensure the students are attempting questions that they have covered, to that point in class.

* The question level performance data is there to give an indication only of how students performed, on each question, in the context of sitting the entire exam paper and is not an indication of how students may perform sitting a question in isolation. The performance data of this series does not necessarily represent a normal series due to small number of entries.

Quick Links to the Data, Papers and Mark Schemes

We have included quick links to make this document easier to navigate. If you are using the MS Word version of this document, you will need to hold the Ctrl key as you click the link.

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Question Performance Data

The tables, on the following pages, contain the data of how the students performed on those question, when they sat the live exam series. In an overall live series, the grade boundaries will be in between the average performance on students at each grade. For example, in Summer 2019 the average marks that grades B and C candidates achieved, across all three, papers were 148.36/300 and 117.89/300 respectively and the Grade B boundary was 134. You can find historical data on grade boundaries here.

Be aware that rearranging the questions, from their order in the Summer 2019 live exam series, does create more uncertainty over the precise location of the grade boundaries. You will need to make a judgement of where you put the exact grade boundaries, especially if you decide to replace questions. Question level data on previous exam series can be found here.

If you are using these for assessment, a single paper gives a less secure indication of performance, and you may want to have students sit the Bronze and Silver together or the Silver and Gold together.

For reference, the average performance at each grade and the grade boundaries, for the Summer 2019 series, was as follows.

2019 A Level Average Performance at each Grade and Grade Boundaries (all figures are out of 300)											
Average A* Student Score	Grade A* Boundary	Average A Student Score	Grade A Boundary	Average B Student Score	Grade B Boundary	Average C Student Score	Grade C Boundary	Average D Student Score	Grade D Boundary	Average E Student Score	Grade E Boundary
243.30	217	188.82	165	148.36	134	117.89	103	87.76	73	58.86	43

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Bronze: Average Performance on Each Question by Each Grade of Student

Question	Max score	A*	A	B	C	D	E	U
1	3	2.90	2.77	2.67	2.56	2.36	2.07	1.53
2	3	2.94	2.86	2.74	2.52	2.17	1.61	0.89
3	8	7.46	6.79	6.13	5.61	5.13	4.55	3.22
4	6	5.62	5.10	4.47	3.90	3.08	1.93	0.67
5	6	5.75	5.24	4.47	3.49	2.31	1.14	0.48
6	9	8.32	7.50	6.38	5.11	3.63	2.13	0.78
7	10	8.34	7.12	6.40	5.82	5.11	4.35	2.87
8	7	6.34	5.37	4.47	3.55	2.62	1.76	0.72
9	9	7.74	6.46	5.35	4.49	3.52	2.61	1.36
10	10	8.11	6.72	5.78	4.99	4.09	2.93	1.42
11	11	8.11	7.25	6.50	5.65	4.76	3.79	2.39
12	5	4.37	3.50	2.83	2.37	1.79	1.11	0.45
13	7	6.12	4.60	3.61	3.09	2.64	2.27	1.59
14	8	6.58	5.49	4.71	3.92	2.88	1.41	0.42
15	12	10.77	8.87	6.70	5.00	3.43	1.96	0.71
Total	**114**	99.47	85.64	73.21	62.07	49.52	35.62	19.50

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Silver: Average Performance on Each Question by Each Grade of Student

Question	Max score	A*	A	B	C	D	E	U
1	6	3.61	2.52	1.96	1.72	1.38	1.08	0.67
2	5	10.83	9.34	7.01	4.66	2.76	1.10	0.24
3	9	6.06	5.04	3.98	2.93	2.06	0.99	0.42
4	7	7.04	6.00	5.23	4.19	2.80	1.30	0.54
5	11	4.77	4.04	3.30	2.69	2.08	1.80	1.41
6	10	9.53	7.94	5.62	3.64	1.82	0.80	0.33
7	5	9.10	6.96	5.02	3.31	1.88	0.88	0.32
8	11	3.11	2.38	1.95	1.79	1.51	1.37	0.90
9	10	2.49	1.65	1.20	0.98	0.64	0.45	0.10
10	3	4.51	3.44	2.82	2.22	1.93	1.35	0.57
11	7	5.73	4.09	3.08	2.34	1.68	1.07	0.51
12	13	4.74	3.66	2.96	2.56	2.01	1.36	0.74
Total	**97**	**79.77**	**61.67**	**47.29**	**36.21**	**25.64**	**16.18**	**7.15**

Gold: Average Performance on Each Question by Each Grade of Student

Question	Max score	A*	A	B	C	D	E	U
1	**2**	6.73	4.44	2.99	2.27	1.65	0.99	0.46
2	**12**	7.39	5.79	4.31	3.00	1.74	0.82	0.25
3	**5**	9.80	7.16	4.93	3.44	2.19	1.16	0.40
4	**2**	2.55	1.70	1.30	1.06	0.80	0.55	0.29
5	**8**	4.39	2.73	1.90	1.39	0.82	0.41	0.15
6	**14**	11.87	7.44	4.34	2.51	1.27	0.55	0.16
7	**14**	5.03	3.08	2.10	1.52	0.93	0.50	0.14
8	**10**	3.94	2.33	1.78	1.43	1.16	0.84	0.43
9	**10**	3.57	2.20	1.53	1.14	0.81	0.49	0.18
10	**5**	2.04	0.92	0.59	0.50	0.42	0.31	0.17
11	**4**	6.75	3.72	2.09	1.35	0.81	0.44	0.19
Total	**91**	**64.06**	**41.51**	**27.86**	**19.61**	**12.60**	**7.06**	**2.82**

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.
2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.
3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sfsignificant figures
- $*$ The answer is printed on the paper
- $\square$ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.

   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.
6. Ignore wrong working or incorrect statements following a correct answer.
7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where} |pq| = |c|$, leading to $x = \ldots$

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where} |pq| = |c| \text{ and } |mn| = |a|$, leading to $x = \ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Extend

Paper Reference **9MA0**

Mathematics

Advanced Paper

2019 Bronze

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks
__/114

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Index

1. $$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that $(x + 3)$ is a factor of $f(x)$, find the value of the constant a.

(3)

(Total for Question 1 is 3 marks)

2.

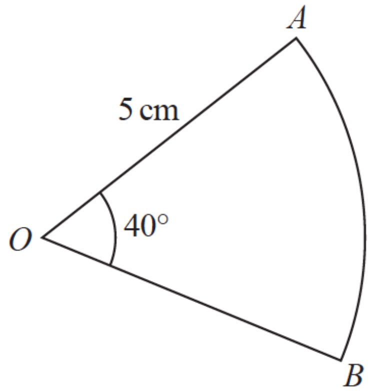

Figure 1

Figure 1 shows a sector AOB of a circle with centre O, radius 5 cm and angle $AOB = 40°$

The attempt of a student to find the area of the sector is shown below.

$$\text{Area of sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2} \times 5^2 \times 40$$
$$= 500 \text{ cm}^2$$

(*a*) Explain the error made by this student.

(1)

(*b*) Write out a correct solution.

(2)

(Total for Question 2 is 3 marks)

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3. Three Bags, A, B and C, each contain 1 red marble and some green marbles.

Bag A contains 1 red marble and 9 green marbles only
Bag B contains 1 red marble and 4 green marbles only
Bag C contains 1 red marble and 2 green marbles only

Sasha selects at random one marble from Bag A.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from Bag B.
If he selects a red marble, he stops selecting.
If the marble is green, he continues by selecting at random one marble from Bag C.

(*a*) Draw a tree diagram to represent this information.

(2)

(*b*) Find the probability that Sasha selects 3 green marbles.

(2)

(*c*) Find the probability that Sasha selects at least 1 marble of each colour.

(2)

(*d*) Given that Sasha selects a red marble, find the probability that he selects it from Bag B.

(2)

(Total for Question 3 is 8 marks)

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4. *In this question position vectors are given relative to a fixed origin O*]

At time t seconds, where $t \geq 0$, a particle, P, moves so that its velocity $\mathbf{v}$ m s^{-1} is given by

$$\mathbf{v} = 6t\mathbf{i} - 5t^{\frac{3}{2}}\mathbf{j}$$

When $t = 0$, the position vector of P is $(-20\mathbf{i} + 20\mathbf{j})$ m.

(*a*) Find the acceleration of P when $t = 4$

(3)

(*b*) Find the position vector of P when $t = 4$

(3)

(Total for Question 4 is 6 marks)

5.

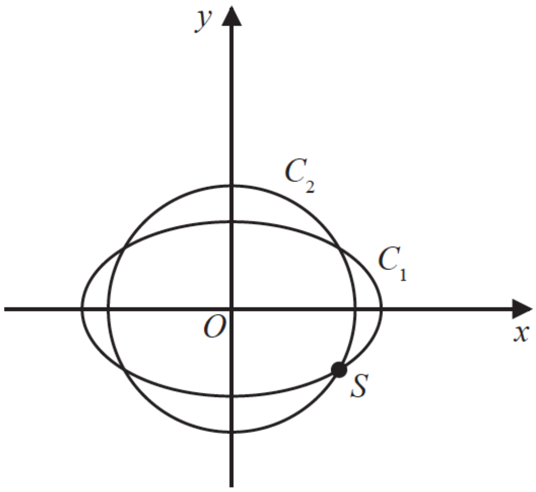

Figure 2

The curve C_1 with parametric equations

$$x = 10\cos t, \qquad y = 4\sqrt{2}\sin t, \qquad 0 \leqslant t < 2\pi$$

meets the circle C_2 with equation

$$x^2 + y^2 = 66$$

at four distinct points as shown in Figure 2.

Given that one of these points, S, lies in the 4th quadrant, find the Cartesian coordinates of S.

(6)

(Total for Question 5 is 6 marks)

6. Barbara is investigating the relationship between average income (GDP per capita), x US dollars, and average annual carbon dioxide (CO_2) emissions, y tonnes, for different countries.

She takes a random sample of 24 countries and finds the product moment correlation coefficient between average annual CO_2, emissions and average income to be 0.446

(*a*) Stating your hypotheses clearly, test, at the 5% level of significance, whether or not the product moment correlation coefficient for all countries is greater than zero.

(3)

Barbara believes that a non-linear model would be a better fit to the data.
She codes the data using the coding $m = \log_{10} x$ and $c = \log_{10} y$ and obtains the model $c = -1.82 + 0.89m$

The product moment correlation coefficient between c and m is found to be 0.882

(*b*) Explain how this value supports Barbara's belief.

(1)

(*c*) Show that the relationship between y and x can be written in the form $y = ax^n$ where a and n are constants to be found.

(5)

(Total for Question 6 is 9 marks)

7.

$$\mathrm{f}(x) = 2x^2 + 4x + 9 \qquad x \in \mathbb{R}$$

(*a*) Write f (x) in the form $a(x + b)^2 + c$, where a, b and c are integers to be found.

(3)

(*b*) Sketch the curve with equation $y = \mathrm{f}(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point.

(3)

(*c*) (i) Describe fully the transformation that maps the curve with equation $y = \mathrm{f}(x)$ onto the curve with equation $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = 2(x - 2)^2 + 4x - 3 \qquad x \in \mathbb{R}$$

(ii) Find the range of the function

$$\mathrm{h}(x) = \frac{21}{2x^2 + 4x + 9} \qquad x \in \mathbb{R}$$

(4)

(Total for Question 7 is 10 marks)

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8. In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

(Total for Question 8 is 7 marks)

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9. Magali is studying the mean total cloud cover, in oktas, for Leuchars in 1987 using data from the large data set. The daily mean total cloud cover for all 184 days from the large data set is summarised in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	1	4	7	10	30	52	52	28

One of the 184 days is selected at random.

(*a*) Find the probability that it has a daily mean total cloud cover of 6 or greater.

(1)

Magali is investigating whether the daily mean total cloud cover can be modelled using a binomial distribution.

She uses the random variable X to denote the daily mean total cloud cover and believes that $X \sim \mathrm{B}(8, 0.76)$

Using Magali's model,

(*b*) (i) find $\mathrm{P}(X \geq 6)$

(2)

(ii) find, to 1 decimal place, the expected number of days in a sample of 184 days with a daily mean total cloud cover of 7

(2)

(*c*) Explain whether or not your answers to part (*b*) support the use of Magali's model.

(1)

There were 28 days that had a daily mean total cloud cover of 8
For these 28 days the daily mean total cloud cover for the **following** day is shown in the table below.

Daily mean total cloud cover (oktas)	0	1	2	3	4	5	6	7	8
Frequency (number of days)	0	0	1	1	2	1	5	9	9

(*d*) Find the proportion of these days when the daily mean total cloud cover was 6 or greater.

(1)

(*e*) Comment on Magali's model in light of your answer to part (*d*).

(2)

(Total for Question 9 is 9 marks)

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10.

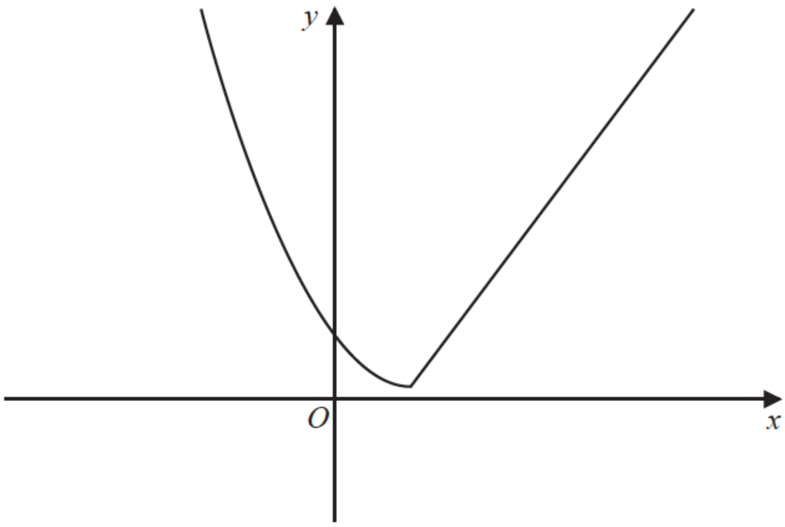

Figure 4

Figure 4 shows a sketch of the graph of $y = g(x)$, where

$$g(x) = \begin{cases} (x - 2)^2 + 1 & x \leqslant 2 \\ 4x - 7 & x > 2 \end{cases}$$

(a) Find the value of gg(0).

(2)

(b) Find all values of x for which

$$g(x) > 28$$

(4)

The function h is defined by

$$h(x) = (x - 2)^2 + 1 \qquad x \leqslant 2$$

(c) Explain why h has an inverse but g does not.

(1)

(d) Solve the equation

$$h^{-1}(x) = -\frac{1}{2}$$

(3)

(Total for Question 10 is 10 marks)

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11.

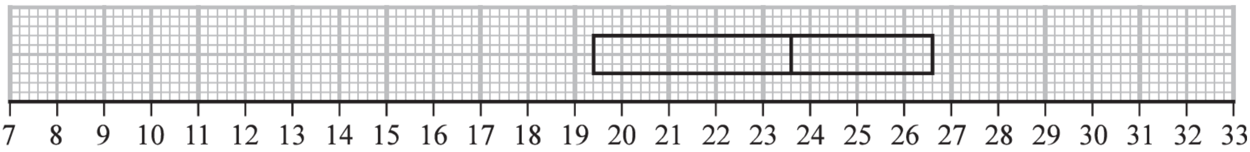

Figure 1

The partially completed box plot in Figure 1 shows the distribution of daily mean air temperatures using the data from the large data set for Beijing in 2015

An outlier is defined as a value
more than $1.5 \times IQR$ below Q_1 or
more than 1.5 x IQR above Q_3

The three lowest air temperatures in the data set are 7.6 °C, 8.1 °C and 9.1 °C
The highest air temperature in the data set is 32.5 °C

(*a*) Complete the box plot in Figure 1 showing clearly any outliers

(4)

(*b*) Using your knowledge of the large data set, suggest from which month the two outliers are likely to have come.

(1)

Using the data from the large data set, Simon produced the following summary statistics for the daily mean air temperature, x °C, for Beijing in 2015

$$n = 184 \qquad \sum x = 4153.6 \, S_{xx} = 4952.906$$

(*c*) Show that, to 3 significant figures, the standard deviation is 5.19 °C

(1)

Simon decides to model the air temperatures with the random variable

$$T \sim \mathrm{N}(22.6, 5.19^2)$$

(*d*) Using Simon's model, calculate the 10th to 90th interpercentile range.

(3)

Simon wants to model another variable from the large data set for Beijing using a normal distribution.

(*e*) State two variables from the large data set for Beijing that are **not** suitable to be modelled by a normal distribution. Give a reason for each answer.

(2)

(Total for Question 11 is 11 marks)

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12.

$$y = \frac{5x^2 + 10x}{(x+1)^2} \qquad x \neq -1$$

(*a*) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{A}{(x+1)^n}$ where A and n are constants to be found.

(4)

(*b*) Hence deduce the range of values for x for which $\dfrac{\mathrm{d}y}{\mathrm{d}x} < 0$

(1)

(Total for Question 12 is 5 marks)

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13. A small factory makes bars of soap.

On any day, the total cost to the factory, £y, of making x bars of soap is modelled to be the sum of two separate elements:

- a fixed cost,
- a cost that is proportional to the number of bars of soap that are made that day.

(*a*) Write down a general equation linking y with x, for this model.

(1)

The bars of soap are sold for £2 each.

On a day when 800 bars of soap are made and sold, the factory makes a profit of £500.

On a day when 300 bars of soap are made and sold, the factory makes a loss of £80.

Using the above information,

(*b*) show that $y = 0.84x + 428$

(3)

(*c*) With reference to the model, interpret the significance of the value 0.84 in the equation.

(1)

Assuming that each bar of soap is sold on the day it is made,

(*d*) find the least number of bars of soap that must be made on any given day for the factory to make a profit that day.

(2)

(Total for Question 13 is 7 marks)

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14. (*a*) Solve, for $-180° \leq \theta \leq 180°$, the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.

[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(6)

(*b*) Deduce the smallest positive solution to the equation

$$5 \sin (2x - 50°) = 9 \tan (x - 25°)$$

(2)

(Total for Question 14 is 8 marks)

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15.

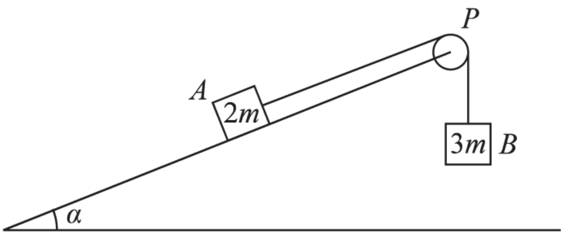

Figure 1

Two blocks, A and B, of masses $2m$ and $3m$ respectively, are attached to the ends of a light string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined at angle a to the horizontal ground, where $\tan \alpha = \frac{5}{12}$

The string passes over a small smooth pulley, P, fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.
Block B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{2}{3}$

The blocks are released from rest with the string taut and A moves up the plane.

The tension in the string immediately after the blocks are released is T.

The blocks are modelled as particles and the string is modelled as being inextensible.

(a) Show that $T = \frac{12mg}{5}$

(8)

After B reaches the ground, A continues to move up the plane until it comes to rest before reaching P.

(b) Determine whether A will remain at rest, carefully justifying your answer.

(2)

(c) Suggest two refinements to the model that would make it more realistic.

(2)

(Total for Question 15 is 12 marks)

TOTAL FOR PAPER IS 114 MARKS

Bronze Mark Scheme

Question	Scheme	Marks	AOs
1	Attempts $f(-3)=3\times(-3)^3+2a\times(-3)^2-4\times-3+5a=0$	M1	3.1a
	Solves linear equation $23a=69\Rightarrow a=...$	M1	1.1b
	$a=3$ cso	A1	1.1b
		(3)	
			(3 marks)

M1: Chooses a suitable method to set up a correct equation in a which may be unsimplified.
This is mainly applying $f(-3)=0$ leading to a correct equation in a.
Missing brackets may be recovered.
Other methods may be seen but they are more demanding
If division is attempted must produce **a correct equation** in a similar way to the $f(-3)=0$ method

	$3x^2$	$(2a-9)x$	$\frac{5a}{3}$
x	$3x^3$	$(2a-9)x^2$	$\frac{5a}{3}x$
3	$9x^2$	$(6a-27)x$	$5a$

$$
\begin{array}{r l}
 & 3x^2+(2a-9)x+23-6a \\
x+3\,\big) & 3x^3+2ax^2-4x+5a \\
 & \underline{3x^3+9x^2} \\
 & (2a-9)x^2-4x \\
 & \underline{(2a-9)x^2+(6a-27)x} \\
 & (23-6a)x+5a \\
 & (23-6a)x+69-18a
\end{array}
$$

So accept $5a=69-18a$ or equivalent, where it implies that the remainder will be 0
You may also see variations on the table below. In this method the terms in x are equated to -4

$$6a - 27 + \frac{5a}{3} = -4$$

M1: This is scored for an attempt at solving a linear equation in a.
For the main scheme it is dependent upon having attempted $\mathrm{f}(\pm 3) = 0$. Allow for a linear equation in a leading to $a = \ldots$. Don't be too concerned with the mechanics of this.

Via division accept $x+3\overline{\big)3x^3 + 2ax^2 - 4x + 5a}$ with quotient $3x^2\ldots$ followed by a remainder in a set $= 0 \Rightarrow a = \ldots$

or two terms in a are equated so that the remainder = 0
FYI the correct remainder via division is $23a + 12 - 81$ oe

A1: $a = 3$ cso

An answer of 3 with no incorrect working can be awarded 3 marks

Question	Scheme	Marks	AOs
2 (a)	Allow explanations such as • student should have worked in radians • they did not convert degrees to radians • 40 should be in radians • θ should be in radians • angle (or θ) should be $\frac{40\pi}{180}$ or $\frac{2\pi}{9}$ • correct formula is $\pi r^2\left(\frac{\theta}{360}\right)$ {where θ is in degrees} • correct formula is $\pi r^2\left(\frac{40}{360}\right)$	B1	2.3
		(1)	
(b) Way 1	{Area of sector = } $\frac{1}{2}(5^2)\left(\frac{2\pi}{9}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
(b) Way 2	{Area of sector = } $\pi(5^2)\left(\frac{40}{360}\right)$	M1	1.1b
	$= \frac{25}{9}\pi$ {cm^2} or awrt 8.73 {cm^2}	A1	1.1b
		(2)	
		(3 marks)	

Notes for Question 3	
(a)	
B1:	Explains that the formula use is only valid when angle AOB is applied in radians. See scheme for examples of suitable explanations.
(b)	**Way 1**
M1:	Correct application of the sector formula using a correct value for θ in radians

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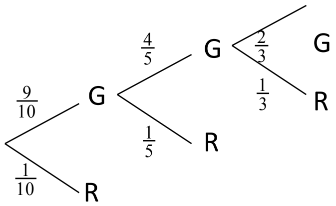
Note:	Allow exact equivalents for θ e.g. $\theta = \dfrac{40\pi}{180}$ or θ in the range [0.68, 0.71]
A1*:	Accept $\dfrac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units
(b)	**Way 2**
M1:	Correct application of the sector formula in degrees
A1:	Accept $\dfrac{25}{9}\pi$ or awrt 8.73 **Note:** Ignore the units.
Note:	Allow exact equivalents such as $\dfrac{50}{18}\pi$
Note:	Allow M1 A1 for $500\left(\dfrac{\pi}{180}\right) = \dfrac{25}{9}\pi\ \{\text{cm}^2\}$ or awrt 8.73 $\{\text{cm}^2\}$

Question	Scheme	Marks	AOs
3(a)		B1 dB1	1.1b 1.1b
		(2)	
(b)	$\dfrac{9}{10}\times\dfrac{4}{5}\times\dfrac{2}{3}$	M1	1.1b
	$=\dfrac{12}{25}\ (=0.48)$	A1	1.1b
		(2)	
(c)	$\dfrac{9}{10}\times\dfrac{1}{5}+\dfrac{9}{10}\times\dfrac{4}{5}\times\dfrac{1}{3}$ or $1-\left(\dfrac{1}{10}+\dfrac{9}{10}\times\dfrac{4}{5}\times\dfrac{2}{3}\right)$	M1	3.1b
	$=\dfrac{21}{50}$ (= 0.42)	A1	1.1b
		(2)	
(d)	[P(Red from *B* | Red selected)] $= \dfrac{\frac{9}{10}\times\frac{1}{5}}{\frac{1}{10}+\frac{9}{10}\times\frac{1}{5}+\frac{9}{10}\times\frac{4}{5}\times\frac{1}{3}}\left[=\dfrac{\frac{9}{50}}{\frac{13}{25}}\right]$	M1	3.1b
	$=\dfrac{9}{26}$	A1	1.1b
		(2)	
			(8 marks)
	Notes		
	Allow decimals or percentages throughout this question.		
(a)	**B1:** for correct shape (3 pairs) and at least **one** label on **at least two** pairs		

	G(reen) and R(ed) allow G and G' **or** R and R' as labels, etc. condone 'extra' pairs if they are labelled with a probability of 0 **dB1:** (dep on previous B1) all correct i.e. for all 6 correct probabilities on the correct branches with at least one label on **each** pair
(b)	**M1:** Multiplication of 3 correct probabilities (allow ft from their tree diagram) **A1:** $\frac{12}{25}$ oe
(c)	**M1:** Either addition of only two correct products (product of two probs + product of three probs) which may ft from their tree diagram or for $1-('\tfrac{1}{10}'+'(b)')$ **A1:** $\frac{21}{50}$ oe
(d)	**M1:** Correct ratio of probabilities **or** correct ft ratio of probabilities e.g. $\frac{'\frac{9}{10}'\times'\frac{1}{5}'}{1-'(b)'}$ or $\frac{'\frac{9}{10}'\times'\frac{1}{5}'}{'\frac{1}{10}'+'(c)'}$ with num < den **A1:** $\frac{9}{26}$ (allow awrt 0.346)

Question	Scheme	Marks	AO
4(a)	Differentiate **v**	M1	1.1a
	$(\mathbf{a}=)6\mathbf{i}-\frac{15}{2}t^{\frac{1}{2}}\mathbf{j}$	A1	1.1b
	$=6\mathbf{i}-15\mathbf{j}\ (\text{m s}^{-2})$	A1	1.1b
		(3)	
4(b)	Integrate **v**	M1	1.1a
	$(\mathbf{r}=)(\mathbf{r}_0)+3t^2\mathbf{i}-2t^{\frac{5}{2}}\mathbf{j}$	A1	1.1b
	$=(-20\mathbf{i}+20\mathbf{j})+(48\mathbf{i}-64\mathbf{j})=28\mathbf{i}-44\mathbf{j}\ (\text{m})$	A1	2.2a

		(3)	
		(6)	

Marks		Notes
		N.B. Accept column vectors throughout and condone missing brackets in working but they must be there in final answers
4a	M1	Use of $\mathbf{a} = \frac{d\mathbf{v}}{dt}$ with attempt to differentiate (both powers decreasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct differentiation in any form
	A1	Correct and simplified. Ignore subsequent working (ISW) if they go on and find the magnitude.
4b	M1	Use of $\mathbf{r} = \int \mathbf{v}\,dt$ with attempt to integrate (both powers increasing by 1) M0 if **i**'s and **j**'s omitted and they don't recover
	A1	Correct integration in any form. Condone $\mathbf{r}_0$ not present
	A1	Correct and simplified.

Question	Scheme		Marks	AOs
5	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ 0 \le t < 2\pi;\ C_2: x^2 + y^2 = 66$			
Way 1	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$		M1	3.1a
	$100(1-\sin^2 t) + 32\sin^2 t = 66$	$100\cos^2 t + 32(1-\cos^2 t) = 66$	M1 A1	2.1 1.1b
	$100 - 68\sin^2 t = 66 \Rightarrow \sin^2 t = \frac{1}{2}$ $\Rightarrow \sin t = \ldots$	$68\cos^2 t + 32 = 66 \Rightarrow \cos^2 t = \frac{1}{2}$ $\Rightarrow \cos t = \ldots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 2	$\{\cos^2 t + \sin^2 t = 1 \Rightarrow\}\ \left(\frac{x}{10}\right)^2 + \left(\frac{y}{4\sqrt{2}}\right)^2 = 1\ \{\Rightarrow 32x^2 + 100y^2 = 3200\}$		M1	3.1a
	$\frac{x^2}{100} + \frac{66-x^2}{32} = 1$	$\frac{66-y^2}{100} + \frac{y^2}{32} = 1$	M1 A1	2.1 1.1b

	$32x^2+6600-100x^2=3200$ $x^2=50 \Rightarrow x=...$	$2112-32y^2+100y^2=3200$ $y^2=16 \Rightarrow y=...$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the corresponding x-coordinate or y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2}, -4)$ or $x=5\sqrt{2}, y=-4$ or $S=(\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
Way 3	$\{C_2 : x^2+y^2=66 \Rightarrow\}$ $x=\sqrt{66}\cos\alpha,\ y=\sqrt{66}\sin\alpha$ $\{C_1=C_2 \Rightarrow\}$ $10\cos t=\sqrt{66}\cos\alpha,\ \ 4\sqrt{2}\sin t=\sqrt{66}\sin\alpha$ $\{\cos^2\alpha+\sin^2\alpha=1\Rightarrow\}$ $\left(\frac{10\cos t}{\sqrt{66}}\right)^2+\left(\frac{4\sqrt{2}\sin t}{\sqrt{66}}\right)^2=1$		M1	3.1a
	then continue with applying the mark scheme for Way 1			
Way 4	$(10\cos t)^2+(4\sqrt{2}\sin t)^2=66$		M1	3.1a
	$100\left(\frac{1+\cos 2t}{2}\right)+32\left(\frac{1-\cos 2t}{2}\right)=66$		M1 A1	2.1 1.1b
	$50+50\cos 2t+16-16\cos 2t=66 \Rightarrow 34\cos 2t+66=66$ $\Rightarrow \cos 2t=...$		dM1	1.1b
	Substitutes their solution back into the original equation(s) to get the value of the x-coordinate and value of the y-coordinate. **Note:** These may not be in the correct quadrant		M1	1.1b
	$S=(5\sqrt{2}, -4)$ or $x=5\sqrt{2}, y=-4$ or $S=(\text{awrt } 7.07, -4)$		A1	3.2a
			(6)	
	Note: Give final A0 for writing $x=5\sqrt{2}, y=-4$ followed by $S=(-4, 5\sqrt{2})$			
				(6 marks)

Notes for Question 5

	Way 1
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 1: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1:	Uses the identity $\sin^2 t+\cos^2 t\equiv 1$ to achieve an equation in $\sin^2 t$ only or $\cos^2 t$ only
A1:	A correct equation in $\sin^2 t$ only or $\cos^2 t$ only
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $\sin t=...$ where $-1\le \sin t\le 1$ or $\cos t=...$ where $-1\le\cos t\le 1$ Condone 3rd M1 for $\sin^2 t=\frac{1}{2}\Rightarrow \sin t=\frac{1}{4}$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S=(5\sqrt{2}, -4)$ or $S=(\text{awrt } 7.07, -4)$
	Way 2
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 2: A complete process of using $\cos^2 t+\sin^2 t\equiv 1$ to convert the parametric equation for C_1 into a Cartesian equation for C_1
M1:	Complete valid attempt to write an equation in terms of x only or y only not involving

	trigonometry
A1:	A correct equation in x only or y only not involving trigonometry
dM1: **Note:**	**dependent on both the previous M marks** Rearranges to make $x = \dots$ or $y = \dots$ their x^2 or their y^2 must be >0 for this mark
M1: **Note:**	See scheme their x^2 and their y^2 must be >0 for this mark
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$
	Way 3
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 3: A complete process of writing C_2 in parametric form, combining the parametric equations of C_1 and C_2 and applying $\cos^2\alpha + \sin^2\alpha \equiv 1$ to give an equation in one variable (i.e. t) only.
	then continue with applying the mark scheme for Way 1
	Way 4
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 4: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. t) only.
M1: **Note:**	Uses the identities $\cos 2t \equiv 2\cos^2 t - 1$ and $\cos 2t \equiv 1 - 2\sin^2 t$ to achieve an equation in $\cos 2t$ only At least one of $\cos 2t \equiv 2\cos^2 t - 1$ or $\cos 2t \equiv 1 - 2\sin^2 t$ must be correct for this mark.
A1:	A correct equation in $\cos 2t$ only
dM1:	**dependent on both the previous M marks** Rearranges to make $\cos 2t = \dots$ where $-1 \le \cos 2t \le 1$
M1:	See scheme
A1:	Selects the correct coordinates for S Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question	Scheme	Marks	AOs
5	$C_1: x = 10\cos t,\ y = 4\sqrt{2}\sin t,\ 0 \le t < 2\pi;\ C_2: x^2 + y^2 = 66$		
Way 5	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66$	M1	3.1a
	$(10\cos t)^2 + (4\sqrt{2}\sin t)^2 = 66(\sin^2 t + \cos^2 t)$	M1	2.1
		A1	1.1b
	$100\cos^2 t + 32\sin^2 t = 66\sin^2 t + 66\cos^2 t \Rightarrow 34\cos^2 t = 34\sin^2 t$ $\Rightarrow \tan t = \dots$	dM1	1.1b
	Substitutes their solution back into the relevant original equation(s) to get the value of the x-coordinate and value of the corresponding y-coordinate. **Note:** These may not be in the correct quadrant	M1	1.1b
	$S = (5\sqrt{2}, -4)$ or $x = 5\sqrt{2},\ y = -4$ or $S = (\text{awrt } 7.07, -4)$	A1	3.2a

		(6)	

	Way 5
M1:	Begins to solve the problem by applying an appropriate strategy. E.g. Way 5: A complete process of combining equations for C_1 and C_2 by substituting the parametric equation into the Cartesian equation to give an equation in one variable (i.e. *t*) only.
M1:	Uses the identity $\sin^2 t + \cos^2 t \equiv 1$ to achieve an equation in $\sin^2 t$ only and $\cos^2 t$ only **with no constant term**
A1:	A correct equation in $\sin^2 t$ and $\cos^2 t$ containing no constant term
dM1:	**dependent on both the previous M marks** Rearranges to make $\tan t = ...$
M1:	See scheme
A1:	Selects the correct coordinates for *S* Allow either $S = (5\sqrt{2}, -4)$ or $S = (\text{awrt } 7.07, -4)$ or $S = (\sqrt{50}, -4)$ or $S = \left(\frac{10}{\sqrt{2}}, -4\right)$

Question	Scheme		Marks	AOs
6(a)	$\text{H}_0 : \rho = 0 \qquad \text{H}_1 : \rho > 0$		B1	2.5
	Critical value 0.3438		M1	1.1a
	(0.446 > 0.3438) so there is evidence that the product moment correlation coefficient (pmcc) is greater than 0/there is positive correlation		A1	2.2b
			(3)	
(b)	The value is close(r) to 1 **or** there is strong(er) (positive) correlation		B1	2.4
			(1)	
(c)	$\log_{10} y = -1.82 + 0.89(\log_{10} x)$	$y = ax^n \rightarrow$ $\log_{10} y = \log_{10}(ax^n)$	M1	1.1b
	$y = 10^{-1.82+0.89(\log_{10} x)}$	$\log_{10} y = \log_{10} a + \log_{10} x^n$	M1	2.1
	$y = 10^{-1.82} \times 10^{0.89(\log_{10} x)}$ $[= 10^{-1.82} \times 10^{(\log_{10} x^{0.89})}]$	$\log_{10} y = \log_{10} a + n\log_{10} x$ $[\log_{10} a = -1.82, n = 0.89]$	M1	1.1b
	$y = 0.015x^{0.89}$	$y = 0.015x^{0.89}$	A1A1	1.1b 1.1b
			(5)	
				(9 marks)

	Notes
(a)	**B1:** for both hypotheses correct in terms of ρ **M1:** for the critical value: sight of 0.3438 or any cv such that 0.25 < |cv| < 0.45 **A1:** a comment suggesting a significant result/ H_0 is rejected on the basis of seeing +0.3438 **and** which mentions "pmcc/correlation/relationship" and "greater than 0/positive" (not just $\rho > 0$) or an answer in context e.g. 'as "income"(o.e.) increases, "CO_2/emissions"(o.e.) increases' A **contradictory** statement scores A0 e.g. 'Accept H_0, therefore positive correlation'
(b)	**B1:** for suitable reason e.g. *r* is close(r) to 1 **or** "strong(er)"/"near perfect" "correlation" Do not allow 'association'
(c)	**For both methods, once an M0 is scored, no further marks can be awarded** **and condone missing base 10 throughout** **Method 1: (working to the model)** **M1:** Correct substitution for both *c* and *m* (may be implied by 2nd M1 mark) **M1:** Making *y* the subject to give an equation in the form $y = 10^{a+b(\log_{10} x)}$ (may be implied by 3rd M1 mark) **M1:** Correct multiplication to give an equation in the form $y = 10^a \times 10^{b(\log_{10} x)}$ (this line implies M1M1M1 provided no previous incorrect working seen) **Method 2: (working from the model)** **M1:** Taking the log of both sides (may be implied by 2nd M1 mark) **M1:** Correct use of addition rule (may be implied by 3rd M1 mark) **M1:** Correct multiplication of power (this line implies M1M1M1 provided no previous incorrect working seen) **A1:** *n* = 0.89 **or** *a* = awrt 0.015 **or** $y = ax^{0.89}$ **or** $y = \text{awrt}0.015x^n$ (**dep on M3**) **A1:** *n* = 0.89 **and** *a* = awrt 0.015 / $y = \text{awrt}0.015x^{0.89}$ (**dep on M3**) do not award the final A1 if answer is given in an incorrect form e.g $y = 0.015 + x^{0.89}$

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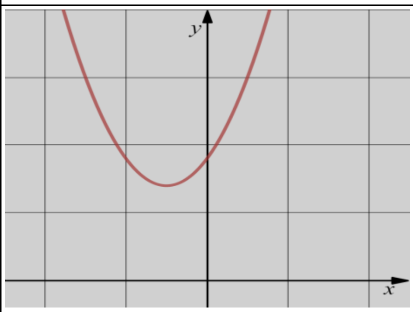
Question	Scheme	Marks	AOs
7 (a)	$2x^2+4x+9=2(x\pm k)^2 \pm ....$ $\quad a=2$	B1	1.1b
	Full method $2x^2+4x+9=2(x+1)^2 \pm ...$ $\quad a=2\ \& b=1$	M1	1.1b
	$2x^2+4x+9=2(x+1)^2+7$	A1	1.1b
		(3)	
(b)	 U shaped curve any position but not through (0,0)	B1	1.2
	y - intercept at $(0,9)$	B1	1.1b
	Minimum at $(-1,7)$	B1ft	2.2a
		(3)	
(c)	(i) Deduces translation with one correct aspect.	M1	3.1a
	Translate $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$	A1	2.2a
	(ii) $\mathrm{h}(x)=\dfrac{21}{"2(x+1)^2+7"} \Rightarrow$ (maximum) value $\dfrac{21}{"7"}(=3)$	M1	3.1a
	$0<\mathrm{h}(x)\leqslant 3$	A1ft	1.1b
		(4)	
			(10 marks)

(a)

B1: Achieves $2x^2+4x+9=2(x\pm k)^2 \pm ....$ or states that $a=2$

M1: Deals correctly with first two terms of $2x^2+4x+9$.

Scored for $2x^2+4x+9=2(x+1)^2 \pm ...$ or stating that $a=2$ and $b=1$

A1: $2x^2+4x+9=2(x+1)^2+7$

Note that this may be done in a variety of ways including equating $2x^2+4x+9$ with the expanded form of $a(x+b)^2+c \equiv ax^2+2abx+ab^2+c$

(b)

B1: For a U-shaped curve in any position not passing through $(0,0)$. Be tolerant of slips of the pen but do not allow if the curve bends back on itself

B1: A curve with a y - intercept on the +ve y axis of 9. The curve cannot just stop at $(0,9)$

Allow the intercept to be marked 9, $(0,9)$ but not $(9,0)$

B1ft: For a minimum at $(-1,7)$ in quadrant 2. This may be implied by -1 and 7 marked on the axes in the correct places. The curve must be a U shape and not a cubic say.

Follow through on a minimum at $(-b,c)$, marked in the correct quadrant, for their $a(x+b)^2+c$

(c)(i)

M1: Deduces translation with one correct aspect or states $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ with no reference to 'translate'.

Allow instead of the word translate, shift or move. $g(x)=f(x-2)-4$ can score M1

For example, possible methods of arriving at this deduction are:

- $f(x)\to g(x)$ is $2x^2+4x+9\to 2(x-2)^2+4(x-2)+5$ So $g(x)=f(x-2)-4$
- $g(x)=2(x-1)^2+3$ New curve has its minimum at $(1,3)$ so $(-1,7)\to(1,3)$
- Using a graphical calculator to sketch $y=g(x)$ and compares to the sketch of $y=f(x)$

In almost all cases you will not allow if the candidate gives two **different types of** transformations.

Eg, stretch and .....

A1: Requires both 'translate' and '$\begin{pmatrix} 2 \\ -4 \end{pmatrix}$' Allow 'shift' or move' instead of translate.

So condone " Move shift 2 (units) to the right and move 4 (units) down

However, for M1 A1, it is possible to reflect in $x=0$ and translate $\begin{pmatrix} 0 \\ -4 \end{pmatrix}$, so please consider all responses.

SC: If the candidate writes translate $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ or " move 2 (units) to the left and 4 (units) up" score M1 A0

(c)(ii)

M1: Correct attempt at finding the maximum value (although it may not be stated as a maximum)

- Uses part (a) to write $h(x)=\dfrac{21}{"2(x+1)^2+7"}$ and attempts to find $\dfrac{21}{\text{their "7"}}$

- Attempts to differentiate, sets $4x+4=0 \to x=-1$ and substitutes into $\mathrm{h}(x)=\dfrac{21}{2x^2+4x+9}$
- Uses a graphical calculator to sketch $y=\mathrm{h}(x)$ and establishes the 'maximum' value $(...,3)$

A1ft: $0<\mathrm{h}(x)\leqslant 3$ Allow for $0<\mathrm{h}\leqslant 3$ $(0,3]$ and $0<y\leqslant 3$ but not $0<x\leqslant 3$

Follow through on their $a(x+b)^2+c$ so award for $0<\mathrm{h}(x)\leqslant \dfrac{21}{c}$

Question	Scheme	Marks	AOs
8 (a)	Uses a model $V = Ae^{\pm kt}$ oe (See next page for other suitable models)	M1	3.3
	Eg. Substitutes $t=0, V=20\,000 \Rightarrow A=20\,000$	M1	1.1b
	Eg. Substitutes $t=1, V=16\,000 \Rightarrow 16\,000=20\,000e^{-1k} \Rightarrow k=..$	dM1	3.1b
	$V=20\,000e^{-0.223t}$	A1	1.1b
		(4)	
(b)	Substitutes $t=10$ in their $V=20\,000e^{-0.223t} \Rightarrow V=(£\,2150)$	M1	3.4
	Eg. The model is reliable as £2150 ≈ £2000	A1	3.5a
		(2)	
(c)	Make the "-0.223" less negative. Alt: Adapt model to for example $V=18\,000e^{-0.223t}+2000$	B1ft	3.3
		(1)	
			(7 marks)

(a) Option 1

M1: For $V = Ae^{\pm kt}$ Do not allow if k is fixed, eg $k=-0.5$

Condone different variables $V \leftrightarrow y$ $t \leftrightarrow x$ for this mark, but for A1 V and t must be used.

M1: Substitutes $t=0 \Rightarrow A=20\,000$ into their exponential model

Candidates may start by simply writing $V=20\,000e^{kt}$ which would be M1 M1

dM1: Substitutes $t=1 \Rightarrow 16\,000=20\,000e^{-1k} \Rightarrow k=..$ via the correct use of logs.

It is dependent upon both previous M's.

A1: $V=20\,000e^{-0.223t}$ (with accuracy to at least 3sf) or $V=20\,000e^{t\ln 0.8}$

A correct linking formula with correct constants must be seen somewhere in the question

(b)

M1: Uses a model of the form $V = Ae^{\pm kt}$ to find the value of V when $t = 10$.

Alternatively substitutes $V=2000$ into their model and finds t

A1: This can only be scored from an acceptable model with correct constants with accuracy to at least 2sf .

Compares V = (£) 2150 with (£) 2 000 and states "reliable as $2150 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".
Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''
Do not allow ''it is not a good model because it is not the same''
In the alternative it is for comparing their value of t with 10 and making a suitable comment as to the reliability of their model with a reason.
$V = 20000\mathrm{e}^{-0.223t} \Rightarrow 2000 = 20000\mathrm{e}^{-0.223t} \Rightarrow t = 10.3$ years.
Deduction Reliable model as the time is approximately the same as 10 years. A candidate can argue that the model is unreliable if they can give a suitable reason.

(c)
B1ft: For a correct statement. Eg states that the value of their ' -0.223 ' should become less negative.
Alt states that the value of their '0.223' should become smaller. If they refer to k then refer to the model and apply the same principles.
Condone the fact that they don't state their -0.223 doesn't lie in the range $(-0.223, 0)$

(a) Option 2
M1: For $V = Ar^t$ or equivalent such as $V = kr^{t-1}$
Condone different variables $V \leftrightarrow y \quad t \leftrightarrow x$ for this mark, but for A1 V and t must be used.
M1: Uses $t = 0 \Rightarrow A = 20000$ in their model. Alternatively uses $(0, 20000)$ and $(1, 16000)$ to give $r = \frac{4}{5}$ oe
You may award if one of the number pair $(0, 20000)$ or $(1, 16000)$ works in an allowable model
dM1: $t = 1 \Rightarrow 16000 = 20000r^1 \Rightarrow r = ..$ Dependent upon both previous M's
In the alternative it would be for using $r = \frac{4}{5}$ with one of the points to find $A = 20000$
You may award if both number pairs $(0, 20000)$ or $(1, 16000)$ work in an allowable model
A1: $V = 20000 \times 0.8^t$ Note that $V = 20000 \times 1.25^{-t}$ $V = 16000 \times 0.8^{t-1}$ and is also correct
(b)
M1: Uses a model of the form $V = Ar^t$ oe to find the value of V when t = 10. Eg. 20000×0.8^{10}
Alternatively substitutes $V = 2000$ into their model and finds t
A1: This can only be scored from an acceptable model with correct constants also allowing an accuracy to 2sf.
Compares (£) 2147 with (£) 2 000 and states "reliable as $2147 \approx 2000$ " or "reasonably good as they are close" or ""OK but a little high".
Allow a candidate to argue that it is unreliable as long as they state a suitable reason. Eg. ''It is too far away from £2000'' or ''It is over £100 away, so it is not good''
Do not allow ''it is not a good model because it is not the same''
(c)
B1ft: States a value of r in the range $(0.8, 1)$ or states would increase the value of "0.8"
They do not need to state that "0.8" must lie in the range $(0.8, 1)$
Condone increase the 0.8. Also allow decrease the "1.25" for $V = 20000 \times 1.25^{-t}$

....
...

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(a) Option 3

M1: They may suggest an exponential model with a lower bound. For example, for $V = Ae^{\pm kt} + 2000$ The bound must be stated but do not allow k to be fixed . Allow as long as the bound < 10 000

M1: $t = 0, V = 20000 \Rightarrow A = 18000$

dM1: $t = 1, V = 16\,000 \Rightarrow 16000 = 2\,000 + 18000e^{k} \Rightarrow k = ..$ Dependent upon both previous M's

A1: $V = 18\,000 \times e^{-0.251t} + 2\,000$

(b)

M1: Uses their model to find the value of V when $t = 10$.

Alternatively substitutes $V = 2000$ into their model and finds t

A1: For $V = 18\,000 \times e^{-0.251 \times 10} + 2\,000 = £3462.83$ Deduction: Unreliable model as £3462.83 is not close to £2 000 This can only be scored from an acceptable model with correct constants

(c)

B1: States make the value of k or the -0.251 greater (or less negative) so that it lies in the range $(-0.251, 0)$

Condone 'make the value of k or the -0.251 greater (or less negative)'

…..
......

It is entirely possible that they start part (a) from a differential equation.

M1: $\dfrac{dV}{dt} = kV \Rightarrow \int \dfrac{dV}{V} = \int k dt \Rightarrow \ln V = kt + c$ M1: $\ln 20000 = c$

dM1: Using $t = 1, V = 16\,000 \Rightarrow k = ..$ A1: $\ln V = -\ln\left(\dfrac{5}{4}\right)t + \ln 20000$

Question	Scheme	Marks	AOs
9 (a)	$\dfrac{132}{184} = 0.71739...$ awrt **<u>0.717</u>**	B1	1.1b
		(1)	
(b)(i)	$P(X \geq 6) = 1 - P(X \leq 5)$ or $P([X=]6) + P([X=]7) + P([X=]8)$	M1	3.4
	$= 1 - 0.296722...$ awrt **<u>0.703</u>**	A1	1.1b
		(2)	
(b)(ii)	$184 \times P(X = 7)$ $[= 184 \times 0.2811...]$	M1	1.1b
	$= 51.7385...$ awrt **<u>51.7</u>**	A1	1.1b
		(2)	
(c)	Part (a) and part (b)(i) are similar **and** the expected number of 7s (51.7 or 0.281) matches with the number of 7s found in the data set (52 or 0.283) so Magali's model is supported.	B1ft	3.5a
		(1)	

(d)	$\frac{23}{28} = 0.82142...$ awrt **0.821**	B1	1.1b
		(1)	
(e)	Any one of… • Part (d)/'0.821' differs from part (a)/(b)(i)/(0.7…) • there is a **greater/different probability of high cloud cover/more likely to have high cloud cover** if the previous day had high cloud cover • **independence(o.e.)** does not hold	B1	2.4
	…therefore Magali's (binomial) model may not be suitable.	dB1	3.5a
		(2)	
			(9 marks)

	Notes
	Allow fractions, decimals or percentages throughout this question.
(a)	Allow equivalent fraction, e.g. $\frac{33}{46}$
(b)(i)	**M1:** for writing or using $1-\text{P}(X \le 5)$ or $\text{P}(X=6)+\text{P}(X=7)+\text{P}(X=8)$ **A1:** awrt 0.703 (correct answer scores 2 out of 2)
(b)(ii)	**M1:** for 184 × $\text{P}(X=7)$ o.e. e.g., 184 ×$[\text{P}(X \le 7)-\text{P}(X \le 6)]$ **A1:** awrt 51.7
(c)	**B1ft:** comparing '0.717' with '0.703' <u>and</u> '51.7 or '0.281' with 52 or 0.283 **and** concluding that Magali's model **is supported** (must be comparing prob. with prob. <u>and</u> days with days). Allow not supported or mixed conclusions if consistent with their f.t. answers in (a) and (b)
(e)	**B1:** Any bullet point **dB1:** (dep on previous B1) for Magali's model may not be suitable (o.e.) Condone not accurate for not suitable **SC:** part (d) is similar to part (a)/(b)(i) **and** a compatible conclusion (i.e. Magali's model is supported) to score B1B1.

Question	Scheme	Marks	AOs
10 (a)	$\text{gg}(0) = \text{g}((0-2)^2+1) = \text{g}(5) = 4(5)-7 = 13$	M1	2.1
		A1	1.1b
		(2)	
(b)	Solves either $(x-2)^2+1=28 \Rightarrow x=...$ or $4x-7=28 \Rightarrow x=...$	M1	1.1b
	At least one critical value $x=2-3\sqrt{3}$ or $x=\frac{35}{4}$ is correct	A1	1.1b
	Solves both $(x-2)^2+1=28 \Rightarrow x=...$ and $4x-7=28 \Rightarrow x=...$	M1	1.1b
	Correct final answer of '$x<2-3\sqrt{3}$, $x>\frac{35}{4}$'	A1	2.1
	Note: Writing awrt -3.20 or a truncated -3.19 or a truncated -3.2 in place of $2-3\sqrt{3}$ is accepted for any of the A marks	**(4)**	
(c)	<u>h is a one-one</u> {function (or mapping) so has an inverse} <u>g is a many-one</u> {function (or mapping) so does not have an inverse}	B1	2.4
		(1)	
(d) Way 1	$\left\{\text{h}^{-1}(x) = -\frac{1}{2} \Rightarrow\right\}$ $x = \text{h}\left(-\frac{1}{2}\right)$	M1 B1 on epen	1.1b
	$x=\left(-\frac{1}{2}-2\right)^2+1$ **Note:** Condone $x=\left(\frac{1}{2}-2\right)^2+1$	M1	1.1b
	$\Rightarrow x=7.25$ only **cso**	A1	2.2a
		(3)	
(d) Way 2	{their $\text{h}^{-1}(x)$ } $=\pm 2\pm\sqrt{x\pm 1}$	M1	1.1b
	Attempts to solve $\pm 2\pm\sqrt{x\pm 1}=-\frac{1}{2} \Rightarrow \pm\sqrt{x\pm 1}=...$	M1	1.1b
	$\Rightarrow x=7.25$ only **cso**	A1	2.2a
		(3)	
		(10 marks)	

Notes for Question 10

(a)	
M1:	Uses a complete method to find $\text{gg}(0)$. E.g. • Substituting $x=0$ into $(0-2)^2+1$ and the result of this into the relevant part of $\text{g}(x)$ • Attempts to substitute $x=0$ into $4((x-2)^2+1)-7$ or $4(x-2)^2-3$
A1:	$\text{gg}(0)=13$
(b)	
M1:	See scheme
A1:	See scheme
M1:	See scheme
A1:	Brings all the strands of the problem together to give a correct solution.
Note:	You can ignore inequality symbols for any of the M marks
Note:	If a 3TQ is formed (e.g. $x^2-4x-23=0$) then a correct method for solving a 3TQ is required for the relevant method mark to be given.
Note:	Writing $(x-2)^2+1=28 \Rightarrow (x-2)+1=\sqrt{28} \Rightarrow x=-1+\sqrt{28}$ (i.e. taking the square-root of each term to solve $(x-2)^2+1=28$ is not considered to be an acceptable method)
Note:	Allow set notation. E.g. $\{x\in\mathbb{R}: x<2-3\sqrt{3} \cup x>8.75\}$ is fine for the final A mark

Notes for Question 10 Continued	
(b)	***continued***
Note:	Give final A0 for $\{x \in \mathbb{R}: x < 2 - 3\sqrt{3} \cap x > 8.75\}$
Note:	Give final A0 for $2 - 3\sqrt{3} > x > 8.75$
Note:	Allow final A1 for their writing a final answer of "$x < 2 - 3\sqrt{3}$ and $x > \frac{35}{4}$"
Note:	Allow final A1 for a final answer of $x < 2 - 3\sqrt{3},\ x > \frac{35}{4}$
Note:	Writing $2 - \sqrt{27}$ in place of $2 - 3\sqrt{3}$ is accepted for any of the A marks
Note:	Allow final A1 for a final answer of $x < -3.20,\ x > 8.75$
Note:	Using 29 instead of 28 is M0 A0 M0 A0
(c)	
B1:	A correct explanation that conveys the underlined points
Note:	A minimal acceptable reason is "h is a one-one and g is a many-one"
Note:	Give B1 for "h^{-1} is one-one and g^{-1} is one-many"
Note:	Give B1 for "h is a one-one and g is not"
Note:	Allow B1 for "g is a many-one and h is not"
(d)	**Way 1**
M1:	Writes $x = \text{h}\left(-\frac{1}{2}\right)$
M1:	See scheme
A1:	Uses $x = \text{h}\left(-\frac{1}{2}\right)$ to deduce that $x = 7.25$ only, **cso**
(d)	**Way 2**
M1:	See scheme
M1:	See scheme
A1:	Use a correct $\text{h}^{-1}(x) = 2 - \sqrt{x-1}$ to deduce that $x = 7.25$ only, **cso**
Note:	Give final A0 cso for $2 + \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A0 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Give final A1 cso for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow -\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$
Note:	Allow final A1 for $2 \pm \sqrt{x-1} = -\frac{1}{2} \Rightarrow \pm\sqrt{x-1} = -\frac{5}{2} \Rightarrow x - 1 = \frac{25}{4} \Rightarrow x = 7.25$

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Question	Scheme
11 (a)	IQR = 26.6 – 19.4 [= 7.2]
	19.4 – 1.5 × '7.2' [= 8.6] or 26.6 + 1.5 × '7.2' [= 37.4]
	Plotting one upper whisker to 32.5 **and** one lower whisker to 8.6 or 9.1
	Plotting 7.6 and 8.1 as the only two outliers
(b)	October (since it is the month with the coldest temperatures between May and October in Beijing)
(c)	$[\sigma =]\sqrt{\frac{4952.906}{184}}$ or e.g. $[\sigma =]\sqrt{\frac{S_{xx}}{n}} = 5.188...$ [=5.19*]
(d)	z = (±) 1.28(16) | $[P_{90} =]29.251...$ or $[P_{10} =]15.948...$
	2 × 1.2816 × 5.19 | '29.251...' – '15.948...'
	= awrt **13.3**
(e)	Daily mean wind speed/Beaufort conversion since it is qualitative Rainfall since it is not symmetric/lots of days with 0 rainfall
	(11 marks)

Notes

(a)	**B1:** for a correct calculation for the IQR (implied by 10.8 **or** 8.6 **or** 37.4 seen) **M1:** for a complete method for either lower outlier limit or upper outlier limit (allow ft on their IQR) (may be implied by the 1st A1 or a lower whisker at 8.6) **A1:** both whiskers plotted correctly (allow ½ square tolerance) **A1:** only two outliers plotted, 7.6 and 8.1 (must be disconnected from whisker) **NOTE:** A fully correct box plot with no incorrect working scores 4/4
(c)	**B1cso*:** Correct expression with square root **or** correct formula and 5.188 or better Allow a complete correct method finding $\sum x^2 = \text{awrt } 98720$ and $\sigma = \sqrt{\frac{98715.9...}{184} - \left(\frac{4153.6}{184}\right)^2}$

(d)	**B1:** Identifying *z*-value for 10th or 90th percentile (allow awrt (±) 1.28) or for identifying $[P_{90}=]29.251...$ (awrt 29.3) or $[P_{10}=]15.948...$ (awrt 15.9) (This may be implied by a correct answer awrt 13.3) **M1:** for 2 × *z* × 5.19 where 1 < *z* < 2 or for their $P_{90}-P_{10}$ where 25< P_{90} <35 and 10< P_{10} <20 **A1:** awrt 13.3
(e)	**B1:** for one variable identified and a correct supporting reason **B1:** for two variables identified and a correct supporting reason for each Allow any two of the following: • Wind speed/Beaufort since the data is non-numeric (o.e.). They need not mention Beaufort provided there is a description of the data as non-numeric (Do not allow wind direction/wind gust) • Rainfall as not symmetric/is skewed/is not bell shaped/lots of 0s /many days with no rain/mean≠mode or median • Date since each data value appears once/it is uniformly distributed • Daily mean pressure since it is not symmetric/is skewed/not bell shaped • Daily mean wind speed since it is not symmetric/is skewed/not bell shaped Do not allow 'not continuous' or 'discrete' as a supporting reason. Ignore extraneous non-contradicting statements

Question	Scheme	Marks	AOs
12 (a)	Correct method used in attempting to differentiate $y=\dfrac{5x^2+10x}{(x+1)^2}$	M1	3.1a
	$\dfrac{dy}{dx}=\dfrac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ oe	A1	1.1b
	Factorises/Cancels term in $(x+1)$ and attempts to simplify $\dfrac{dy}{dx}=\dfrac{(x+1)\times(10x+10)-(5x^2+10x)\times 2}{(x+1)^3}=\dfrac{A}{(x+1)^3}$	M1	2.1
	$\dfrac{dy}{dx}=\dfrac{10}{(x+1)^3}$	A1	1.1b
		(4)	
(b)	For $x<-1$	B1ft	2.2a

	Follow through on their $\frac{dy}{dx}=\frac{A}{(x+1)^n}, n=1,3$		
		(1)	
			(5 marks)

(a)

M1: Attempts to use a correct rule to differentiate Eg: Use of quotient (& chain) rules on $y=\frac{5x^2+10x}{(x+1)^2}$

Alternatively uses the product (and chain) rules on $y=(5x^2+10x)(x+1)^{-2}$

Condone slips but expect $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{(x+1)^4}$

$(A,B,C,D>0)$ or $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(Ax+B)-(5x^2+10x)\times(Cx+D)}{((x+1)^2)^2}$ $(A,B,C,D>0)$

using the quotient rule

or $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(Ax+B)+(5x^2+10x)\times C(x+1)^{-3}$ $(A,B,C\neq 0)$ using the product rule.

Condone missing brackets and slips for the M mark. For instance if they quote $u=5x^2+10$, $v=(x+1)^2$ and don't make the differentiation easier, they can be awarded this mark for applying the correct rule.

Also allow where they quote the correct formula, give values of u and v, but only have v rather than v^2 the denominator.

A1: A correct (unsimplified) answer

Eg. $\left(\frac{dy}{dx}\right)=\frac{(x+1)^2\times(10x+10)-(5x^2+10x)\times 2(x+1)}{(x+1)^4}$ or equivalent via the quotient rule.

OR $\left(\frac{dy}{dx}\right)=(x+1)^{-2}\times(10x+10)+(5x^2+10x)\times -2(x+1)^{-3}$ or equivalent via the product rule

M1: A valid attempt to proceed to the given form of the answer.

It is dependent upon having a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v^2}$ and proceeding to $\frac{A}{(x+1)^3}$

It can also be scored on a quotient rule of $\pm\frac{v\mathrm{d}u-u\mathrm{d}v}{v}$ and proceeding to $\frac{A}{(x+1)}$

You may see candidates expanding terms in the numerator. FYI
$10x^3+30x^2+30x+10-10x^3-30x^2-20x$

but under this method they must reach the same expression as required by the main method.

Using the product rule expect to see a common denominator being used correctly before the above

A1: $\frac{dy}{dx}=\frac{10}{(x+1)^3}$ There is no requirement to see $\frac{dy}{dx}=$ and they can recover from missing brackets/slips.

(b)

B1ft: Score for deducing the correct answer of $x<-1$ This can be scored independent of their answer to part (a). Alternatively score for a correct **ft** answer for their $\frac{dy}{dx}=\frac{A}{(x+1)^n}$ where $A<0$ and $n=1,3$ award for $x>-1$. So for example if $A>0$ and $n=1,3 \Rightarrow x<-1$

Question	Scheme	Marks	AOs
Alt via division	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in form $y=A\pm\frac{B}{(x+1)^2}$ $A,B\neq 0$	M1	3.1a
	Writes $y=\frac{5x^2+10x}{(x+1)^2}$ in the form $y=5-\frac{5}{(x+1)^2}$	A1	1.1b
	Uses the chain rule $\Rightarrow \frac{dy}{dx}=\frac{C}{(x+1)^3}$ (May be scored from $A=0$)	M1	2.1
	$\frac{dy}{dx}=\frac{10}{(x+1)^3}$ which cannot be awarded from incorrect value of A	A1	1.1b
		(4)	
(b)	For $x<-1$ or correct follow through	B1ft	2.2a
		(1)	
		(5 marks)	

Question	Scheme	Marks	AOs
13	£y is the total cost of making x bars of soap Bars of soap are sold for £2 each		
(a)	$y=kx+c$ {where k and c are constants}	B1	3.3
	Note: Work for (a) cannot be recovered in (b) or (c)	**(1)**	

(b) **Way 1**	Either • $x=800 \Rightarrow y=2(800)-500$ $\{=1100 \Rightarrow (x,y)=(800, 1100)\}$ • $x=300 \Rightarrow y=2(300)+80$ $\{=680 \Rightarrow (x,y)=(300, 680)\}$	M1	3.1b
	Applies (800, their 1100) and (300, their 680) to give two equations $1100=800k+c$ and $680=300k+c \Rightarrow k, c=...$	dM1	1.1b
	Solves correctly to find $k=0.84$, $c=428$ and states $y=0.84x+428$ *	A1*	2.1
	Note: the answer $y=0.84x+428$ must be stated in (b)	**(3)**	
(b) **Way 2**	Either • $x=800 \Rightarrow y=2(800)-500$ $\{=1100 \Rightarrow (x,y)=(800, 1100)\}$ • $x=300 \Rightarrow y=2(300)+80$ $\{=680 \Rightarrow (x,y)=(300, 680)\}$	M1	3.1b
	Complete method for finding both $k=...$ and $c=...$ e.g. $k=\dfrac{1100-680}{800-300}$ $\{=0.84\}$ $(800, 1100) \Rightarrow 1100=800(0.84)+c \Rightarrow c=...$	dM1	1.1b
	Solves to find $k=0.84$, $c=428$ and states $y=0.84x+428$ *	A1*	2.1
	Note: the answer $y=0.84x+428$ must be stated in (b)	**(3)**	
(b) **Way 3**	Either • $x=800 \Rightarrow y=2(800)-500$ $\{=1100 \Rightarrow (x,y)=(800, 1100)\}$ • $x=300 \Rightarrow y=2(300)+80$ $\{=680 \Rightarrow (x,y)=(300, 680)\}$	M1	3.1b
	$\{y=0.84x+428 \Rightarrow\}$ $x=800 \Rightarrow y=(0.84)(800)+428=1100$ $x=300 \Rightarrow y=(0.84)(300)+428=680$	dM1	1.1b
	Hence $y=0.84x+428$ *	A1*	2.1
		(3)	
(c)	Allow any of $\{0.84$, in £s$\}$ represents • the ***cost*** of {making} each extra bar {of soap} • the direct ***cost*** of {making} a bar {of soap} • the marginal ***cost*** of {making} a bar {of soap} • the ***cost*** of {making} a bar {of soap} (Condone this answer) **Note: Do not allow** • $\{0.84$, in £s$\}$ is the profit per bar {of soap} • $\{0.84$, in £s$\}$ is the (selling) price per bar {of soap}	B1	3.4
		(1)	
(d) **Way 1**	{Let n be the least number of bars required to make a profit}		
	$2n=0.84n+428 \Rightarrow n=...$ (Condone $2x=0.84x+428 \Rightarrow x=...$)	M1	3.4
	Answer of 369 {bars}	A1	3.2a
		(2)	
(d) **Way 2**	• Trial 1: $n=368 \Rightarrow y=(0.84)(368)+428 \Rightarrow y=737.12$ $\{$revenue $=2(368)=736$ or loss $=1.12\}$	M1	3.4
	• Trial 2: $n=369 \Rightarrow y=(0.84)(369)+428 \Rightarrow y=737.96$ $\{$revenue $=2(369)=738$ or profit $=0.04\}$ leading to an answer of 369 {bars}	A1	3.2a
		(2)	
			(7 marks)

Notes for Question 13	
(a)	
B1:	Obtains a correct form of the equation. E.g. $y = kx + c$; $k \neq 0, c \neq 0$. **Note:** Must be seen in (a)
Note:	Ignore how the constants are labelled – as long as they appear to be constants. e.g. k, c, m etc.
(b)	**Way 1**
M1:	Translates the problem into the model by finding either • $y = 2(800) - 500$ for $x = 800$ • $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no errors in their working
Note	Allow 1st M1 for any of • $1600 - y = 500$ • $600 - y = -80$
(b)	**Way 2**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** See scheme
A1:	See scheme – no error in their working
(b)	**Way 3**
M1:	Translates the problem into the model by finding either $y = 2(800) - 500$ for $x = 800$ $y = 2(300) + 80$ for $x = 300$
dM1:	**dependent on the previous M mark** Uses the model to test both points $(800, \text{their } 1100)$ and $(300, \text{their } 680)$
A1:	Confirms $y = 0.84x + 428$ is true for both $(800, 1100)$ and $(300, 680)$ and gives a conclusion
Note:	Conclusion could be "$y = 0.84x + 428$" or "QED" or "proved"
Note:	Give 1st M0 for $500 = 800k + c$, $80 = 300k + c$ $\Rightarrow k = \dfrac{500-80}{800-300} = 0.84$
(c)	
B1:	see scheme
Note:	Also condone B1 for "rate of change of cost", "cost of {making} a bar", "constant of proportionality for cost per bar of soap" or "rate of increase in cost",
Note:	Do not allow reasons such as "price increase or decrease", "rate of change of the bar of soap" or "decrease in cost"
Note:	Give B0 for incorrect use of units. E.g. Give B0 for "the cost of making each extra bar of soap is £84" Condone the use of £0.84p

Notes for Question 13 Continued	
(d)	**Way 1**
M1:	Using the model and constructing an argument leading to a critical value for the number of bars of soap sold. See scheme.
A1:	369 only. Do not accept decimal answers.
(d)	**Way 2**
M1:	Uses either 368 or 369 to find the cost $y = ...$
A1:	Attempts both trial 1 and trial 2 to find both the cost $y = ...$ and arrives at an answer of 369 only. Do not accept decimal answers.
Note:	You can ignore inequality symbols for the method mark in part (d)
Note:	Give M1 A1 for no working leading to 369 {bars}
Note:	Give final A0 for $x > 369$ or $x > 368$ or $x \geq 369$ without $x = 369$ or 369 stated as their final answer
Note:	Condone final A1 for **in words** "at least 369 bars must be made/sold"
Note:	**Special Case:** Assuming a profit of £1 is required and achieving $x = 370$ scores special case M1A0

Question	Scheme	Marks	AOs
14 (a)	$5\sin 2\theta = 9\tan\theta \Rightarrow 10\sin\theta\cos\theta = 9 \times \dfrac{\sin\theta}{\cos\theta}$ $A\cos^2\theta = B$ or $C\sin^2\theta = D$ or $P\cos^2\theta\sin\theta = Q\sin\theta$	M1	3.1a
	For a correct simplified equation in one trigonometric function Eg $10\cos^2\theta = 9$ $10\sin^2\theta = 1$ oe	A1	1.1b
	Correct order of operations For example $10\cos^2\theta = 9 \Rightarrow \theta = \arccos(\pm)\sqrt{\dfrac{9}{10}}$	dM1	2.1
	Any one of the four values awrt $\theta = \pm 18.4°, \pm 161.6°$	A1	1.1b
	All four values $\theta =$ awrt $\pm 18.4°, \pm 161.6°$	A1	1.1b
	$\theta = 0°, \pm 180°$	B1	1.1b
		(6)	
(b)	Attempts to solve $x - 25° = -18.4°$	M1	1.1b
	$x = 6.6°$	A1ft	2.2a
		(2)	
			(8 marks)

(a)

M1: Scored for the whole strategy of attempting to form an equation in one function of the form given in the scheme. For this to be awarded there must be an attempt at using $\sin 2\theta = \ldots\sin\theta\cos\theta$, $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and possibly $\pm 1 \pm \sin^2\theta = \pm\cos^2\theta$ to form an equation in one "function" usually $\sin^2\theta$ or $\cos^2\theta$

Allow for this mark equations of the form $P\cos^2\theta\sin\theta = Q\sin\theta$ oe

A1: Uses the correct identities $\sin 2\theta = 2\sin\theta\cos\theta$ **and** $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ to form a correct simplified equation in one trigonometric function. It is usually one of the equations given in the scheme, but you may see equivalent correct equations such as $10 = 9\sec^2\theta$ which is acceptable, but in almost all cases it is for a correct equation in $\sin\theta$ or $\cos\theta$

dM1: Uses the correct order of operations for their equation, usually in terms of just $\sin\theta$ or $\cos\theta$, to find at least one value for θ (Eg. square root before invcos). It is dependent upon the previous M.

Note that some candidates will use $\cos^2\theta = \dfrac{\pm\cos 2\theta \pm 1}{2}$ and the same rules apply.

Look for correct order of operations.

A1: Any one of the four values awrt $\pm 18.4°, \pm 161.6°$. Allow awrt 0.32 (rad) or 2.82 (rad)

A1: All four values awrt $\pm 18.4°, \pm 161.6°$ and no other values apart from $0°, \pm 180°$

B1: $\theta = 0°, \pm 180°$ This can be scored independent of method.

(b)

M1: Attempts to solve $x - 25° = "\theta"$ where θ is a solution of their part (a)

A1ft: For awrt $x = 6.6°$ but you may ft on their $\theta + 25°$ where $-25 < \theta < 0$

If multiple answers are given, the correct value for their θ must be chosen

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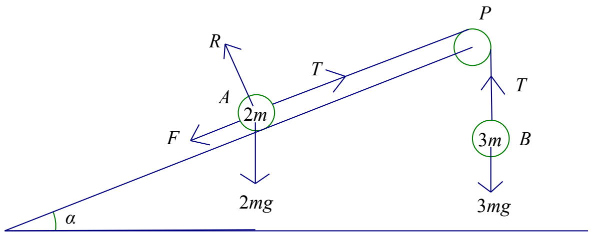
Question	Scheme	Marks	AO
15 (a)			
	$R = 2mg\cos\alpha$	B1	3.4
	$F = \frac{2}{3}R$	B1	1.2
	Equation of motion for A:	M1	3.3
	$T - F - 2mg\sin\alpha = 2ma$	A1	1.1b
	Equation of motion for B:	M1	3.3
	$3mg - T = 3ma$	A1	1.1b
	Complete strategy to find an equation in T, m and g only.	M1	3.1b
	$T = \frac{12mg}{5}$ *	A1*	2.2a
		(8)	
(b)	$(F_{max} =)\frac{16mg}{13} > \frac{10mg}{13}$	M1	2.1
	…… so A will not move.	A1	2.2a
		(2)	
(c)	• Extensible string • Weight of string • Friction at pulley e.g. rough pulley • Allow for the dimensions of the blocks e.g. “Do not model blocks as particles”; “(include) air resistance”;“include rotational effects of forces on blocks i.e. spin”	B1 B1	3.5c 3.5c
		(2)	
		(12)	

Marks		**Notes**		
15a	B1	Normal reaction between A and the plane seen or implied, $\cos\alpha$ does not need to be substituted.		
	B1	$F = \frac{2}{3}R$ seen or implied anywhere, including part (b)		
	M1	Form an equation of motion for A. Must include all relevant terms. Must be the correct mass but condone consistent missing m's. Condone sign errors and sin/cos confusion		
	A1	Correct unsimplified equation (F does not need to be substituted). Allow consistent use of $(-a)$ **N.B**. If $T - 2mg = 2ma$ is seen with no working, M0A0 unless both B1 marks have been scored.		
	M1	Form an equation of motion for B. Must be the correct mass on RHS but condone consistent missing m's. Condone sign errors and sin/cos confusion.		
	A1	Correct unsimplified equation (F does not need to be substituted). Allow consistent use of $(-a)$		
		N.B. Allow the 'whole system' equation to replace the equation for A or B. $3mg - F - 2mg\sin\alpha = 5ma$ Must be the correct mass on RHS but condone consistent missing m's. Condone sign errors and sin/cos confusion.		
	M1	Complete method to give an equation in T, m and g only. **N.B.** Allow θ in the equation if they have defined what θ is: e.g. $\theta = \tan^{-1}(\frac{5}{12})$ This is an <u>independent</u> mark but they must have two simultaneous equations in T and a unless one of the equations is the whole system equation in which case one equation will be in T and a and the other equation will be in a only.		
	A1*	Obtain the **given answer** from correct working using EXACT trig ratios. (not available if using a decimal angle)		
15b	M1	Comparison of their F_{max} $(\frac{2}{3}R)$ and their component of weight down the slope, must be comparing numerical values. oe e.g. if they consider the difference **N.B.** Allow comparison of μ and $\tan\alpha$ with numerical values		
	A1	Correctly justified conclusion and no errors seen **N.B.** If they equate their difference to an 'ma' term then A0		

15c	B1 B1	Deduct 1 mark for each extra (more than 2) incorrect answer up to a maximum of 2 incorrect answers. Ignore extra correct answers. e.g. two correct, one incorrect B1 B0 one correct, one incorrect B1 B0 one correct, two incorrect B0 B0 Ignore incorrect reasons or consequences. Ignore any mention of wind or a general reference to friction.

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Extend | Paper Reference **9MA0**

Mathematics

Advanced Paper

2019 Silver

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks __/97

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 12 questions in this question paper.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Index

1. (*a*) Find the first three terms, in ascending powers of x, of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to $\sqrt{2}$

Possible values of x that could be substituted into this expansion are:

- $x = -14$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{18}} = \dfrac{\sqrt{2}}{6}$
- $x = 2$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$
- $x = -\dfrac{1}{2}$ because $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{\sqrt{\dfrac{9}{2}}} = \dfrac{\sqrt{2}}{3}$

(*b*) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of x should not be used

(1)

(ii) state, giving a reason, which of the three values of x would lead to the most accurate approximation to $\sqrt{2}$

(1)

(Total for Question 1 is 6 marks)

---

2. Given that $a > b > 0$ and that a and b satisfy the equation

$$\log a - \log b = \log(a - b)$$

(*a*) show that

$$a = \frac{b^2}{b-1}$$

(3)

(*b*) Write down the full restriction on the value of b, explaining the reason for this restriction.

(2)

(Total for Question 2 is 5 marks)

---

3. A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of V km h^{-1}.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

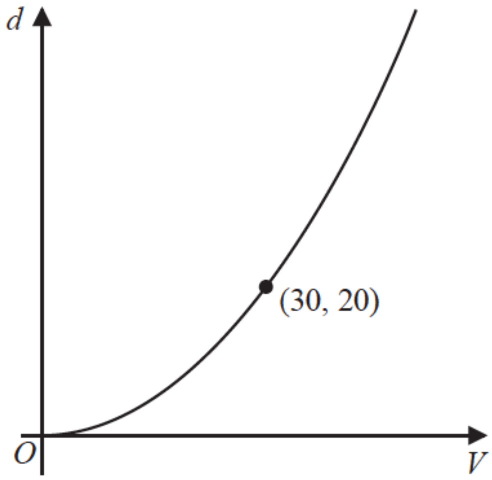

Figure 5

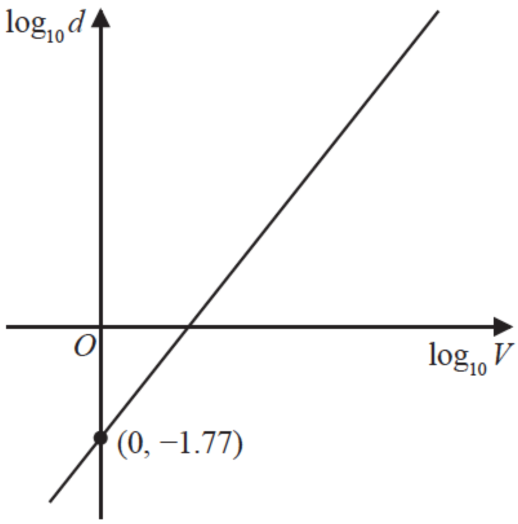

Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with k » 0.017

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

(Total for Question 3 is 9 marks)

---

4. (*a*) Prove

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta \qquad\qquad \theta \neq (90n)°,\ n \in \mathbb{Z}$$

(4)

(*b*) Hence solve, for $90° < \theta < 180°$, the equation

$$\frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta} = 4$$

giving any solutions to one decimal place.

(3)

(Total for Question 4 is 7 marks)

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5. The curve C with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \qquad x \in \mathbb{R}, \quad x \neq -3, x \neq 2$$

where p and q are constants, passes through the point $\left(3, \dfrac{1}{2}\right)$ and has two vertical asymptotes
with equations $x = 2$ and $x = -3$

(*a*) (i) Explain why you can deduce that $q = 4$

(ii) Show that $p = 15$

(3)

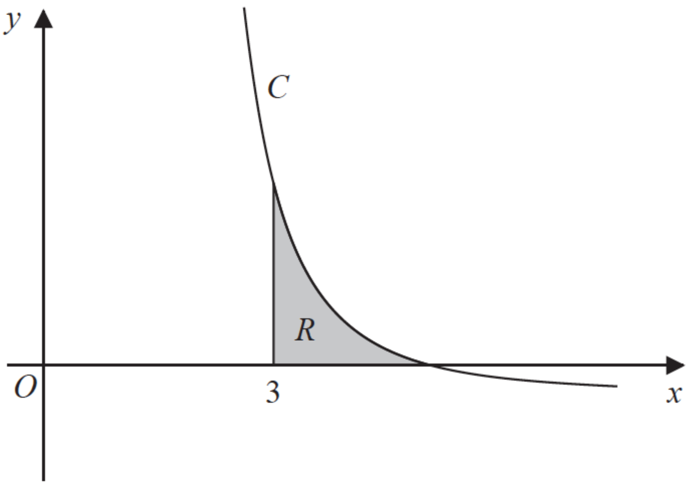

Figure 4

Figure 4 shows a sketch of part of the curve C. The region R, shown shaded in Figure 4, is bounded by the curve C, the x-axis and the line with equation $x = 3$

(*b*) Show that the exact value of the area of R is $a \ln 2 + b \ln 3$, where a and b are rational constants to be found.

(8)

(Total for Question 5 is 11 marks)

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6.

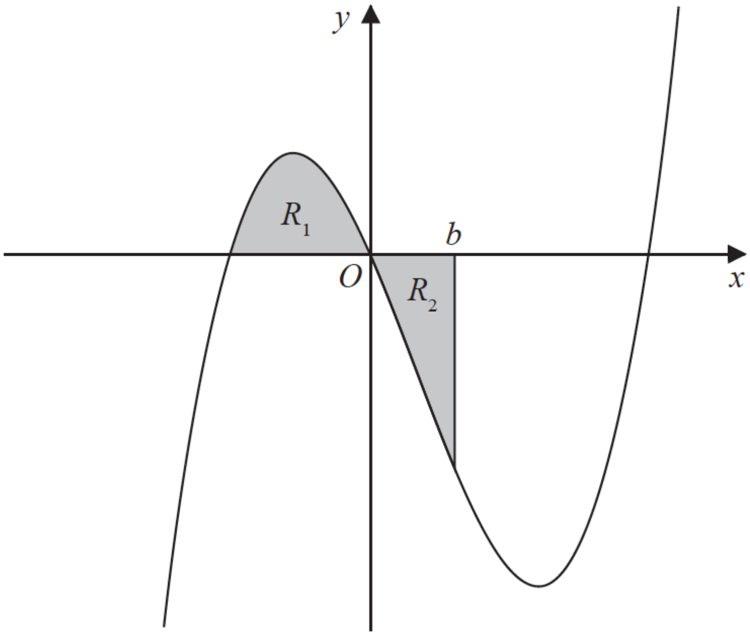

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = x(x + 2)(x - 4)$.

The region R_1 shown shaded in Figure 2 is bounded by the curve and the negative x-axis.

(*a*) Show that the exact area of R_1 is $\dfrac{20}{3}$

(4)

The region R_2 also shown shaded in Figure 2 is bounded by the curve, the positive x-axis and the line with equation $x = b$, where b is a positive constant and $0 < b < 4$

Given that the area of R_1 is equal to the area of R_2

(*b*) verify that b satisfies the equation

$$(b + 2)^2\,(3b^2 - 20b + 20) = 0$$

(4)

The roots of the equation $3b^2 - 20b + 20 = 0$ are 1.225 and 5.442 to 3 decimal places. The value of b is therefore 1.225 to 3 decimal places.

(*c*) Explain, with the aid of a diagram, the significance of the root 5.442

(2)

(Total for Question 6 is 10 marks)

7.

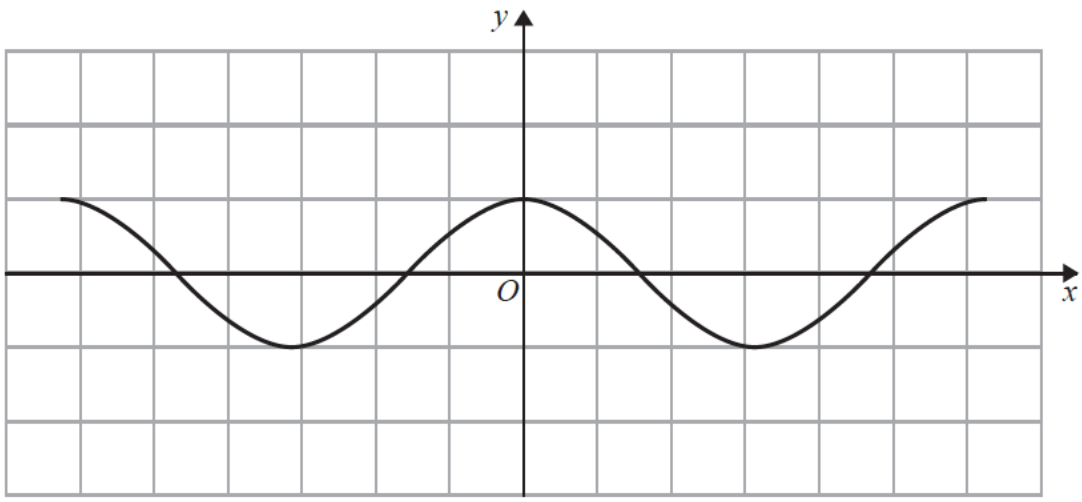

Figure 1

Figure 1 shows a plot of part of the curve with equation $y = \cos x$ where x is measured in radians.
Diagram 1, on the opposite page, is a copy of Figure 1.

(*a*) Copy and use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is α, and that α is small,

(*b*) use the small angle approximation for $\cos x$ to estimate the value of α to 3 decimal places.

(3)

(Total for Question 7 is 5 marks)

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8.

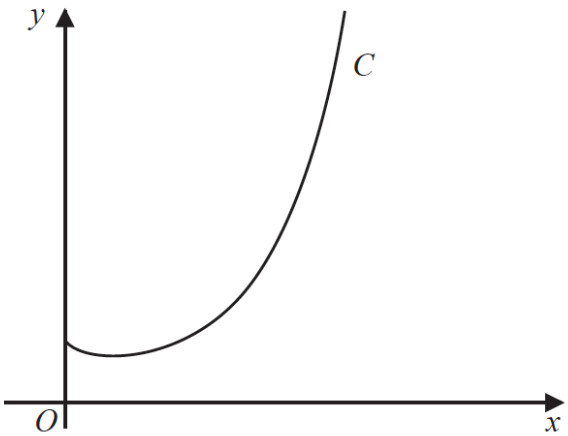

Figure 8

Figure 8 shows a sketch of the curve C with equation $y = x^x, \quad x > 0$.

(a) Find, by firstly taking logarithms, the x coordinate of the turning point of C.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(5)

The point $P(\alpha, 2)$ lies on C.

(b) Show that $1.5 < \alpha < 1.6$

(2)

A possible iteration formula that could be used in an attempt to find α is

$$x_{n+1} = 2x_n^{1-x_n}$$

Using this formula with $x_1 = 1.5$

(c) find x_4 to 3 decimal places,

(2)

(d) describe the long-term behaviour of x_n

(2)

(Total for Question 8 is 11 marks)

9.

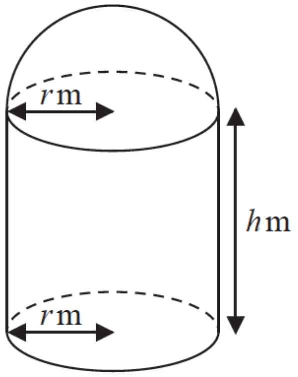

Figure 9

[A sphere of radius r has volume $\frac{4}{3}\pi r^3$ and surface area $4\pi r^2$]

A manufacturer produces a storage tank.

The tank is modelled in the shape of a hollow circular cylinder closed at one end with a hemispherical shell at the other end as shown in Figure 9.

The walls of the tank are assumed to have negligible thickness.

The cylinder has radius r metres and height h metres and the hemisphere has radius r metres.

The volume of the tank is 6 m^3.

(a) Show that, according to the model, the surface area of the tank, in m^2, is given by

$$\frac{12}{r} + \frac{5}{3}\pi r^2$$

(4)

The manufacturer needs to minimise the surface area of the tank.

(b) Use calculus to find the radius of the tank for which the surface area is a minimum.

(4)

(c) Calculate the minimum surface area of the tank, giving your answer to the nearest integer.

(2)

(Total for Question 9 is 10 marks)

10. Given

$$2^x \times 4^y = \frac{1}{2\sqrt{2}}$$

express y as a function of x.

(3)

(Total for Question 10 is 3 marks)

---

11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(*a*) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,

(2)

(*b*) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leq r \leq 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(*c*) estimate the total time, in minutes and seconds, that she will take to complete the race.

(4)

(Total for Question 11 is 7 marks)

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12. A machine puts liquid into bottles of perfume. The amount of liquid put into each bottle, D ml, follows a normal distribution with mean 25 ml

Given that 15% of bottles contain less than 24.63 ml

(a) find, to 2 decimal places, the value of k such that $P(24.63 < D < k) = 0.45$

(5)

A random sample of 200 bottles is taken.

(b) Using a normal approximation, find the probability that fewer than half of these bottles contain between 24.63 ml and k ml

(3)

The machine is adjusted so that the standard deviation of the liquid put in the bottles is now 0.16 ml

Following the adjustments, Hannah believes that the mean amount of liquid put in each bottle is less than 25 ml

She takes a random sample of 20 bottles and finds the mean amount of liquid to be 24.94 ml

(c) Test Hannah's belief at the 5% level of significance.
You should state your hypotheses clearly.

(5)

(Total for Question 12 is 13 marks)

TOTAL FOR PAPER IS 97 MARKS

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Silver Mark Scheme

1 (a)	$\dfrac{1}{\sqrt{4-x}} = (4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} \times (1 \pm ......$	M1	2.1
	Uses a "correct" binomial expansion for their $(1+ax)^n = 1 + nax + \dfrac{n(n-1)}{2}a^2x^2 +$	M1	1.1b
	$\left(1-\dfrac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\dfrac{1}{2}\right)\left(-\dfrac{x}{4}\right) + \dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\dfrac{x}{4}\right)^2$	A1	1.1b
	$\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{2} + \dfrac{1}{16}x + \dfrac{3}{256}x^2$	A1	1.1b
		(4)	
(b) (i)	States $x = -14$ and gives a valid reason. Eg explains that the expansion is not valid for $\lvert x\rvert > 4$	B1	2.4
		(1)	
(b)(ii)	States $x = -\dfrac{1}{2}$ and gives a valid reason. Eg. explains that it is closest to zero	B1	2.4
		(1)	
		(6 marks)	

(a)

M1: For the strategy of expanding $\dfrac{1}{\sqrt{4-x}}$ using the binomial expansion.

You must see $4^{-\frac{1}{2}}$ oe and an expansion which may or may not be combined.

M1: Uses a correct binomial expansion for their $(1 \pm ax)^n = 1 \pm nax \pm \dfrac{n(n-1)}{2}a^2x^2 +$

Condone sign slips and the "a" not being squared in term 3. Condone $a = \pm 1$
Look for an attempt at the correct binomial coefficient for their n, being combined with the correct power of ax

A1: $\left(1-\dfrac{x}{4}\right)^{-\frac{1}{2}} = 1 + \left(-\dfrac{1}{2}\right)\left(-\dfrac{x}{4}\right) + \dfrac{\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)}{2}\left(-\dfrac{x}{4}\right)^2$ unsimplified

FYI the simplified form is $1 + \dfrac{x}{8} + \dfrac{3x^2}{128}$ Accept the terms with commas between.

A1: $\dfrac{1}{\sqrt{4-x}} = \dfrac{1}{2} + \dfrac{1}{16}x + \dfrac{3}{256}x^2$ Ignore subsequent terms. Allow with commas between.

Note: Alternatively $(4-x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}} + \left(-\frac{1}{2}\right)4^{-\frac{3}{2}}(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}4^{-\frac{5}{2}}(-x)^2 + ..$

M1: For $4^{-\frac{1}{2}} + ...$ **M1:** As above but allow slips on the sign of x and the value of n **A1:** Correct unsimplified (as above) **A1:** As main scheme

(b) Any evaluations of the expansions are irrelevant.

Look for a suitable value and a suitable reason for both parts.

(b)(i)

B1: Requires $x = -14$ with a suitable reason.

Eg. $x = -14$ as the expansion is only valid for $|x| < 4$ or equivalent.

Eg ‘ $x = -14$ as $|-14| > 4$" or ' I cannot use $x = -14$ as $\left|\frac{-14}{4}\right| > 1$ '

Eg. ‘ $x = -14$ as is outside the range $|x| < 4$ ’

Do not allow ‘ -14 is too big’ or ‘ $x = -14, |x| < 4$ ’ either way around without some reference to the validity of the expansion.

(b)(ii)

B1: Requires $x = -\frac{1}{2}$ with a suitable reason.

Eg. $x = -\frac{1}{2}$ as it is ‘the smallest/smaller value’ or ‘ $x = -\frac{1}{2}$ as the value closest to zero’ (that will give the more accurate approximation). The bracketed statement is not required.

Question	Scheme	Marks	AOs
2 (a)	States $\log a - \log b = \log \frac{a}{b}$	B1	1.2
	Proceeds from $\frac{a}{b} = a - b \to ...... \to ab - a = b^2$	M1	1.1b
	$ab - a = b^2 \to a(b-1) = b^2 \Rightarrow a = \frac{b^2}{b-1}$ *	A1*	2.1
		(3)	
(b)	States either $b > 1$ or $b \neq 1$ with reason $\frac{b^2}{b-1}$ is not defined at $b = 1$ oe	B1	2.2a
	States $b > 1$ and explains that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$	B1	2.4
		(2)	
			(5 marks)

(a)

B1: States or uses $\log a - \log b = \log\frac{a}{b}$. This may be awarded anywhere in the question and may be implied by a **starting line** of $\frac{a}{b} = a - b$ oe. Alternatively takes $\log b$ to the rhs and uses the addition law $\log(a-b) + \log b = \log(a-b)b$. Watch out for $\log a - \log b = \frac{\log a}{\log b} = \log\left(\frac{a}{b}\right)$ which could score 010

M1: Attempts to make '*a*' the subject. Awarded for proceeding from $\frac{a}{b} = a - b$ to a point where the two terms in a are on the same side of the equation and the term in b is on the other.

A1*: CSO. Shows clear reasoning and correct mathematics leading to $a = \frac{b^2}{b-1}$. Bracketing must be correct.

Allow a candidate to proceed from $ab - a = b^2$ to $a = \frac{b^2}{b-1}$ without the intermediate line.

(b)

B1: For deducing $b \neq 1$ as $a \to \infty$ oe such as "you cannot divide by 0" or correctly deducing that $b > 1$.

They may state that b cannot be less than 1.

B1: For $b > 1$ and explaining that as $a > 0 \Rightarrow \frac{b^2}{b-1} > 0 \Rightarrow b > 1$ (as b^2 is positive)

As a minimum accept that $b > 1$ as a cannot be negative.

Note that $a > b > 1$ is a correct statement but not sufficient on its own without an explanation.

……

Alt (a)

Note that it is possible to attempt part (a) by substituting $a = \frac{b^2}{b-1}$ into both sides of the given identity.

$$\log a - \log b = \log(a-b) \Rightarrow \log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b^2}{b-1} - b\right)$$

B1: Score for $\log\left(\frac{b^2}{b-1}\right) - \log b = \log\left(\frac{b}{b-1}\right)$

M1: Attempts to write $\frac{b^2}{b-1} - b$ as a single fraction $\frac{b^2}{b-1} - b = \frac{b^2 - b(b-1)}{b-1}$

Allow as two separate fractions with the same common denominator

A1*: Achieves lhs and rhs as $\log\left(\frac{b}{b-1}\right)$ **and** makes a comment such as "hence true"

Question	Scheme	Marks	AOs
3 (a) **Way 1**	$\{d = kV^n \Rightarrow\}$ $\log_{10} d = \log_{10} k + n\log_{10} V$ or $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ seen or used as part of their argument	M1	2.1
	Alludes to $d = kV^n$ and gives a full explanation by comparing their result with a linear model e.g. $Y = MX + C$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
9 (a) **Way 2**	$\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$ or $\log_{10} d = \log_{10} k + n\log_{10} V$ seen or used as part of their argument	M1	2.1
	$\{d = kV^n \Rightarrow\}$ $\log_{10} d = \log_{10}(kV^n)$ $\Rightarrow$ $\log_{10} d = \log_{10} k + \log_{10} V^n$ $\Rightarrow$ $\log_{10} d = \log_{10} k + n\log_{10} V$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(a) **Way 3**	Starts from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$	M1	2.1
	$\log_{10} d = m\log_{10} V + c$ $\Rightarrow$ $d = 10^{m\log_{10} V + c}$ $\Rightarrow$ $d = 10^c V^m$ $\Rightarrow d = kV^n$ **or** $\log_{10} d = m\log_{10} V - 1.77$ $\Rightarrow$ $d = 10^{m\log_{10} V - 1.77}$ $\Rightarrow$ $d = 10^{-1.77} V^m$ $\Rightarrow d = kV^n$	A1	2.4
	$\{k =\}$ $10^{-1.77} = 0.017$ or $\log 0.017 = -1.77$ linked together in the same part of the question	B1 *	1.1b
		(3)	
(b)	$\{d = 20, V = 30 \Rightarrow\}$ $20 = k(30)^n$ or $\log_{10} 20 = \log_{10} k + n\log_{10} 30$	M1	3.4
	$20 = k(30)^n$ $\Rightarrow$ $\log 20 = \log k + n\log 30 \Rightarrow n = \dfrac{\log 20 - \log k}{\log 30}$ $\Rightarrow n = ...$ $\log_{10} 20 = \log_{10} k + n\log_{10} 30 \Rightarrow n = \dfrac{\log_{10} 20 - \log_{10} k}{\log_{10} 30}$ $\Rightarrow n = ...$	M1	1.1b
	$\{n = \text{awrt } 2.08 \Rightarrow\}$ $d = (0.017)V^{2.08}$ **or** $\log_{10} d = -1.77 + 2.08\log_{10} V$	A1	1.1b
	Note: You can recover the A1 mark for a correct model equation given in part (c)	**(3)**	
(c)	$d = (0.017)(60)^{2.08}$	M1	3.4
	• $13.333... + 84.918... = 98.251...$ $\Rightarrow$ Sean stops in time	M1	3.1b
	• $100 - 13.333... = 86.666...$ & $d = 84.918$ $\Rightarrow$ Sean stops in time	A1ft	3.2a
		(3)	
			(9 marks)
	ADVICE: Ignore labelling (a), (b), (c) when marking this question **Note:** Give B0 in (a) for $10^{-1.77} = 0.01698...$ without reference to 0.017 in the same part		

Notes for Question 3	
Note:	In their solution to (a) and/or (b) condone writing $\log$ in place of $\log_{10}$
(a)	**Way 1**
M1:	See scheme

A1:	See scheme
B1*:	See scheme
(a)	**Way 2**
M1:	See scheme
A1:	Starts from $d = kV^n$ (which they do not have to state) and progresses to $\log_{10} d = \log_{10} k + n\log_{10} V$ with an intermediate step in their working.
B1*:	See scheme
(a)	**Way 3**
M1:	Starts their argument from $\log_{10} d = m\log_{10} V + c$ or $\log_{10} d = m\log_{10} V - 1.77$
A1:	Mathematical explanation is seen by showing any of either • $\log_{10} d = m\log_{10} V + c \rightarrow d = 10^c V^m$ or $d = kV^n$ • $\log_{10} d = m\log_{10} V - 1.77 \rightarrow d = 10^{-1.77} V^m$ or $d = kV^n$ with no errors seen in their working
B1*:	See scheme
Note:	Allow B1 for $\log_{10} 0.017 = -1.77$ or $\log 0.017 = -1.77$
(b)	
M1:	Applies $V = 30$ and $d = 20$ to their model **(correct way round)**
M1:	Applies $(V, d) = (30, 20)$ or $(20, 30)$ and applies logarithms correctly leading to $n = ...$
A1:	$d = (0.017)V^{2.08}$ or $\log_{10} d = -1.77 + 2.08\log_{10} V$ or $\log_{10} d = \log_{10}(0.017) + 2.08\log_{10} V$
Note:	Allow $k =$ awrt 0.017 and/or $n =$ awrt 2.08 in their final model equation
Note:	M0 M1 A0 is a possible score for (b)
(c)	
M1:	Applies $V = 60$ to their exponential model or their logarithmic model
M1:	Uses their model in a correct problem-solving process of either • adding a "thinking distance" to their value of their d to find an overall stopping distance • applying 100 – "thinking distance" and finds their value of d
Note:	$\frac{1}{75}$ or 48 are examples of acceptable thinking distances
A1ft:	**Either** adds 13.3… to their d to find a total stopping distance and gives a correct ft conclusion **or** finds their d and a comparative 86.666…(m) or awrt 87 (m) and gives a correct ft conclusion
Note:	The thinking distance must be dimensionally correct for the M1 mark. i.e. $0.8\times$ their velocity
Note:	A thinking distance of awrt 13 and a value of d in the range [81.5, 88.5] are required for A1ft
Note:	Allow "Sean stops in time" or "Yes he stops in time" or "he misses the puddle" as relevant conclusions.
Note:	A mark of M0 M1 A0 is possible in (c)

Question	Scheme	Marks	AOs
4	$\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} \equiv 2\cot 2\theta$		
(a) **Way 1**	$\{\text{LHS} =\}\ \dfrac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \dfrac{\cos(3\theta - \theta)}{\sin\theta\cos\theta}\ \left\{= \dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1

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		(4)	
(a) Way 2	$\{\text{LHS} = \}\ \dfrac{\cos 2\theta \cos\theta - \sin 2\theta \sin\theta}{\sin\theta} + \dfrac{\sin 2\theta \cos\theta + \cos 2\theta \sin\theta}{\cos\theta}$		
	$= \dfrac{\cos 2\theta \cos^2\theta - \sin 2\theta \sin\theta\cos\theta + \sin 2\theta \cos\theta \sin\theta + \cos 2\theta \sin^2\theta}{\sin\theta\cos\theta}$	M1	3.1a
	$= \dfrac{\cos 2\theta(\cos^2\theta + \sin^2\theta)}{\sin\theta\cos\theta}\ \left\{= \dfrac{\cos 2\theta}{\sin\theta\cos\theta}\right\}$	A1	2.1
	$= \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *	dM1	1.1b
		A1 *	2.1
		(4)	
(a) Way 3	$\{\text{RHS} = \}\ \dfrac{2\cos 2\theta}{\sin 2\theta} = \dfrac{2\cos(3\theta - \theta)}{\sin 2\theta} = \dfrac{2(\cos 3\theta \cos\theta + \sin 3\theta \sin\theta)}{\sin 2\theta}$	M1	3.1a
		A1	2.1
	$= \dfrac{2(\cos 3\theta \cos\theta + \sin 3\theta \sin\theta)}{2\sin\theta\cos\theta}$	dM1	1.1b
	$= \dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta}$ *	A1 *	2.1
		(4)	
(b) Way 1	$\left\{\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow 2\left(\dfrac{1}{\tan 2\theta}\right) = 4$	M1	1.1b
	Rearranges to give $\tan 2\theta = k;\ k \neq 0$ and applies $\arctan k$	dM1	1.1b
	$\left\{90° < \theta < 180°,\ \tan 2\theta = \dfrac{1}{2} \Rightarrow\right\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
(b) Way 2	$\left\{\dfrac{\cos 3\theta}{\sin\theta} + \dfrac{\sin 3\theta}{\cos\theta} = 4 \Rightarrow\right\}\ 2\cot 2\theta = 4 \Rightarrow \dfrac{2}{\tan 2\theta} = 4$	M1	1.1b
	$\dfrac{2}{\left(\dfrac{2\tan\theta}{1-\tan^2\theta}\right)} = 4 \Rightarrow 2(1 - \tan^2\theta) = 8\tan\theta$ $\Rightarrow \tan^2\theta + 4\tan\theta - 1 = 0 \Rightarrow \tan\theta = \dfrac{-4 \pm \sqrt{(4)^2 - 4(1)(-1)}}{2(1)}$ $\{\Rightarrow \tan\theta = -2 \pm \sqrt{5}\} \Rightarrow \tan\theta = k;\ k \neq 0 \Rightarrow$ applies $\arctan k$	dM1	1.1b
	$\{90° < \theta < 180°,\ \tan\theta = -2 - \sqrt{5} \Rightarrow\}$		
	Only one solution of $\theta = 103.3°$ (1 dp) or awrt $103.3°$	A1	2.2a
		(3)	
			(7 marks)

Notes for Question 4	
(a)	**Way 1 and Way 2**
M1:	Correct valid method forming a common denominator of $\sin\theta\cos\theta$ i.e. correct process of $\dfrac{(...)\cos\theta + (...)\sin\theta}{\cos\theta\sin\theta}$
A1:	Proceeds to show that the numerator of their resulting fraction simplifies to $\cos(3\theta - \theta)$ or $\cos 2\theta$

dM1:	**dependent on the previous M mark** Applies a correct $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to the common denominator $\sin\theta\cos\theta$
A1*	Correct proof
Note:	Writing $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta}{\sin\theta\cos\theta} + \dfrac{\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$ is considered a correct valid method of forming a common denominator of $\sin\theta\cos\theta$ for the 1st M1 mark
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos 4\theta+\sin 4\theta}{\sin\theta\cos\theta}$
Note:	Give 1st M0 e.g. for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$ but allow 1st M1 for $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\cos^2 3\theta+\sin^2 3\theta}{\sin\theta\cos\theta}$
Note:	Allow 2nd M1 for stating a correct $\sin 2\theta = 2\sin\theta\cos\theta$ and for attempting to apply it to the common denominator $\sin\theta\cos\theta$
(a)	**Way 3**
M1:	Starts from RHS and proceeds to expand $\cos 2\theta$ in the form $\cos 3\theta\cos\theta \pm \sin 3\theta\sin\theta$
A1:	Shows, as part of their proof, that $\cos 2\theta = \cos 3\theta\cos\theta + \sin 3\theta\sin\theta$
dM1:	**dependent on the previous M mark** Applies $\sin 2\theta \equiv 2\sin\theta\cos\theta$ to their denominator
A1*:	Correct proof
Note:	Allow 1st M1 1st A1 (together) for any of LHS $\rightarrow \dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ or LHS $\rightarrow \dfrac{\cos 2\theta(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}$ or LHS $\rightarrow \cos 2\theta(\cot\theta+\tan\theta)$ or LHS $\rightarrow \cos 2\theta\left(\dfrac{1+\tan^2\theta}{\tan\theta}\right)$ (i.e. where $\cos 2\theta$ has been factorised out)
Note:	Allow 1st M1 1st A1 for progressing as far as LHS $= ... = \cot x - \tan x$
Note:	The following is a correct alternative solution $\dfrac{\cos 3\theta}{\sin\theta}+\dfrac{\sin 3\theta}{\cos\theta} = \dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta} = \dfrac{\frac{1}{2}(\cos 4\theta+\cos 2\theta)-\frac{1}{2}(\cos 4\theta-\cos 2\theta)}{\sin\theta\cos\theta}$ $= \dfrac{\cos 2\theta}{\sin\theta\cos\theta} = \dfrac{\cos 2\theta}{\frac{1}{2}\sin 2\theta} = 2\cot 2\theta$ *
Note:	E.g. going from $\dfrac{\cos 2\theta\cos^2\theta - \sin 2\theta\sin\theta\cos\theta + \sin 2\theta\cos\theta\sin\theta + \cos 2\theta\sin^2\theta}{\sin\theta\cos\theta}$ to $\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$ with no intermediate working is 1st A0

Notes for Question 4 Continued	
(b)	**Way 1**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Rearranges to give $\tan 2\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$
Note:	Give M0M0A0 for writing, for example, $\tan 2\theta = 2$ with no evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
Note:	1^{st} M1 can be implied by seeing $\tan 2\theta = \frac{1}{2}$
Note:	Condone 2^{nd} M1 for applying $\frac{1}{2}\arctan\left(\frac{1}{2}\right)$ $\{= 13.28...\}$
(b)	**Way 2**
M1:	Evidence of applying $\cot 2\theta = \frac{1}{\tan 2\theta}$
dM1:	**dependent on the previous M mark** Applies $\tan 2\theta \equiv \frac{2\tan\theta}{1-\tan^2\theta}$, forms and uses a correct method for solving a 3TQ to give $\tan\theta = k,\ k \neq 0,$ and applies $\arctan k$
A1:	Uses $90° < \theta < 180°$ to deduce the only solution $\theta =$ awrt $103.3°$
Note:	Give M1 dM1 A1 for no working leading to $\theta =$ awrt $103.3°$ and no other solutions
Note:	Give M1 dM1 A0 for no working leading to $\theta =$ awrt $103.3°$ and other solutions which can be either outside or inside the range $90° < \theta < 180°$

5 (a)	(i) **Explains** $2x - q = 0$ when $x = 2$ oe Hence $q = 4$ *	B1*	2.4
	(ii) Substitutes $\left(3, \frac{1}{2}\right)$ into $y = \frac{p-3x}{(2x-4)(x+3)}$ and solves	M1	1.1b
	$\frac{1}{2} = \frac{p-9}{(2)\times(6)} \Rightarrow p - 9 = 6 \Rightarrow p = 15$ *	A1*	2.1
		(3)	
(b)	Attempts to write $\frac{15-3x}{(2x-4)(x+3)}$ in PF's and integrates using lns between 3 and another value of x.	M1	3.1a
	$\frac{15-3x}{(2x-4)(x+3)} = \frac{A}{(2x-4)} + \frac{B}{(x+3)}$ leading to A and B	M1	1.1b
	$\frac{15-3x}{(2x-4)(x+3)} = \frac{1.8}{(2x-4)} - \frac{2.4}{(x+3)}$ or $\frac{0.9}{(x-2)} - \frac{2.4}{(x+3)}$ oe	A1	1.1b

	I = $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = m\ln(2x-4)+n\ln(x+3)+(c)$	M1	1.1b
	I = $\int \frac{15-3x}{(2x-4)(x+3)}\,dx = 0.9\ln(2x-4)-2.4\ln(x+3)$ oe	A1ft	1.1b
	Deduces that Area Either $\int_3^5 \frac{15-3x}{(2x-4)(x+3)}\,dx$ Or $[.............]_3^5$	B1	2.2a
	Uses correct ln work seen at least once for $\ln 6 = \ln 2 + \ln 3$ **or** $\ln 8 = 3\ln 2$ $[0.9\ln(6)-2.4\ln(8)]-[0.9\ln(2)-2.4\ln(6)]$ $= 3.3\ln 6 - 7.2\ln 2 - 0.9\ln 2$	dM1	2.1
	$= 3.3\ln 3 - 4.8\ln 2$	A1	1.1b
		(8)	
			(11marks)

(a)

B1*: Is able to link $2x-q=0$ and $x=2$ to explain why $q=4$

Eg "The asymptote $x=2$ is where $2x-q=0$ so $4-q=0 \Rightarrow q=4$"

"The curve is not defined when $2\times 2-q=0 \Rightarrow q=4$"

There **must be some words** explaining why $q=4$ and in most cases, you should see a reference to either "the asymptote $x=2$", "the curve is not defined at $x=2$", 'the denominator is 0 at $x=2$"

M1: Substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{p-3x}{(2x-4)(x+3)}$ and solves

Alternatively substitutes $\left(3,\frac{1}{2}\right)$ into $y=\frac{15-3x}{(2x-4)(x+3)}$ and shows $\frac{1}{2}=\frac{6}{(2)\times(6)}$ oe

A1*: Full proof showing all necessary steps $\frac{1}{2}=\frac{p-9}{(2)\times(6)} \Rightarrow p-9=6 \Rightarrow p=15$

In the alternative there would have to be some recognition that these are equal eg ✓ hence $p=15$

(b)

M1: Scored for an overall attempt at using PF's and integrating with lns seen with sight of limits 3 and another value of x.

M1: $\dfrac{15-3x}{(2x-4)(x+3)}=\dfrac{A}{(2x-4)}+\dfrac{B}{(x+3)}$ leading to A and B

A1: $\dfrac{15-3x}{(2x-4)(x+3)}=\dfrac{1.8}{(2x-4)}-\dfrac{2.4}{(x+3)}$, or for example $\dfrac{0.9}{(x-2)}-\dfrac{2.4}{(x+3)}$, $\dfrac{9}{(10x-20)}-\dfrac{12}{(5x+15)}$ oe

Must be written in PF form, not just for correct A and B

M1: Area $R=\displaystyle\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=m\ln(2x-4)+n\ln(x+3)$

OR $\displaystyle\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=m\ln(x-2)+n\ln(x+3)$

Note that $\displaystyle\int\frac{l}{(x-2)}\mathrm{d}x\to l\ln(kx-2k)$ and $\displaystyle\int\frac{m}{(x+3)}\mathrm{d}x\to m\ln(nx+3n)$

A1ft: $=\displaystyle\int\frac{15-3x}{(2x-4)(x+3)}\mathrm{d}x=0.9\ln(2x-4)-2.4\ln(x+3)$ oe. FT on their A and B

B1: Deduces that the limits for the integral are 3 and 5. It cannot just be awarded from 5 being marked on Figure 4. So award for sight of $\displaystyle\int_3^5\frac{15-3x}{(2x-4)(x+3)}(\mathrm{d}x)$ or $[.............]_3^5$ having performed an integral which may be incorrect

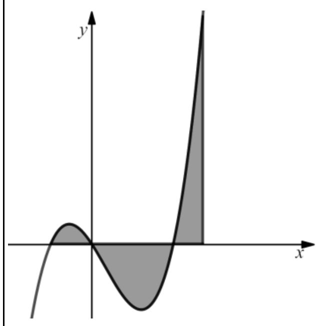
dM1: Uses correct ln work seen at least once eg $\ln 6=\ln 2+\ln 3$, $\ln 8=3\ln 2$ or $m\ln 6k-m\ln 2k=m\ln 3$

This is an attempt to get either of the above ln's in terms of ln2 and/or ln3

It is dependent upon the correct limits and having achieved $m\ln(2x-4)+n\ln(x+3)$ oe

A1: $=3.3\ln 3-4.8\ln 2$ oe

Question	Scheme	Marks	AOs
6 (a)	$y=x(x+2)(x-4)=x^3-2x^2-8x$	B1	1.1b
	$\displaystyle\int x^3-2x^2-8x\,\mathrm{d}x\to\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2$	M1	1.1b
	Attempts area using the correct strategy $\displaystyle\int_{-2}^{0}y\,\mathrm{d}x$	dM1	2.2a
	$\left[\dfrac{1}{4}x^4-\dfrac{2}{3}x^3-4x^2\right]_{-2}^{0}=(0)-\left(4-\dfrac{-16}{3}-16\right)\;=\dfrac{20}{3}$ *	A1*	2.1
		(4)	
(b)	For setting 'their' $\dfrac{1}{4}b^4-\dfrac{2}{3}b^3-4b^2=\pm\dfrac{20}{3}$	M1	1.1b
	For correctly deducing that $3b^4-8b^3-48b^2+80=0$	A1	2.2a

	Attempts to factorise $3b^4-8b^3-48b^2\pm 80=(b+2)(b+2)(3b^2...b...20)$	M1	1.1b
	Achieves $(b+2)^2(3b^2-20b+20)=0$ with no errors	A1*	2.1
		(4)	
(c)	 States that between $x=-2$ and $x=5.442$ the area above the x-axis = area below the x-axis	B1 B1	1.1b 2.4
		(2)	
			(10 marks)

(a)

B1: Expands $x(x+2)(x-4)$ to x^3-2x^2-8x (They may be in a different order)

M1: Correct attempt at integration of their cubic seen in at least two terms.

Look for an expansion to a cubic and $x^n \to x^{n+1}$ seen at least twice

dM1: For a correct strategy to find the area of R_1

It is dependent upon the previous M and requires a substitution of -2 into $\pm$ their integrated function. The limit of 0 may not be seen. Condone $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{0}=\frac{20}{3}$ oe for this mark

A1*: For a rigorous argument leading to area of $R_1=\frac{20}{3}$ For this to be awarded the integration must be correct and the limits must be the correct way around and embedded or calculated values must be seen.

Eg. Look for $-\left(4+\frac{16}{3}-16\right)$ or $-\left(\frac{1}{4}(-2)^4-\frac{2}{3}(-2)^3-4(-2)^2\right)$ oe before you see the $\frac{20}{3}$

Note: It is possible to do this integration by parts.

(b)

M1: For setting their $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=\pm\frac{20}{3}$ or $\left[\frac{1}{4}x^4-\frac{2}{3}x^3-4x^2\right]_{-2}^{b}=0$

A1: Deduces that $3b^4-8b^3-48b^2+80=0$. Terms may be in a different order but expect integer coefficients. It must have followed $\frac{1}{4}b^4-\frac{2}{3}b^3-4b^2=-\frac{20}{3}$ oe.

Do not award this mark for $\frac{1}{4}b^4 - \frac{2}{3}b^3 - 4b^2 + \frac{20}{3} = 0$ unless they attempt the second part of this question by expansion and then divide the resulting expanded expression by 12

M1: Attempts to factorise $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)(b+2)(3b^2 ... b ... 20)$ via repeated division or inspection. FYI $3b^4 - 8b^3 - 48b^2 + 80 = (b+2)(3b^3 - 14b^2 - 20b + 40)$ Allow an attempt via inspection $3b^4 - 8b^3 - 48b^2 \pm 80 = (b^2 + 4b + 4)(3b^2 ... b ... 20)$ but do not allow candidates to just write out $3b^4 - 8b^3 - 48b^2 \pm 80 = (b+2)^2(3b^2 - 20b + 20)$ which is really just copying out the given answer.

Alternatively attempts to expand $(b+2)^2(3b^2 - 20b + 20)$ achieving terms of a quartic expression

A1*: Correctly reaches $(b+2)^2(3b^2 - 20b + 20) = 0$ with no errors and must have $= 0$

In the alternative obtains both equations in the same form **and states that they are same**. Allow ✓ QED etc here.

(c) Please watch for candidates who answer this on Figure 2 which is fine

B1: Sketches the curve and a vertical line to the right of 4 ($x = 5.442$ may not be labelled.)

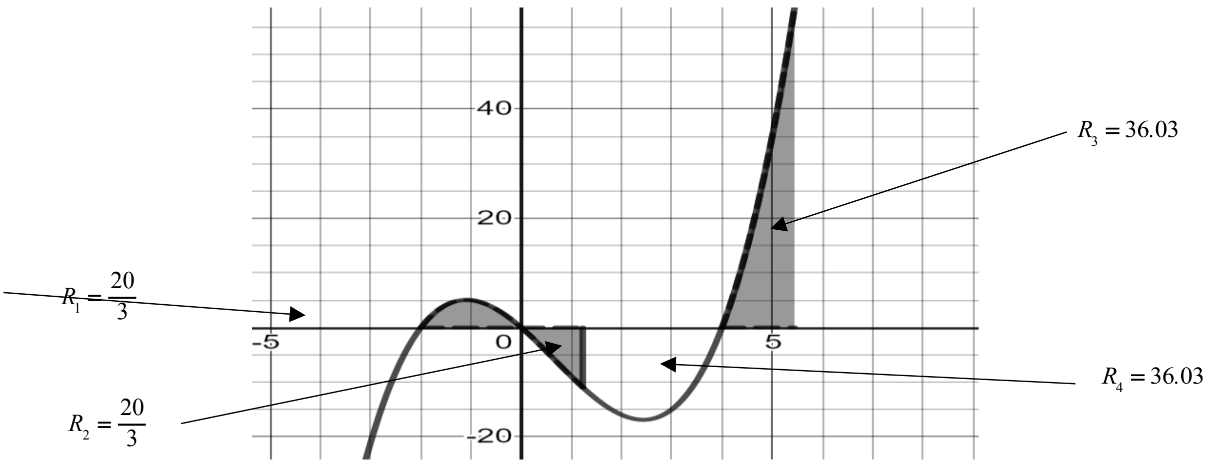
B1: Explains that (between $x = -2$ and $x = 5.442$) the area above the x-axis = area below the x -axis with appropriate areas shaded or labelled**.**

Alternatively states that the area between 1.225 and 4 is the same as the area between 4 and 5.442

Another correct statement is that the net area between 0 and 5.442 is $-\frac{20}{3}$

Look carefully at what is written. There are many correct statements/ deductions.

Eg. " (area between 0 and 4) - (area between 4 and 5.442) = 20/3". Diagram below for your information.



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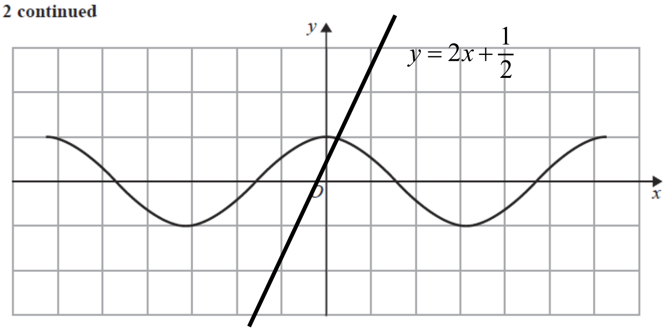
Question	Scheme	Marks	AOs
7(a)	2 continued  Diagram 1	B1	3.1a
	For an allowable linear graph and explaining that there is only one intersection	B1	2.4
		(2)	
(b)	$\cos x - 2x - \frac{1}{2} = 0 \Rightarrow 1 - \frac{x^2}{2} - 2x - \frac{1}{2} = 0$	M1	1.1b
	Solves their $x^2 + 4x - 1 = 0$	dM1	1.1b
	Allow awrt 0.236 but accept $-2+\sqrt{5}$	A1	1.1b
		(3)	
			(5 marks)

(

B1: Draws $y = 2x + \frac{1}{2}$ on Figure 1 or Diagram 1 with an attempt at the correct gradient and the correct intercept. Look for a straight line with an intercept at $\approx \frac{1}{2}$ and a further point at $\approx \left(\frac{1}{2}, 1\frac{1}{2}\right)$

Allow a tolerance of 0.25 of a square in either direction on these two points. It must appear in quadrants 1, 2 and 3.

B1: There must be an allowable linear graph on Figure 1 or Diagram1 for this to be awarded
Explains that as there is only one intersection so there is just one root.
This requires a reason and a minimal conclusion.
The question asks candidates to explain but as a bare minimum allow one "intersection"

Note: An allowable linear graph is one with intercept of $\pm\frac{1}{2}$ with one intersection with $\cos x$ **OR** gradient of ± 2 with one intersection with $\cos x$

(b)

M1: Attempts to use the small angle approximation $\cos x = 1 - \frac{x^2}{2}$ in the given equation.
The equation must be in a single variable but may be recovered later in the question.

dM1: Proceeds to a 3TQ in a single variable and attempts to solve. See General Principles
The previous M must have been scored. Allow completion of square or formula or calculator.
Do not allow attempts via factorisation unless their equation does factorise. You may have to use your calculator to check if a calculator is used.

A1: Allow $-2+\sqrt{5}$ or awrt 0.236.

Do not allow this where there is another root given and it is not obvious that 0.236 has been chosen.

Question	Scheme	Marks	AOs
8 (a) **Way 1**	$\{y = x^x \Rightarrow\}$ $\ln y = x \ln x$	B1	1.1a
	$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x$	M1	1.1b
		A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}$ $\frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(a) **Way 2**	$\{y = x^x \Rightarrow\}$ $y = \mathrm{e}^{x \ln x}$	B1	1.1a
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{x}{x} + \ln x\right)\mathrm{e}^{x \ln x}$	M1	1.1b
		A1	2.1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}$ $\frac{x}{x} + \ln x = 0$ or $1 + \ln x = 0 \Rightarrow \ln x = k \Rightarrow x = ...$	M1	1.1b
	$x = \mathrm{e}^{-1}$ or awrt 0.368	A1	1.1b
	Note: $k \neq 0$	**(5)**	
(b) **Way 1**	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$1.8... < 2$ and $2.1... > 2$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
(c)	Attempts $x_{n+1} = 2x_n^{\,1-x_n}$ at least once with $x_1 = 1.5$ Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63	M1	1.1b
	$\{x_4 = 1.67313... \Rightarrow\}$ $x_4 = 1.673$ (3 dp) **cao**	A1	1.1b
		(2)	
(d)	Give 1st B1 for any of • oscillates • periodic • non-convergent • divergent • fluctuates • goes up and down • 1, 2, 1, 2, 1, 2 • alternates (condone) Give B1 B1 for any of • periodic {sequence} with period 2 • oscillates between 1 and 2	B1	2.5
	Condone B1 B1 for any of • fluctuates between 1 and 2 • keep getting 1, 2 • alternates between 1 and 2 • goes up and down between 1 and 2 • 1, 2, 1, 2, 1, 2, …	B1	2.5
		(2)	
			(11 marks)

Note	**A common solution**
	A maximum of 3 marks (i.e. B1 1st M1 and 2nd M1) can be given for the solution $\log y = x \log x \Rightarrow \frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \log x$ $\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}$ $1 + \log x = 0 \Rightarrow x = 10^{-1}$

	• 1st B1 for $\log y = x\log x$ • 1st M1 for $\log y \to \lambda \frac{1}{y}\frac{dy}{dx}$; $\lambda \neq 0$ or $x\log x \to 1 + \log x$ or $\frac{x}{x} + \log x$ • 2nd M1 can be given for $1 + \log x = 0 \Rightarrow \log x = k \Rightarrow x = ...$; $k \neq 0$

Question	Scheme	Marks	AOs
8 (b) Way 2	For $x^x - 2$, attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$-0.16... < 0$ and $0.12... > 0$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
8 (b) Way 3	For $\ln y = x\ln x$, attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp	M1	1.1b
	$0.608... < 0.69...$ and $0.752... > 0.69...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	
8 (b) Way 4	For $\log y = x\log x$, attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp	M1	1.1b
	$0.264... < 0.301...$ and $0.326... > 0.301...$ and as C is continuous then $1.5 < \alpha < 1.6$	A1	2.1
		(2)	

Notes for Question 8

(a)	**Way 1**
B1:	$\ln y = x\ln x$. Condone $\log_x y = x\log_x x$ or $\log_x y = x$
M1:	For either $\ln y \to \frac{1}{y}\frac{dy}{dx}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{1}{y}\frac{dy}{dx} = 1 + \ln x$ or $\frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \ln x$ or $\frac{dy}{dx} = y(1 + \ln x)$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give no marks for no working leading to 0.368
Note:	Give M0 A0 M0 A0 for $\ln y = x\ln x \to x = 0.368$ with no intermediate working
(a)	**Way 2**
B1:	$y = e^{x\ln x}$
M1:	For either $y = e^{x\ln x} \Rightarrow \frac{dy}{dx} = f(\ln x)e^{x\ln x}$ or $x\ln x \to 1 + \ln x$ or $\frac{x}{x} + \ln x$
A1:	Correct differentiated equation. i.e. $\frac{dy}{dx} = \left(\frac{x}{x} + \ln x\right)e^{x\ln x}$ or $\frac{dy}{dx} = (1 + \ln x)e^{x\ln x}$ or $\frac{dy}{dx} = x^x(1 + \ln x)$
M1:	Sets $1 + \ln x = 0$ and rearranges to make $\ln x = k \Rightarrow x = ...$; k is a constant and $k \neq 0$
A1:	$x = e^{-1}$ or awrt 0.368 only (with no other solutions for x)
Note:	Give B1 M1 A0 M1 A1 for the following solution:

	$\{y = x^x \Rightarrow\}$ $\ln y = x\ln x \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \ln x \Rightarrow 1 + \ln x = 0 \Rightarrow x = \mathrm{e}^{-1}$ or awrt 0.368
Notes for Question 8 Continued	
(b)	**Way 1**
M1:	Attempts both $1.5^{1.5} = 1.8...$ and $1.6^{1.6} = 2.1...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} =$ awrt $1.8...$ and $1.6^{1.6} =$ awrt $2.1...$, reason (e.g. $1.8... < 2$ and $2.1... > 2$ or states C cuts through $y = 2$), C continuous and conclusion
(b)	**Way 2**
M1:	Attempts both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5^{1.5} - 2 = -0.16...$ and $1.6^{1.6} - 2 = 0.12...$ correct to awrt 1 dp, reason (e.g. $-0.16... < 0$ and $0.12... > 0$, sign change or states C cuts through $y = 0$), C continuous and conclusion
(b)	**Way 3**
M1:	Attempts both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ and at least one result is correct to awrt 1 dp
A1:	Both $1.5\ln 1.5 = 0.608...$ and $1.6\ln 1.6 = 0.752...$ correct to awrt 1 dp, reason (e.g. $0.608... < 0.69...$ and $0.752... > 0.69...$ or states they are either side of $\ln 2$), C continuous and conclusion.
(b)	**Way 4**
M1:	Attempts both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ and at least one result is correct to awrt 2 dp
A1:	Both $1.5\log 1.5 = 0.264...$ and $1.6\log 1.6 = 0.326...$ correct to awrt 2 dp, reason (e.g. $0.264... < 0.301...$ and $0.326... > 0.301...$ or states they are either side of $\log 2$), C continuous and conclusion.
(c)	
M1:	An attempt to use the given or their formula once. Can be implied by $2(1.5)^{1-1.5}$ or awrt 1.63
A1:	States $x_4 = 1.673$ **cao** (to 3 dp)
Note:	Give M1 A1 for stating $x_4 = 1.673$
Note:	M1 can be implied by stating their final answer $x_4 =$ awrt 1.673
Note:	$x_2 = 1.63299...$, $x_3 = 1.46626...$, $x_4 = 1.67313...$
(d)	
B1:	see scheme
B1:	see scheme
Note:	Only marks of B1B0 or B1B1 are possible in (d)
Note:	Give B0 B0 for "Converges in a cob-web pattern" or "Converges up and down to α"

Question	Scheme	Marks	AOs
9 (a)	States or uses $6 = \pi r^2 h + \frac{2}{3}\pi r^3$	B1	1.1a
	$\Rightarrow h = \frac{6}{\pi r^2} - \frac{2}{3}r,\ \pi h = \frac{6}{r^2} - \frac{2}{3}\pi r,\ \pi rh = \frac{6}{r} - \frac{2}{3}\pi r^2,\ rh = \frac{6}{\pi r} - \frac{2}{3}r^2$		
	$A = \pi r^2 + 2\pi rh + 2\pi r^2\ \{\Rightarrow A = 3\pi r^2 + 2\pi rh\}$		
	$A = 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$	M1	3.1a
		A1	1.1b

	$A = 3\pi r^2 + \frac{12}{r} - \frac{4}{3}\pi r^2 \;\Rightarrow\; A = \frac{12}{r} + \frac{5}{3}\pi r^2$ *	A1*	2.1
		(4)	
(b)	$\left\{A = 12r^{-1} + \frac{5}{3}\pi r^2 \Rightarrow\right\}\; \frac{\mathrm{d}A}{\mathrm{d}r} = -12r^{-2} + \frac{10}{3}\pi r$	M1 A1	3.4 1.1b
	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r} = 0 \Rightarrow\right\}\; -\frac{12}{r^2} + \frac{10}{3}\pi r = 0 \Rightarrow -36 + 10\pi r^3 = 0 \Rightarrow r^{\pm 3} = ...\left\{= \frac{18}{5\pi}\right\}$	M1	2.1
	$r = 1.046447736... \Rightarrow r =$ 1.05 (m) (3 sf) or awrt 1.05 (m)	A1	1.1b
	Note: Give final A1 for correct exact values for r	**(4)**	
(c)	$A_{\min} = \frac{12}{(1.046...)} + \frac{5}{3}\pi(1.046...)^2$	M1	3.4
	$\{A_{\min} = 17.20... \Rightarrow\}\; A = 17\ (\mathrm{m}^2)$ or $A =$ awrt 17 (m^2)	A1ft	1.1b
		(2)	
			(10 marks)

Notes for Question 9

(a)	
B1:	See scheme
M1:	Complete process of substituting their $h = ...$ or $\pi h = ...$ or $\pi rh = ...$ or $rh = ...$, where $'...' = \mathrm{f}(r)$ into an expression for the surface area which is of the form $A = \lambda\pi r^2 + \mu\pi rh$; $\lambda, \mu \neq 0$
A1:	Obtains correct simplified or un-simplified $\{A =\}\; 2\pi r^2 + 2\pi r\left(\frac{6}{\pi r^2} - \frac{2}{3}r\right) + \pi r^2$
A1*:	Proceeds, using rigorous and careful reasoning, to $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Condone the lack of $A = ...$ or $S = ...$ for any one of the A marks or for both of the A marks
(b)	
M1:	Uses the model (or their model) and differentiates $\frac{\lambda}{r} + \mu r^2$ to give $\alpha r^{-2} + \beta r$; $\lambda, \mu, \alpha, \beta \neq 0$
A1:	$\left\{\frac{\mathrm{d}A}{\mathrm{d}r} =\right\}\; -12r^{-2} + \frac{10}{3}\pi r$ o.e.
M1:	Sets their $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ and rearranges to give $r^{\pm 3} = k,\ k \neq 0$ (**Note:** k can be positive or negative)
Note:	This mark can be implied. Give M1 (and A1) for $-36 + 10\pi r^3 = 0 \rightarrow r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$
A1:	$r =$ awrt 1.05 (ignoring units) or $r =$ awrt 105 cm
Note:	Give M0 A0 M0 A0 where $r =$ 1.05 (m) (3 sf) or awrt 1.05 (m) is found from no working.
Note:	Give final A1 for correct exact values for r. E.g. $r = \left(\frac{18}{5\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{36}{10\pi}\right)^{\frac{1}{3}}$ or $r = \left(\frac{3.6}{\pi}\right)^{\frac{1}{3}}$

Notes for Question 9 Continued	
Note:	Give final M0 A0 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464$
Note:	Give final M1 A1 for $-\frac{12}{r^2}+\frac{10}{3}\pi r > 0 \Rightarrow r > 1.0464... \Rightarrow r = 1.0464...$
(c)	
M1:	Substitutes their $r = 1.046...$, found from solving $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ in part (b), into the model with equation $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
Note:	Give M0 for substituting their r which has been found from solving $\frac{\mathrm{d}^2A}{\mathrm{d}r^2} = 0$ or from using $\frac{\mathrm{d}^2A}{\mathrm{d}r^2}$ into the model with equation $A = \frac{12}{r} + \frac{5}{3}\pi r^2$
A1ft:	$\{A =\}$ 17 or $\{A =\}$ awrt 17 (ignoring units)
Note:	You can only follow through on values of r for $0.6 \le$ their $r \le 1.3$ (and where their r has been found from solving $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ in part (b))
Note:	Give M1 A1 for $A = 17$ (m^2) or $A =$ awrt 17 (m^2) from no working

r	A	A (nearest integer)
0.6	21.88495…	awrt 22
0.7	19.70849…	awrt 20
0.8	18.35103…	awrt 18
0.9	17.57448…	awrt 18
1.0	17.23598…	awrt 17
1.1	17.24463…	awrt 17
1.2	17.53982…	awrt 18
1.3	18.07958…	awrt 18
1.05	17.20124…	awrt 17
1.04644…	17.20105…	awrt 17

Question	**Scheme**	**Marks**	**AOs**
10	$2^x \times 4^y = \frac{1}{2\sqrt{2}} \left\{= \frac{\sqrt{2}}{4}\right\}$		
Special Case	If 0 marks are scored on application of the mark scheme then allow Special Case B1 M0 A0 (total of 1 mark) for any of • $2^x \times 4^y \to 2^{x+2y}$ • $2^x \times 4^y \to 4^{\frac{1}{2}x+y}$ • $\frac{1}{2^x 2\sqrt{2}} \to 2^{-x-\frac{3}{2}}$ • $\log 2^x + \log 4^y \to x\log 2 + y\log 4$ or $x\log 2 + 2y\log 2$ • $\ln 2^x + \ln 4^y \to x\ln 2 + y\ln 4$ or $x\ln 2 + 2y\ln 2$		

	• $y = \log\left(\dfrac{1}{2^x\, 2\sqrt{2}}\right)$ o.e. {base of 4 omitted}		
Way 1	$2^x \times 2^{2y} = 2^{-\frac{3}{2}}$	B1	1.1b
	$2^{x+2y} = 2^{-\frac{3}{2}} \Rightarrow x + 2y = -\dfrac{3}{2} \Rightarrow y = \ldots$	M1	2.1
	E.g. $y = -\dfrac{1}{2}x - \dfrac{3}{4}$ or $y = -\dfrac{1}{4}(2x+3)$	A1	1.1b
		(3)	
Way 2	$\log(2^x \times 4^y) = \log\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\dfrac{1}{2\sqrt{2}}\right)$ $\Rightarrow x\log 2 + y\log 4 = \log 1 - \log(2\sqrt{2}) \Rightarrow y = \ldots$	M1	2.1
	$y = \dfrac{-\log(2\sqrt{2}) - x\log 2}{\log 4}$ $\left\{\Rightarrow y = -\dfrac{1}{2}x - \dfrac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 3	$\log(2^x \times 4^y) = \log\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log 2^x + \log 4^y = \log\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow \log 2^x + y\log 4 = \log\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow y = \ldots$	M1	2.1
	$y = \dfrac{\log\left(\dfrac{1}{2\sqrt{2}}\right) - \log(2^x)}{\log 4}$ $\left\{\Rightarrow y = -\dfrac{1}{2}x - \dfrac{3}{4}\right\}$	A1	1.1b
		(3)	
Way 4	$\log_2(2^x \times 4^y) = \log_2\left(\dfrac{1}{2\sqrt{2}}\right)$	B1	1.1b
	$\log_2 2^x + \log_2 4^y = \log_2\left(\dfrac{1}{2\sqrt{2}}\right) \Rightarrow x + 2y = -\dfrac{3}{2} \Rightarrow y = \ldots$	M1	2.1
	E.g. $y = -\dfrac{1}{2}x - \dfrac{3}{4}$ or $y = -\dfrac{1}{4}(2x+3)$	A1	1.1b
		(3)	
			(3 marks)

Question	Scheme	Marks	AOs
Way 5	$4^{\frac{1}{2}x} \times 4^y = 4^{-\frac{3}{4}}$	B1	1.1b
	$4^{\frac{1}{2}x+y} = 4^{-\frac{3}{4}} \Rightarrow \dfrac{1}{2}x + y = -\dfrac{3}{4} \Rightarrow y = \ldots$	M1	2.1

	E.g. $y=-\frac{1}{2}x-\frac{3}{4}$ or $y=-\frac{1}{4}(2x+3)$	A1	1.1b
		(3)	

Notes for Question 10	
	Way 1
B1:	Writes a correct equation in powers of 2 only
M1:	Complete process of writing a correct equation in powers of 2 only and using correct index laws to obtain y written as a function of x.
A1:	$y=-\frac{1}{2}x-\frac{3}{4}$ o.e.
	Way 2, Way 3 and Way 4
B1:	Writes a correct equation involving logarithms
M1:	Complete process of writing a correct equation involving logarithms and using correct log laws to obtain y written as a function of x.
A1:	$y=\frac{-\log(2\sqrt{2})-x\log 2}{\log 4}$ or $y=\frac{-\ln(2\sqrt{2})-x\ln 2}{\ln 4}$ or $y=\frac{\log\left(\frac{1}{2\sqrt{2}}\right)-\log(2^x)}{\log 4}$ or $y=-\frac{1}{2}x-\frac{3}{4}$ or $y=-\frac{1}{4}(2x+3)$ o.e.
	Way 5
B1:	Writes a correct equation in powers of 4 only
M1:	Complete process of writing a correct equation in powers of 4 only and using correct index laws to obtain y written as a function of x.
A1:	$y=-\frac{1}{2}x-\frac{3}{4}$ o.e.
Note:	Allow equivalent results for A1 where y is written as a function of x
Note:	You can ignore subsequent working following on from a correct answer.
Note:	Allow B1 for $2^x\times 4^y=\frac{1}{2\sqrt{2}} \Rightarrow 4^y=\frac{1}{2^x2\sqrt{2}} \Rightarrow \log_4(4^y)=\log_4\left(\frac{1}{2^x2\sqrt{2}}\right)$ followed by M1 A1 for $y=\log_4\left(\frac{1}{2^x\,2\sqrt{2}}\right)$ or $y=\log_4\left(\frac{2^{-x}}{2\sqrt{2}}\right)$ or $y=\log_4\left(\frac{\sqrt{2}}{4(2^x)}\right)$ or $y=-\log_4\left(2^{x+\frac{3}{2}}\right)$ or $y=-\log_4(\sqrt{2}\,(2^{x+1}))$

Question	Scheme	Marks	AOs
11 (a)	Total time for 6 km = 24 minutes + $6\times1.05+6\times1.05^2$ minutes	M1	3.4
	$=36.915$ minutes $=36$ minutes 55 seconds *	A1*	1.1b
		(2)	
(b)	**5** th km is $6\times1.05=6\times1.05^1$ **6**th km is $6\times1.05\times1.05=6\times1.05^2$ **7**th km is $6\times1.05\times1.05\times1.05=6\times1.05^3$	B1	3.4

	Hence the time for the r^{th} km is $6\times1.05^{r-4}$		
		(1)	
(c)	Attempts the total time for the race = Eg. 24 minutes + $\sum_{r=5}^{r=20} 6\times1.05^{r-4}$ minutes	M1	3.1a
	Uses the series formula to find an allowable sum Eg. Time for 5 th to 20th km $=\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}=(149.04)$	M1	3.4
	Correct calculation that leads to the **total time** Eg. Total time = 24 + $\dfrac{6.3\left(1.05^{16}-1\right)}{1.05-1}$	A1	1.1b
	Total time = awrt 173 minutes and 3 seconds	A1	1.1b
		(4)	
			(7 marks)

(a)

M1: For using model to calculate the total time.

Sight of 24 minutes + $6\times1.05+6\times1.05^2$ or equivalent is required. Eg 24 + 6.3 + 6.615

Alternatively in seconds 24 minutes + 378 sec (6min 18 s) +396.9 (6 min 37 s)

A1*: 36 minutes 55 seconds following 36.915, 24+ 6.3+6.615 , 24 + $6\times1.05+6\times1.05^2$ or equivalent working in seconds

(b) Must be seen in (b)

B1: As seen in scheme. For making the link between the r th km and the index of 1.05 Or for EXPLAINING that "the time taken per km (6 mins) only starts to increase by 5% after the first 4 km"

(c) The correct sum formula $\dfrac{a\left(r^n-1\right)}{r-1}$, if seen, must be correct in part (c) for all relevant marks

M1: For the overall strategy of finding the total time.

Award for adding 18, 24, 30.3 or awrt 36.9 and the sum of a geometric sequence

So award the mark for expressions such as $6\times4+\sum6\times1.05^n$ or $24+\dfrac{6\left(1.05^{20}-1\right)}{1.05-1}$

The geometric sequence formula, must be used with $r=1.05$ oe but condone slips on a and n

M1: For an attempt at using a correct sum formula for a GP to find an allowable sum

The value of r must be 1.05 oe such as 105% but you should allow a slip on the value of n used for their value of a (See below: We are going to allow the correct value of n or one less)

If you don't see a calculation it may be implied by sight of one of the values seen below

Allow for $a=6, \quad n=17$ or 16 Eg. $\frac{6\left(1.05^{17}-1\right)}{1.05-1}=(155.0)$ or

$\frac{6\left(1.05^{16}-1\right)}{1.05-1}=(141.9)$

Allow for $a=6.3, n=16$ or 15 Eg $\frac{6.3\left(1.05^{16}-1\right)}{1.05-1}=(149.0)$ or

$\frac{6.3\left(1.05^{15}-1\right)}{1.05-1}=(135.9)$

Allow for $a=6.615, \; n=15$ or 14 Eg $\frac{6.615\left(1.05^{15}-1\right)}{1.05-1}=(142.7)$ or

$\frac{6.615\left(1.05^{14}-1\right)}{1.05-1}=(129.6)$

A1: For a correct calculation that will find the **total time**. It does not need to be processed

Allow for example, amongst others, $24+\frac{6.3\left(1.05^{16}-1\right)}{1.05-1}$, $18+\frac{6\left(1.05^{17}-1\right)}{1.05-1}$,

$30.3+\frac{6.615\left(1.05^{15}-1\right)}{1.05-1}$

A1: For a total time of awrt 173 minutes and 3 seconds

This answer alone can be awarded 4 marks as long as there is some evidence of where it has come from.

....

Candidates that list values: Handy Table for Guidance

Km	Time per km	Total Time
1	6.0000	
2	6.0000	12
3	6.0000	18
4	6.0000	24
5	6.3000	30.3
6	6.6150	36.915
7	6.9458	43.86075
8	7.2930	51.15379
9	7.6577	58.81148
10	8.0406	66.85205
11	8.4426	75.29465
12	8.8647	84.15939
13	9.3080	93.46736
14	9.7734	103.2407
15	10.2620	113.5028
16	10.7751	124.2779
17	11.3139	135.5918

18	11.8796	147.4714
19	12.4736	159.945
20	13.0972	173.0422

M1: For a correct overall strategy which would involve adding four sixes followed by at least 16 other values

The values which may be written in the form 6×1.05^2 or as numbers

Can be implied by $6+6+6+6+(6\times1.05)+.....+(6\times1.05^{16})$

M1: For an attempt to add the numbers from (6×1.05) to (6×1.05^{16}). This could be done on a calculator in which case

expect to see awrt 149

Alternatively, if written out, look for 16 values with 8 correct or follow through correct to 1 dp

A1: Awrt 173 minutes

A1: Awrt 173 minutes and 3 seconds

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Question	Scheme
12(a)	$\frac{24.63-25}{'\sigma'}=-1.0364$
	$[\sigma=]0.357$ (must come from compatible signs)
	P($D > k$) = 0.4 or P($D < k$) = 0.6
	$\frac{k-25}{'0.357'}=0.2533$
	k = awrt **<u>25.09</u>**
(b)	[Y ~ B(200, 0.45) →] W ~ N(90, 49.5)
	$\mathrm{P}(Y<100)\approx\mathrm{P}(W<99.5)\left[=\mathrm{P}\left(Z<\frac{99.5-90}{\sqrt{49.5}}\right)\right]$
	= 0.9115… awrt **<u>0.912</u>**
(c)	$\mathrm{H}_0:\mu=25 \qquad \mathrm{H}_1:\mu<25$
	$[\bar{D}\sim]\mathrm{N}\left(25,\frac{0.16^2}{20}\right)$
	$\mathrm{P}(\bar{D}<24.94)[=\mathrm{P}(Z<-1.677...)]=0.046766...$
	p = 0.047 < 0.05 <u>or</u> z = −1.677… < −1.6449 <u>or</u> 24.94 < 24.94115… <u>or</u> reject H_0 /in the critical region/significant
	There is sufficient evidence to support <u>Hannah's</u> <u>belief</u>.
	(13 marks)

Notes	
(a)	**M1:** for standardising 24.63, 25 and 'σ' (ignore label) and setting = to z where 1 < |z| < 2 **A1:** [σ =] awrt 0.36. Do **not** award this mark if signs are not compatible. **B1:** for either correct probability statement (may be implied by correct answer) this mark may be scored for a correct region shown on a diagram **M1:** for a correct expression with z = awrt 0.253 (may be implied by correct answer)

	A1: awrt 25.09 (Correct answer with no incorrect working scores 5 out of 5)
(b)	**B1:** setting up normal distribution approximation of binomial N(90, 49.5) (may be implied by a correct answer) Look out for e.g. $\sigma = \frac{3\sqrt{22}}{2}$ or $\sigma = \text{awrt } 7.04$ **M1:** attempting a probability using a continuity correction i.e. P(W < 100.5), P(W < 99.5) or P(W < 98.5) condone $\leq$ (The continuity correction may be seen in a standardisation). **A1:** awrt 0.912 [Note: 0.911299… from binomial scores 0 out of 3]
(c)	**B1:** for both hypotheses in terms of μ **M1:** selecting suitable model must see N(ormal), mean 25, sd = $\frac{0.16}{\sqrt{20}}$ (o.e.) or var = $\frac{4}{3125}$ (o.e.) Condone N(25, $\frac{0.16}{\sqrt{20}}$) if $\frac{0.16}{\sqrt{20}}$ then used as s.d. **A1:** p value = awrt 0.047 <u>or</u> **test statistic** awrt −1.68 <u>or</u> **CV** awrt 24.941 (any of these values imply the M1 provided they do not come from Normal mean = 24.94) **M1:** a correct comparison (including compatible signs) **or** correct non-contextual conclusion (f.t. their p value, test statistic or critical value in the comparison) M1 may be implied by a correct contextual statement **NB** Any contradictory non contextual statements/comparisons score M0A0 e.g. 'p < 0.05, not significant' **A1:** correct conclusion in context mentioning <u>Hannah's belief</u> <u>or</u> the mean <u>amount</u>/<u>liquid</u> in each bottle is now <u>less than 25</u>ml (**dep on M1A1M1**)

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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Extend

Paper Reference **9MA0**

Mathematics

Advanced Paper

2019 Gold

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

__/89

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. A particle, P, moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})$ m s^{-2}

At time $t = 0$, the particle is at the point A and is moving with velocity $(-\mathbf{i} + 4\mathbf{j})$ m s^{-1}

At time $t = T$ seconds, P is moving in the direction of vector $(3\mathbf{i} - 4\mathbf{j})$

(*a*) Find the value of T.

(4)

At time $t = 4$ seconds, P is at the point B.

(*b*) Find the distance AB.

(4)

(Total for Question 1 is 8 marks)

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2. $$f(x) = 10e^{-0.25x} \sin x, \ x \geq 0$$

(*a*) Show that the x coordinates of the turning points of the curve with equation $y = f(x)$ satisfy the equation $\tan x = 4$

(4)

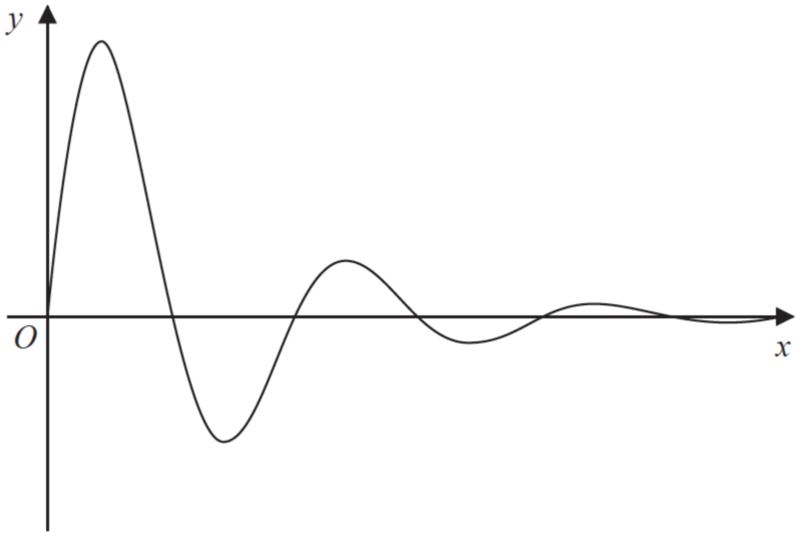

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$.

(*b*) Sketch the graph of H against t where

$$H(t) = |10e^{-0.25x} \sin t| \qquad t \geq 0$$

showing the long-term behaviour of this curve.

(2)

The function $H(t)$ is used to model the height, in metres, of a ball above the ground t seconds after it has been kicked.

Using this model, find

(*c*) the maximum height of the ball above the ground between the first and second bounce.

(3)

(*d*) Explain why this model should not be used to predict the time of each bounce.

(1)

(Total for Question 2 is 10 marks)

3.

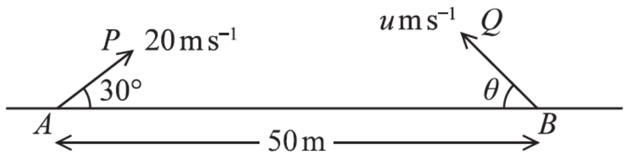

Figure 3

The points A and B lie 50 m apart on horizontal ground.

At time $t = 0$ two small balls, P and Q, are projected in the vertical plane containing AB.

Ball P is projected from A with speed 20 m s^{-1} at 30° to AB.

Ball Q is projected from B with speed u m s^{-1} at angle θ to BA, as shown in Figure 3.

At time $t = 2$ seconds, P and Q collide.

Until they collide, the balls are modelled as particles moving freely under gravity.

(a) Find the velocity of P at the instant before it collides with Q.

(6)

(b) Find

(i) the size of angle θ,

(ii) the value of u.

(6)

(c) State one limitation of the model, other than air resistance, that could affect the accuracy of your answers.

(1)

(Total for Question 3 is 13 marks)

4. The speed of a small jet aircraft was measured every 5 seconds, starting from the time it turned onto a runway, until the time when it left the ground.

The results are given in the table below with the time in seconds and the speed in m s^{-1}.

Time (s)	0	5	10	15	20	25
Speed (m s^{-1})	2	5	10	18	28	42

Using all of this information,

(*a*) estimate the length of runway used by the jet to take off.

(3)

Given that the jet accelerated smoothly in these 25 seconds,

(*b*) explain whether your answer to part (*a*) is an underestimate or an overestimate of the length of runway used by the jet to take off.

(1)

(Total for Question 4 is 4 marks)

---

5. (i) Find the value of

$$\sum_{r=4}^{\infty} 20 \times \left(\frac{1}{2}\right)^r$$

(3)

(ii) Show that

$$\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) = 2$$

(3)

(Total for Question 5 is 6 marks)

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6. (*a*) Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{4 - \sqrt{h}} = -8 \ln |4 - \sqrt{h}| - 2\sqrt{h} + k$$

where k is a constant

(6)

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4 - \sqrt{h})}{20}$$

where h is the height in metres and t is the time, measured in years, after the tree is planted.

(*b*) Find, according to the model, the range in heights of trees in this species.

(2)

One of these trees is one metre high when it is first planted.

According to the model,

(*c*) calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3 significant figures.

(7)

(Total for Question 6 is 15 marks)

7. The curve C, in the standard Cartesian plane, is defined by the equation

$$x = 4 \sin 2y \qquad \frac{-p}{4} < y < \frac{p}{4}$$

The curve C passes through the origin O

(a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the origin.

(2)

(b) (i) Use the small angle approximation for $\sin 2y$ to find an equation linking x and y for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i).

(2)

(c) Show that, for all points (x, y) lying on C,

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{a\sqrt{b - x^2}}$$

where a and b are constants to be found.

(3)

(Total for Question 7 is 7 marks)

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8.

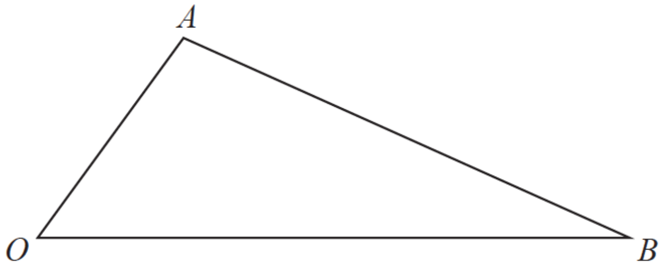

Figure 7

Figure 7 shows a sketch of triangle OAB.

The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$.

The point M is the midpoint of AB.

The straight line through C and M cuts OB at the point N.

Given $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

(*a*) Find $\overrightarrow{CM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(2)

(*b*) Show that $\overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$, where λ is a scalar constant.

(2)

(*c*) Hence prove that $ON : NB = 2 : 1$

(2)

(Total for Question 8 is 6 marks)

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9. (i) Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

(4)

(ii) "Given $x \in \mathbb{R}$, the value of $|3x - 28|$ is greater than or equal to the value of $(x - 9)$."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)

(Total for Question 9 is 6 marks)

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10.

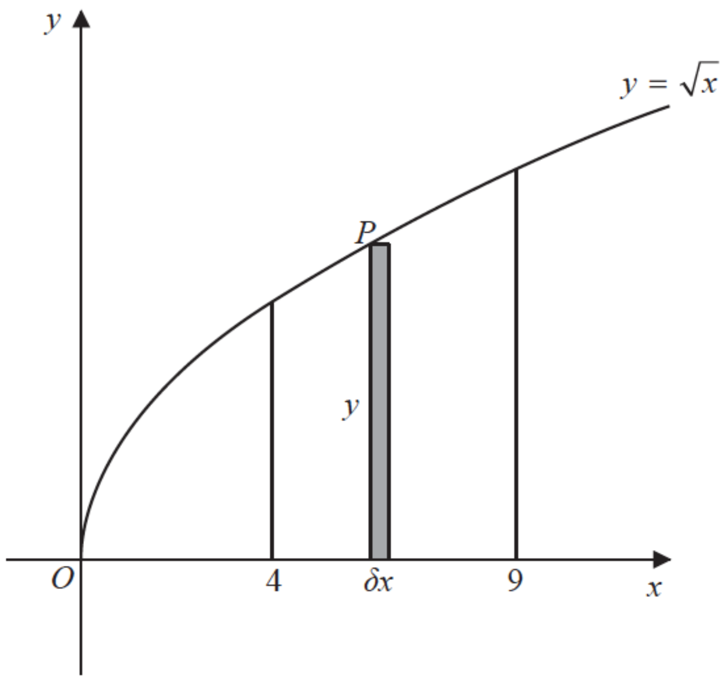

Figure 3

Figure 3 shows a sketch of the curve with equation $y = \sqrt{x}$

The point $P(x, y)$ lies on the curve.

The rectangle, shown shaded on Figure 3, has height y and width δx.

Calculate

$$\lim_{dx \to 0} \sum_{x=4}^{9} \sqrt{x}\,dx$$

(3)

(Total for Question 10 is 3 marks)

11.

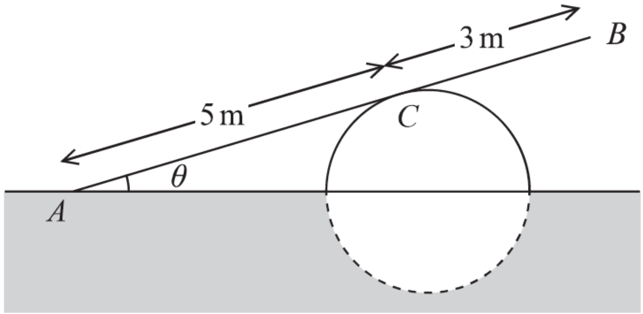

Figure 2

A ramp, AB, of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A.

The point of contact between the ramp and the drum is C, where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan\theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

(*a*) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp.

(1)

(*b*) Find the magnitude of the resultant force acting on the ramp at A.

(9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B,

(*c*) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C.

(1)

(Total for Question 11 is 11 marks)

TOTAL FOR PAPER IS 91 MARKS

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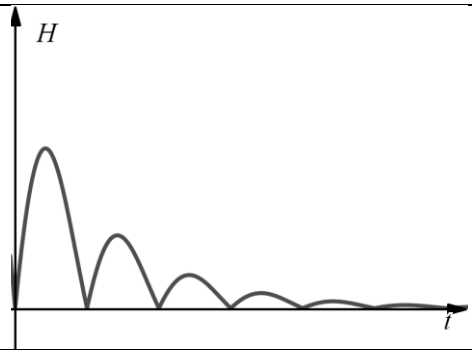
Gold Mark Scheme

Question	Scheme	Marks	AO
1(a)	$(\mathbf{v} =)\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$	M1	3.1a
	$(\mathbf{v} =)(-\mathbf{i}+4\mathbf{j})+(2\mathbf{i}-3\mathbf{j})t$	A1	1.1b
	$\dfrac{4-3T}{-1+2T}=\dfrac{-4}{3}$ oe	M1	3.1a
	$T=8$	A1	1.1b
		(4)	
(b)	$(\mathbf{s} =)\,\mathbf{C}t+(2\mathbf{i}-3\mathbf{j})\frac{1}{2}t^2 \quad (+\,\mathbf{D})$	M1	3.1a
	$(\mathbf{s} =)(-\mathbf{i}+4\mathbf{j})t+\frac{1}{2}(2\mathbf{i}-3\mathbf{j})t^2 \quad (+\,\mathbf{D})$	A1	1.1b
	$AB=\sqrt{12^2+8^2}$ **N.B. Beware you may see 4(2i – 3j) which leads to** $\sqrt{(8^2+12^2)}$ this is M0A0M0A0.	M1	3.1a
	$=4\sqrt{13}\,(=14.422051....)$ (m)	A1**cso**	1.1b
		(4)	
		(8)	

Marks		Notes
1a	M1	Use of $\mathbf{v}=\mathbf{u}+\mathbf{a}t$ **OR** integration to give an expression of the form $\mathbf{C}+(2\mathbf{i}-3\mathbf{j})t$ **, where C is a non-zero constant <u>vector</u>** M0 if **u** and **a** are reversed Condone use of **a** = (2**i** + 3**j**) for this M mark
	A1	Any correct unsimplified expression seen or implied
	M1	Correct use of ratios, <u>using a velocity vector</u> (must be using $\frac{-4}{3}$) to give equation <u>in T only</u> M0 if they equate $4-3T=-4$ and/or $-1+2T=3$ and therefore M0 if they then divide to produce their equation
	A1	Correct only

		N.B. (i) Can score the second M1A1 if they get $T = 8$, using a calculator to solve two simultaneous equations, but if answer is wrong, and no equation in T only, second M0 (ii) Can score M1A1 M1A1 if they get $T = 8$, using trial and error, but if they don't get $T = 8$, can only score max M1A1M0A0
1b	M1	Use of $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with **a = (2i – 3j)** **OR** integration to give an expression of the form $\mathbf{C}t + (2\mathbf{i} - 3\mathbf{j})\frac{1}{2}t^2$ **, where C is their non-zero constant <u>vector</u> from (a)** Condone use of **a** = (2**i** + 3**j**) for this M mark **OR** any other complete method using vector **suva***t* equations
	A1	Correct unsimplified expression seen or implied
	M1	Use of $t = 4$ in their **s** (which must be a **displacement vector**) and then Pythagoras with the root sign **N.B.** This M mark can be implied by a correct answer, otherwise we need to see Pythagoras used, with the root sign, for the M mark.
	A1**cso**	Any surd form or 14 or better

2 (a)	$\mathrm{f}(x) = 10\mathrm{e}^{-0.25x}\sin x$		
	$\Rightarrow \mathrm{f}'(x) = -2.5\mathrm{e}^{-0.25x}\sin x + 10\mathrm{e}^{-0.25x}\cos x$ oe	M1 A1	1.1b 1.1b
	$\mathrm{f}'(x) = 0 \Rightarrow -2.5\cancel{\mathrm{e}^{-0.25x}}\sin x + 10\cancel{\mathrm{e}^{-0.25x}}\cos x = 0$	M1	2.1
	$\frac{\sin x}{\cos x} = \frac{10}{2.5} \Rightarrow \tan x = 4\,*$	A1*	1.1b
		(4)	
(b)	"Correct" shape for 2 loops	M1 A1	1.1b 1.1b

	 Fully correct with decreasing heights		
		(2)	
(c)	Solves $\tan x = 4$ and substitutes answer into $H(t)$	M1	3.1a
	$H(4.47) = \left\|10e^{-0.25\times4.47}\sin 4.47\right\|$	M1	1.1b
	awrt 3.18 (metres)	A1	3.2a
		(3)	
(d)	The **times between** each bounce should not stay the same when the heights of each bounce is getting smaller	B1	3.5b
		(1)	
			(10 marks)

(a)

M1: For attempting to differentiate using the product rule condoning slips, for example the power of e .

So for example score expressions of the form $\pm...e^{-0.25x}\sin x \pm ...e^{-0.25x}\cos x$ M1

Sight of $vdu - udv$ however is M0

A1: $f'(x) = -2.5e^{-0.25x}\sin x + 10e^{-0.25x}\cos x$ which may be unsimplified

M1: For clear reasoning in setting their $f'(x) = 0$, factorising/ cancelling out the $e^{-0.25x}$ term leading to a trigonometric equation in only $\sin x$ and $\cos x$

Do not allow candidates to substitute $x = \arctan 4$ into $f'(x)$ to score this mark.

A1*: Shows the steps $\frac{\sin x}{\cos x} = \frac{10}{2.5}$ or equivalent leading to $\Rightarrow \tan x = 4*$. $\frac{\sin x}{\cos x}$ must be seen.

(b)

M1: Draws at least two "loops". The height of the second loop should be lower than the first loop.

Condone the sight of rounding where there should be cusps

A1: At least 4 loops with decreasing heights and no rounding at the cusps.

The intention should be that the graph should 'sit' on the x -axis but be tolerant.

It is possible to overwrite Figure 3, but all loops must be clearly seen.

(c)

M1: Understands that to solve the problem they are required to substitute an answer to $\tan t = 4$ into $H(t)$

This can be awarded for an attempt to substitute $t =$ awrt 1.33 or $t =$ awrt 4.47 into $H(t)$

$H(t) = 6.96$ implies the use of $t = 1.33$ Condone for this mark only, an attempt to substitute $t =$ awrt 76° or awrt 256° into $H(t)$

M1: Substitutes $t=$ awrt 4.47 into $H(t)=\left|10\mathrm{e}^{-0.25t}\sin t\right|$. Implied by awrt 3.2

A1: Awrt 3.18 metres. Condone the lack of units. If two values are given the correct one must be seen to have been chosen

It is possible for candidates to sketch this on their graphical calculators and gain this answer. If there is no incorrect working seen and 3.18 is given, then award 111 for such an attempt.

(d)

B1: Makes reference to the fact that the time between each bounce should not stay the same when the heights of each bounce is getting smaller.

Look for " time (or gap) between the bounces will change"

'bounces would not be equal times apart'

'bounces would become more frequent'

But do not accept 'the times between each bounce would be longer or slower'

Do not accept explanations such as there are other factors that would affect this such as "wind resistance", friction etc

Question	Scheme	Marks	AO
	In this question mark parts (a) and (b) together.		
3(a)	Horizontal speed $=20\cos 30°$	B1	3.4
	Vertical velocity at $t=2$	M1	3.4
	$=20\sin 30°-2g$	A1	1.1b
	$\theta=\tan^{-1}\left(\pm\dfrac{9.6}{10\sqrt{3}}\right)$	M1	1.1b
	Speed $=\sqrt{100\times 3+9.6^2}$ or e.g. speed $=\dfrac{9.6}{\sin\theta}$	M1	1.1b
	19.8 or 20 $\left(\mathrm{m\,s^{-1}}\right)$ at 29.0° or 29° to the horizontal oe	A1	2.2a
		(6)	
(b)	Using sum of horizontal distances $=50$ at $t=2$	M1	3.3
	$(u\cos\theta)\times 2+(20\cos 30°)\times 2=50$ $(u\cos\theta=25-20\cos 30°)$	A1	1.1b
	Vertical distances equal	M1	3.4
	$\Rightarrow(20\sin 30°)\times 2-\dfrac{g}{2}\times 4=(u\sin\theta)\times 2-\dfrac{g}{2}\times 4$ $(20\sin 30°=u\sin\theta)$	A1	1.1b
	Solving for both θ and u	M1	3.1b
	$\theta=52°$ or better (52.47756849….°) $u=13$ or better (12.6085128…)	A1	2.2a

		(6)	
(c)	It does not take account of the fact that they are not particles (moving freely under gravity) It does not take account of the size(s) of the balls It does not take account of the spin of the balls It does not take account of the wind g is not exactly 9.8 m s^{-2} **N.B.** If they refer to the mass or weight of the balls give B0	B1	3.5b
		(1)	
		(13)	

Marks		**Notes**
3a	B1	Seen or implied, possibly on a diagram
	M1	Use of $v = u + at$ or any other complete method <u>using $t = 2$</u> Condone sign errors and sin/cos confusion.
	A1	Correct unsimplified equation in v or v^2
	M1	Correct use of trig to find a relevant angle for the direction. Must have found a horizontal and a vertical velocity component
	M1	Use Pythagoras or trig to find the magnitude Must have found a horizontal and a vertical velocity component
	A1	Or equivalent. Need magnitude **and** direction stated or implied in a diagram. (0.506 or 0.51 rads)
3b	M1	First equation,in terms of u and θ (could be implied by subsequent working), using the horizontal motion with $t = 2$ used Condone sign errors and sin/cos confusion
	A1	Correct unsimplified equation – any equivalent form
	M1	Second equation, in terms of u and θ (could be implied by subsequent working), using the vertical motion – equating distances or just vertical components of velocities. Condone sign errors and sin/cos confusion

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	A1	Correct unsimplified equation – any equivalent form
	M1	Complete strategy: all necessary equations formed and solve for u and θ **N.B.** This is an independent method mark but can only be earned if 50 m has been used in their solution.
	A1	Both values correct. (Here we accept 2SF or better, since the g's cancel) Allow radians for θ: 0.92 or better (0.915906..) rads.
3c	B1	Any factor **related to the model** as stated in the question. Penalise incorrect extras but ignore consequences e.g. 'AB (or the ground) is not horizontal' should be penalised or 'they do not move in a vertical plane' should be penalised

Question	Scheme	Marks	AOs
4	<table><tr><td>Time (s)</td><td>0</td><td>5</td><td>10</td><td>15</td><td>20</td><td>25</td></tr><tr><td>Speed (m s^{-1})</td><td>2</td><td>5</td><td>10</td><td>18</td><td>28</td><td>42</td></tr></table>		
(a)	Uses an allowable method to estimate the area under the curve. E.g. **Way 1:** an attempt at the trapezium rule (see below) **Way 2:** $\{s=\}\left(\dfrac{2+42}{2}\right)(25)\ \{=550\}$ **Way 3:** $42=2+25(a)\Rightarrow a=1.6\Rightarrow s=2(25)+(0.5)(1.6)(25)^2\{=550\}$ **Way 4:** $\{d=\}(2)(5)+5(5)+10(5)+18(5)+28(5)\ \{=63(5)=315\}$ **Way 5:** $\{d=\}\ 5(5)+10(5)+18(5)+28(5)+42(5)\ \{=103(5)=515\}$ **Way 6:** $\{d=\}\ \dfrac{315+515}{2}\ \{=415\}$ **Way 7:** $\{d=\}\left(\dfrac{2+5+10+18+28+42}{6}\right)(25)\ \{=437.5\}$	M1	3.1a
	$\dfrac{1}{2}\times(5)\times[2+2(5+10+18+28)+42]$ or $\dfrac{1}{2}\times["315"+"515"]$	M1	1.1b
	$=415\ \{\text{m}\}$	A1	1.1b
		(3)	
(b) **Alt 1**	<u>**Uses a Way 1, Way 2, Way 3, Way 5, Way 6 or Way 7 method in (a)**</u>		
	Overestimate **and** a relevant explanation e.g. • {top of} trapezia lie above the curve • Area of trapezia > area under curve • An appropriate diagram which gives reference to the extra area • Curve is convex • $\dfrac{\text{d}^2y}{\text{d}x^2}>0$ • Acceleration is {continually} increasing • The gradient of the curve is {continually} increasing	B1ft	2.4

	• All the rectangles are above the curve (Way 5)		
		(1)	
(b) Alt 2	**Uses a Way 4 method in (a)**		
	Underestimate **and** a relevant explanation e.g. • All the rectangles are below the curve	B1ft	2.4
		(1)	
			(4 marks)

Notes for Question 2

(a)	
M1:	A low-level problem-solving mark for using an allowable method to estimate the area under the curve. E.g.
	Way 1: See scheme. Allow $\lambda(2+2(5+10+18+28)+42)$; $\lambda > 0$ for 1st M1
	Way 2: Uses $s = \left(\frac{u+v}{2}\right)t$ which is equivalent to finding the area of a large trapezium
	Way 3: Complete method using a uniform acceleration equation.
	Way 4: Sums rectangles lying below the curve. Condone a slip on one of the speeds.
	Way 5: Sums rectangles lying above the curve. Condone a slip on one of the speeds.
	Way 6: Average the result of Way 3 and Way 4. Equivalent to Way 1.
	Way 7: Applies (average speed) $\times$ (time)

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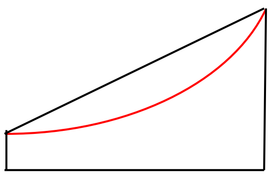
Notes for Question 4 Continued	
(a)	***continued***
M1:	Correct trapezium rule method with $h=5$. Condone a slip on one of the speeds. The '2' and '42' should be in the correct place in the [......].
A1:	415
Note:	Units do not have to be stated
Note:	Give final A0 for giving a final answer with incorrect units. e.g. give final A0 for 415 km or $415\,\text{m}\,\text{s}^{-1}$
Note:	Only the 1st M1 can only be scored for Way 2, Way 3, Way 4, Way 5 and Way 7 methods
Note:	Full marks in part (a) can only be scored by using a Way 1 or a Way 6 method.
Note:	Give M0 M0 A0 for $\{d=\}\ 2(5)+5(5)+10(5)+18(5)+28(5)+42(5)\ \ \{=105(5)=525\}$ (i.e. using too many rectangles)
Note	Condone M1 M0 A0 for $\left[\frac{(2+10)}{2}(10)+\frac{(10+18)}{2}(5)+\frac{(18+28)}{2}(5)+\frac{(28+42)}{2}(5)\right]=395\text{ m}$
Note:	Give M1 M1 A1 for $5\left[\frac{(2+5)}{2}+\frac{(5+10)}{2}+\frac{(10+18)}{2}+\frac{(18+28)}{2}+\frac{(28+42)}{2}\right]=415\text{ m}$
Note:	Give M1 M1 A1 for $\frac{5}{2}(2+42)+5(5+10+18+28)=415\text{ m}$
Note:	**Bracketing mistake:**
	Unless the final calculated answer implies that the method has been applied correctly
	give M1 M0 A0 for $\frac{5}{2}(2)+2(5+10+18+28)+42\ \ \{=169\}$
	give M1 M0 A0 for $\frac{5}{2}(2+42)+2(5+10+18+28)\ \ \{=232\}$
Note:	Give M0 M0 A0 for a Simpson's Rule Method
(b)	**Alt 1**
B1ft:	**This mark depends on both an answer to part (a) being obtained and the first M in part (a)** See scheme
Note:	Allow the explanation "curve concaves upwards"
Note:	Do not allow explanations such as "curve is concave" or "curve concaves downwards"
Note:	Do not allow explanation "gradient of the curve is positive"
Note:	Do not allow explanations which refer to "friction" or "air resistance"
Note:	The diagram opposite is sufficient as an explanation. It must show the top of a trapezium lying above the curve. 
(b)	**Alt 2**
B1ft:	**This mark depends on both an answer to part (a) being obtained and the first M in part (a)** See scheme
Note:	Do not allow explanations which refer to "friction" or "air-resistance"

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Question	Scheme	Marks	AOs
5 (i) Way 1	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = 20\left(\frac{1}{2}\right)^4 + 20\left(\frac{1}{2}\right)^5 + 20\left(\frac{1}{2}\right)^6 + \dots$		
	$= \frac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= (1.25)(2)\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 2	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=1}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=1}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \frac{10}{1-\frac{1}{2}} - (10+5+2.5)$ or $= \frac{10}{1-\frac{1}{2}} - \frac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 20 - 17.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(i) Way 3	$\sum_{r=4}^{\infty} 20\times\left(\frac{1}{2}\right)^r = \sum_{r=0}^{\infty} 20\times\left(\frac{1}{2}\right)^r - \sum_{r=0}^{3} 20\times\left(\frac{1}{2}\right)^r$		
	$= \frac{20}{1-\frac{1}{2}} - (20+10+5+2.5)$ or $= \frac{20}{1-\frac{1}{2}} - \frac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$	M1	1.1b
		M1	3.1a
	$\{= 40 - 37.5\} = 2.5$ o.e.	A1	1.1b
		(3)	
(ii) Way 1	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\}$		
	$= \log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + \dots\dots + \log_5\left(\frac{50}{49}\right) = \log_5\left(\frac{3}{2}\times\frac{4}{3}\times\dots\times\frac{50}{49}\right)$	M1	1.1b
		M1	3.1a
	$= \log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$ *	A1*	2.1
		(3)	
(ii) Way 2	$\left\{\sum_{n=1}^{48} \log_5\left(\frac{n+2}{n+1}\right) =\right\} \sum_{n=1}^{48}(\log_5(n+2) - \log_5(n+1))$	M1	1.1b
	$= (\log_5 3 + \log_5 4 + \dots\dots + \log_5 50) - (\log_5 2 + \log_5 3 + \dots\dots + \log_5 49)$	M1	3.1a
	$= \log_5 50 - \log_5 2$ or $\log_5\left(\frac{50}{2}\right)$ or $\log_5(25) = 2$*	A1*	2.1
		(3)	
			(6 marks)

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Notes for Question 5	
(i)	**Way 1**
M1:	Applies $\dfrac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a complete strategy of applying $\dfrac{20(\frac{1}{2})^4}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 2**
M1:	Applies $\dfrac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\dfrac{10}{1-\frac{1}{2}} - (10 + 5 + 2.5)$ or $\dfrac{10}{1-\frac{1}{2}} - \dfrac{10(1-(\frac{1}{2})^3)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
(i)	**Way 3**
M1:	Applies $\dfrac{a}{1-r}$ for their r (where $-1 <$ their $r < 1$) and their value for a
M1:	Finds the infinite sum by using a completely correct strategy of applying $\dfrac{20}{1-\frac{1}{2}} - (20+10 + 5 + 2.5)$ or $\dfrac{20}{1-\frac{1}{2}} - \dfrac{20(1-(\frac{1}{2})^4)}{1-\frac{1}{2}}$
A1:	2.5 o.e.
Note:	Give M1 M1 A1 for a correct answer of 2.5 from no working in (i)
(ii)	**Way 1**
M1:	Some evidence of applying the addition law of logarithms as part of a valid proof
M1:	Begins to solve the problem by just writing (or by combining) at least three terms including • **either** the first two terms and the last term • **or** the first term and the last two terms
Note:	The 2nd mark can be gained by writing any of • listing $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right), \log_5\left(\frac{50}{49}\right)$ or $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{49}{48}\right), \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{4}{3}\right) + ... + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2}\right) + ... + \log_5\left(\frac{49}{48}\right) + \log_5\left(\frac{50}{49}\right)$ • $\log_5\left(\frac{3}{2} \times \frac{4}{3} \times ... \times \frac{50}{49}\right)$ *{this will also gain the 1st M1 mark}* • $\log_5\left(\frac{3}{2} \times ... \times \frac{49}{48} \times \frac{50}{49}\right)$ *{this will also gain the 1st M1 mark}*
A1*:	Correct proof leading to a correct answer of 2
Note:	Do not allow the 2nd M1 if $\log_5\left(\frac{3}{2}\right), \log_5\left(\frac{4}{3}\right)$ are listed and $\log_5\left(\frac{50}{49}\right)$ is used for the first time in their applying the formula $S_{48} = \frac{48}{2}\left(\log_5\left(\frac{3}{2}\right) + \log_5\left(\frac{50}{49}\right)\right)$

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Note:	**<u>Listing all 48 terms</u>** Give M0 M1 A0 for $\log_5\left(\frac{3}{2}\right)+\log_5\left(\frac{4}{3}\right)+\log_5\left(\frac{5}{4}\right)+......+\log_5\left(\frac{50}{49}\right)=2$ {lists all terms} Give M0 M0 A0 for $0.2519...+0.1787...+0.1386...+......+0.0125...=2$ {all terms in decimals}

Notes for Question 5	
(ii)	**Way 2**
M1:	Uses the subtraction law of logarithms to give $\log_5\left(\frac{n+2}{n+1}\right)\to\log_5(n+2)-\log_5(n+1)$
M1:	Begins to solve the problem by writing at least three terms for each of $\log_5(n+2)$ and $\log_5(n+1)$ including • **either** the first two terms and the last term for both $\log_5(n+2)$ and $\log_5(n+1)$ • **or** the first term and the last two terms for both $\log_5(n+2)$ and $\log_5(n+1)$
Note:	This mark can be gained by writing any of • $(\log_5 3+\log_5 4+......+\log_5 50)-(\log_5 2+\log_5 3+......+\log_5 49)$ • $(\log_5 3+......+\log_5 49+\log_5 50)-(\log_5 2+......+\log_5 48+\log_5 49)$ • $(\log_5 3+\log_5 4+......+\log_5 50)-(\log_5 2+\log_5 3+......+\log_5 49)$ • $(\log_5 3-\log_5 2)+(\log_5 4-\log_5 3)+......+(\log_5 50-\log_5 49)$ • $\log_5 3-\log_5 2,\ ......, \log_5 49-\log_5 48,\ \log_5 50-\log_5 49$
A1*:	Correct proof leading to a correct answer of 2
Note:	The base of 5 can be omitted for the M marks in part (ii), but the base of 5 must be included in the final line (as shown on the mark scheme) of their solution.
Note:	If a student uses a mixture of a Way 1 or Way 2 method, then award the best Way 1 mark only or the best Way 2 mark only.
Note:	Give M1 M0 A0 (1st M for implied use of subtraction law of logarithms) for $\sum_{n=1}^{48}\log_5\left(\frac{n+2}{n+1}\right)=91.8237...-89.8237...=2$
Note:	Give M1 M1 A1 for $\sum_{n=1}^{48}\log_5\left(\frac{n+2}{n+1}\right)=\sum_{n=1}^{48}(\log_5(n+2)-\log_5(n+1))$ $=\log_5(3\times4\times.......\times50)-\log_5(2\times3\times.......\times49)$ $=\log_5\left(\frac{50!}{2}\right)-\log_5(49!)$ or $=\log_5(25\times49!)-\log_5(49!)$ $=\log_5 25=2$

Question	Scheme	Marks	AOs

6 (a)	$\{u = 4 - \sqrt{h} \Rightarrow\}\ \frac{du}{dh} = -\frac{1}{2}h^{-\frac{1}{2}}$ or $\frac{dh}{du} = -2(4-u)$ or $\frac{dh}{du} = -2\sqrt{h}$	B1	1.1b
	$\left\{\int \frac{dh}{4-\sqrt{h}} = \right\}\ \int \frac{-2(4-u)}{u}\,du$	M1	2.1
	$= \int\left(-\frac{8}{u} + 2\right)du$	M1	1.1b
	$= -8\ln u + 2u\ \{+c\}$	M1	1.1b
		A1	1.1b
	$= -8\ln\lvert 4-\sqrt{h}\rvert + 2(4-\sqrt{h}) + c = -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} + k$ *	A1*	2.1
		(6)	
(b)	$\left\{\frac{dh}{dt} = \frac{t^{0.25}(4-\sqrt{h})}{20} = 0 \Rightarrow\right\}\ 4-\sqrt{h} = 0$	M1	3.4
	Deduces any of $0 < h < 16,\ 0 \le h < 16,\ 0 < h \le 16,\ 0 \le h \le 16,$ $h < 16,\ h \le 16$ or all values up to 16	A1	2.2a
		(2)	
(c) Way 1	$\int \frac{1}{(4-\sqrt{h})}\,dh = \int \frac{1}{20}t^{0.25}\,dt$	B1	1.1b
	$-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \frac{1}{25}t^{1.25}\ \{+c\}$	M1	1.1b
		A1	1.1b
	$\{t=0,\ h=1 \Rightarrow\}\ -8\ln(4-1) - 2\sqrt{(1)} = \frac{1}{25}(0)^{1.25} + c$	M1	3.4
	$\Rightarrow c = -8\ln(3) - 2 \Rightarrow\ -8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$ $\{h = 12 \Rightarrow\}\ -8\ln\lvert 4-\sqrt{12}\rvert - 2\sqrt{12} = \frac{1}{25}t^{1.25} - 8\ln(3) - 2$	dM1	3.1a
	$t^{1.25} = 221.2795202... \Rightarrow t = \sqrt[1.25]{221.2795...}$ or $t = (221.2795...)^{0.8}$	M1	1.1b
	$t = 75.154... \Rightarrow t = 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
(c) Way 2	$\int_1^{12} \frac{20}{(4-\sqrt{h})}\,dh = \int_0^T t^{0.25}\,dt$	B1	1.1b
	$\left[20(-8\ln\lvert 4-\sqrt{h}\rvert - 2\sqrt{h})\right]_1^{12} = \left[\frac{4}{5}t^{1.25}\right]_0^T$	M1	1.1b
		A1	1.1b
	$20(-8\ln(4-\sqrt{12}) - 2\sqrt{12}) - 20(-8\ln(4-1) - 2\sqrt{1}) = \frac{4}{5}T^{1.25} - 0$	M1	3.4
		dM1	3.1a
	$T^{1.25} = 221.2795202... \Rightarrow T = \sqrt[1.25]{221.2795...}$ or $T = (221.2795...)^{0.8}$	M1	1.1b
	$T = 75.154... \Rightarrow T = 75.2$ (years) (3 sf) or awrt 75.2 (years)	A1	1.1b
	Note: You can recover work for part (c) in part (b)	**(7)**	
			(15 marks)

Notes for Question 6	
(a)	
B1:	See scheme. Allow $\mathrm{d}u = -\frac{1}{2}h^{-\frac{1}{2}}\mathrm{d}h$, $\mathrm{d}h = -2(4-u)\mathrm{d}u$, $\mathrm{d}h = -2\sqrt{h}\,\mathrm{d}u$ o.e.
M1: **Note:**	Complete method for applying $u = 4-\sqrt{h}$ to $\int \frac{\mathrm{d}h}{4-\sqrt{h}}$ to give an expression of the form $\int \frac{k(4-u)}{u}\mathrm{d}u$; $k \neq 0$ Condone the omission of an integral sign and/or $\mathrm{d}u$
M1:	Proceeds to obtain an integral of the form $\int \left(\frac{A}{u}+B\right)\{\mathrm{d}u\}$; $A, B \neq 0$
M1:	$\int \left(\frac{A}{u}+B\right)\{\mathrm{d}u\} \to D\ln u + Eu$; $A, B, D, E \neq 0$; with or without a constant of integration
A1:	$\int \left(-\frac{8}{u}+2\right)\{\mathrm{d}u\} \to -8\ln u + 2u$; with or without a constant of integration
A1*:	**dependent on all previous marks** Substitutes $u = 4-\sqrt{h}$ into their integrated result and completes the proof by obtaining the printed result $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$. Condone the use of brackets instead of the modulus sign.
Note:	They must combine $2(4)$ and their $+c$ correctly to give $+k$
Note:	Going from $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + c$ to $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$, with no intermediate working or with no incorrect working is required for the final A1* mark.
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + c + 8$ and stating $k = c+8$
Note:	Allow A1* for correctly reaching $-8\ln\left\|4-\sqrt{h}\right\| + 2(4-\sqrt{h}) + k = -8\ln\left\|4-\sqrt{h}\right\| - 2\sqrt{h} + k$
	Alternative (integration by parts) method for the 2nd M, 3rd M and 1st A mark $\left\{\int \frac{-2(4-u)}{u}\mathrm{d}u = \int \frac{2u-8}{u}\mathrm{d}u\right\} = (2u-8)\ln u - \int 2\ln u\,\mathrm{d}u = (2u-8)\ln u - 2(u\ln u - u)\,\{+c\}$
2nd M1:	Proceeds to obtain an integral of the form $(Au+B)\ln u - \int A\ln u\,\{\mathrm{d}u\}$; $A, B \neq 0$
3rd M1:	Integrates to give $D\ln u + Eu$; $D, E \neq 0$; which can be simplified or un-simplified with or without a constant of integration.
Note:	Give 3rd M1 for $(2u-8)\ln u - 2(u\ln u - u)$ because it is an un-simplified form of $D\ln u + Eu$
1st A1:	Integrates to give $(2u-8)\ln u - 2(u\ln u - u)$ or $-8\ln u + 2u$ o.e. with or without a constant of integration.
(b)	
M1: **Note:**	Uses the context of the model and has an understanding that the tree keeps growing until $\frac{\mathrm{d}h}{\mathrm{d}t} = 0 \Rightarrow 4-\sqrt{h} = 0$. Alternatively, they can write $\frac{\mathrm{d}h}{\mathrm{d}t} > 0 \Rightarrow 4-\sqrt{h} > 0$ Accept $h = 16$ or 16 used in their inequality statement for this mark.
A1:	See scheme
Note:	A correct answer can be given M1 A1 from any working.

Notes for Question 6	
(c)	**Way 1**
B1:	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working. Condone absence of integral signs.
M1:	Integrates $t^{0.25}$ to give $\lambda t^{1.25}$; $\lambda \neq 0$
A1:	Correct integration. E.g. $-8\ln\left\|4-\sqrt{h}\right\|-2\sqrt{h} = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\|-2\sqrt{h}) = \frac{4}{5}t^{1.25}$ $-8\ln\left\|4-\sqrt{h}\right\|+2(4-\sqrt{h}) = \frac{1}{25}t^{1.25}$ or $20(-8\ln\left\|4-\sqrt{h}\right\|+2(4-\sqrt{h})) = \frac{4}{5}t^{1.25}$ with or without a constant of integration, e.g. k, c or A
Note:	There is no requirement for modulus signs.
M1:	Some evidence of ***applying*** both $t=0$ and $h=1$ to their model (which can be a changed equation) which contains a constant of integration, e.g. k, c or A
dM1:	**dependent on the previous M mark** Complete process of finding their constant of integration, followed by applying $h=12$ and their constant of integration to their changed equation
M1:	Rearranges their equation to make $t^{\text{their } 1.25} = ...$ followed by a correct method to give $t = ...$; $t > 0$
Note:	$t^{\text{their } 1.25} = ...$ can be negative, but their '$t = ...$' must be positive
Note:	"their 1.25" cannot be 0 or 1 for this mark
Note:	Do not give this mark if $t^{\text{their } 1.25} = ...$ (usually $t^{0.25} = ...$) is a result of substituting $t=12$ (or $t=11$) into the given $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20}$. **Note:** They will usually write $\frac{\mathrm{d}h}{\mathrm{d}t}$ as either 12 or 11.
A1:	awrt 75.2
(c)	**Way 2**
B1:	Separates the variables correctly. dh and dt should not be in the wrong positions, although this mark can be implied by later working.
Note:	Integral signs and limits are not required for this mark.
M1:	Same as Way 1 (ignore limits)
A1:	Same as Way 1 (ignore limits)
M1:	Applies limits of 1 and 12 to their model (i.e. to their changed expression in h) and subtracts
dM1	**dependent on the previous M mark** Complete process of applying limits of 1 and 12 and 0 and T (or 't') appropriately to their changed equation
M1:	Same as Way 1
A1:	Same as Way 1

Question	Scheme	Marks	AOs
7 (a)	Attempts to differentiate $x = 4\sin 2y$ and inverts $\frac{\mathrm{d}x}{\mathrm{d}y} = 8\cos 2y \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{8\cos 2y}$	M1	1.1b
	At (0,0) $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{8}$	A1	1.1b
		(2)	
	(i) Uses $\sin 2y \approx 2y$ when y is small to obtain $x \approx 8y$	B1	1.1b

(b)	(ii) The value found in (a) is the gradient of the line found in (b)(i)	B1	2.4
		(2)	
(c)	Uses their $\frac{dy}{dx}$ as a function of y and, using both $\sin^2 2y+\cos^2 2y=1$ and $x=4\sin 2y$ in an attempt to write $\frac{dy}{dx}$ or $\frac{dx}{dy}$ as a function of x Allow for $\frac{dy}{dx}=k\frac{1}{\cos 2y}=..\frac{1}{\sqrt{1-(..x)^2}}$	M1	2.1
	A correct answer $\frac{dy}{dx}=\frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$ or $\frac{dx}{dy}=8\sqrt{1-\left(\frac{x}{4}\right)^2}$	A1	1.1b
	and in the correct form $\frac{dy}{dx}=\frac{1}{2\sqrt{16-x^2}}$	A1	1.1b
		(3)	
			(7 marks)

(a)

M1: Attempts to differentiate $x=4\sin 2y$ and inverts.

Allow for $\frac{dx}{dy}=k\cos 2y \Rightarrow \frac{dy}{dx}=\frac{1}{k\cos 2y}$ or $1=k\cos 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx}=\frac{1}{k\cos 2y}$

Alternatively, changes the subject and differentiates

$x=4\sin 2y \to y=...\arcsin\left(\frac{x}{4}\right) \to \frac{dy}{dx}=\frac{...}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

It is possible to approach this from $x=8\sin y\cos y \Rightarrow \frac{dx}{dy}=\pm 8\sin^2 y\pm 8\cos^2 y$ before inverting

A1: $\frac{dy}{dx}=\frac{1}{8}$ Allow both marks for sight of this answer as long as no incorrect working is seen (See below)

Watch for candidates who reach this answer via $\frac{dx}{dy}=8\cos 2x \Rightarrow \frac{dy}{dx}=\frac{1}{8\cos 2x}$ This is M0 A0

(b)(i)

B1: Uses $\sin 2y \approx 2y$ when y is small to obtain $x=8y$ oe such as $x=4(2y)$.

Do not allow $\sin 2y \approx 2\theta$ to get $x=8\theta$ but allow recovery in (b)(i) or (b)(ii)

Double angle formula is B0 as it does not satisfy the demands of the question.

(b)(ii)

B1: Explains the relationship between the answers to (a) and (b) (i).

For this to be scored the first three marks, in almost all cases, must have been awarded and the statement must refer to both answers

Allow for example "The gradients are the same $\left(=\frac{1}{8}\right)$" ‘both have $m=\frac{1}{8}$ ’

Do not accept the statement that 8 and $\frac{1}{8}$ are reciprocals of each other unless further correct work explains the relationship in terms of $\frac{\mathrm{d}x}{\mathrm{d}y}$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$

(c)

M1: Uses their $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of y and, using both $\sin^2 2y+\cos^2 2y=1$ and $x=4\sin 2y$, attempts to write $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ as a function of x. The $\frac{\mathrm{d}y}{\mathrm{d}x}$ may not be seen and may be implied by their calculation.

A1: A correct (un-simplified) answer for $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ Eg. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{8\sqrt{1-\left(\frac{x}{4}\right)^2}}$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{16-x^2}}$ The $\frac{\mathrm{d}y}{\mathrm{d}x}$ must be seen at least once in part (c) of this solution

....

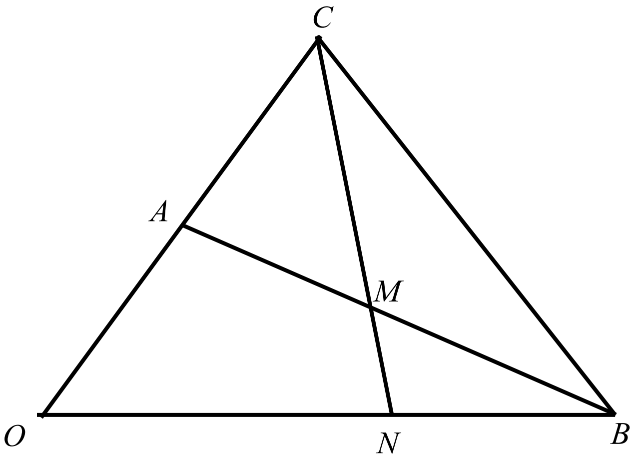
Alt to (c) using arcsin

M1: Alternatively, changes the subject and differentiates $x=4\sin 2y \rightarrow y=\ldots\arcsin\left(\frac{x}{4}\right)\rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\ldots}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$

Condone a lack of bracketing on the $\frac{x}{4}$ which may appear as $\frac{x^2}{4}$

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1/8}{\sqrt{1-\left(\frac{x}{4}\right)^2}}$ oe

A1: $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2\sqrt{16-x^2}}$

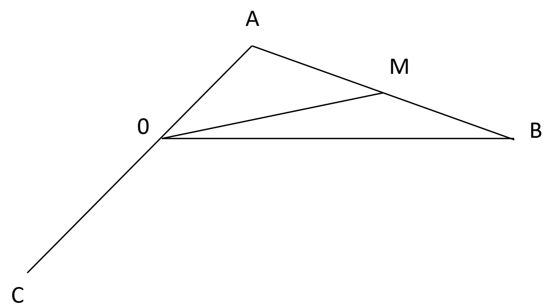
Question	Scheme	Marks	AOs
8			
	$\overrightarrow{OA} = \mathbf{a},\ \overrightarrow{OB} = \mathbf{b}$		
(a)	$\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \overrightarrow{CA} + \frac{1}{2}\overrightarrow{AB} \Rightarrow\right\}\ \overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$	M1	3.1a
	$\left\{\overrightarrow{CM} = \overrightarrow{CB} + \overrightarrow{BM} = \overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA} \Rightarrow\right\}\ \overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$		
	$\Rightarrow \overrightarrow{CM} = -\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ **(*needs to be simplified and seen in (a) only*)**	A1	1.1b
		(2)	
(b)	$\overrightarrow{ON} = \overrightarrow{OC} + \overrightarrow{CN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$	M1	1.1b
	$\overrightarrow{ON} = 2\mathbf{a} + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$ *	A1*	2.1
		(2)	
(c) Way 1	$\left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = \ldots$	M1	2.2a
	$\lambda = \frac{4}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1*	2.1
		(2)	
(c) Way 2	$\overrightarrow{ON} = \mu\mathbf{b} \Rightarrow \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} = \mu\mathbf{b}$		
	$\mathbf{a}:\ \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = \ldots\ \left\{\mathbf{b}:\ \frac{1}{2}\lambda = \mu\ \ \&\ \lambda = \frac{4}{3} \Rightarrow \mu = \frac{2}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}\ \left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1*	2.1
		(2)	
			(6 marks)

Question	Scheme	Marks	AOs
8 (c) Way 3	$\overrightarrow{OB} = \overrightarrow{ON} + \overrightarrow{NB} \Rightarrow \mathbf{b} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b} + K\mathbf{b}$		
	$\mathbf{a}: \left(2 - \frac{3}{2}\lambda\right) = 0 \Rightarrow \lambda = ...$ $\left\{\mathbf{b}:\ 1 = \frac{1}{2}\lambda + K\ \ \&\ \ \lambda = \frac{4}{3} \Rightarrow K = \frac{1}{3}\right\}$	M1	2.2a
	$\lambda = \frac{4}{3}$ or $K = \frac{1}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ or $\overrightarrow{NB} = \frac{1}{3}\mathbf{b} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	
8 (c) Way 4	$\overrightarrow{ON} = \mu\mathbf{b}\ \&\ \overrightarrow{CN} = k\overrightarrow{CM} \Rightarrow \overrightarrow{CO} + \overrightarrow{ON} = k\overrightarrow{CM}$		
	$-2\mathbf{a} + \mu\mathbf{b} = k\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$		
	$\mathbf{a}: -2 = -\frac{3}{2}k \Rightarrow k = \frac{4}{3}$, $\mathbf{b}:\ \mu = \frac{1}{2}k \Rightarrow \mu = \frac{1}{2}\left(\frac{4}{3}\right) = ...$	M1	2.2a
	$\mu = \frac{2}{3} \Rightarrow \overrightarrow{ON} = \frac{2}{3}\mathbf{b}$ $\left\{\Rightarrow \overrightarrow{NB} = \frac{1}{3}\mathbf{b}\right\} \Rightarrow ON : NB = 2:1$ *	A1	2.1
		(2)	

Notes for Question 10

(a)	
M1:	Valid attempt to find $\overrightarrow{CM}$ using a combination of known vectors **a** and **b**
A1:	A simplified correct answer for $\overrightarrow{CM}$
Note:	Give M1 for $\overrightarrow{CM} = -\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\overrightarrow{CM} = (-2\mathbf{a} + \mathbf{b}) + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or for $\left\{\overrightarrow{CM} = \overrightarrow{OM} - \overrightarrow{OC} \Rightarrow\right\}$ $\overrightarrow{CM} = \frac{1}{2}(\mathbf{a} + \mathbf{b}) - 2\mathbf{a}$ **only o.e.**
(b)	
M1:	Uses $\overrightarrow{ON} = \overrightarrow{OC} + \lambda\overrightarrow{CM}$
A1*:	Correct proof
Note:	<u>**Special Case**</u>
	Give SC M1 A0 for the solution $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \lambda\overrightarrow{CM}$
	$\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \lambda\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right)$ $\left\{= \left(\frac{1}{2} - \frac{3}{2}\lambda\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\lambda\right)\mathbf{b}\right\}$
Note:	<u>**Alternative 1:**</u> Give M1 A1 for the following alternative solution: $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \overrightarrow{MN} \Rightarrow \overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AM} + \mu\overrightarrow{CM}$ $\overrightarrow{ON} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) + \mu\left(-\frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \left(\frac{1}{2} - \frac{3}{2}\mu\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}\mu\right)\mathbf{b}$ $\mu = \lambda - 1 \Rightarrow$ $\overrightarrow{ON} = \left(\frac{1}{2} - \frac{3}{2}(\lambda - 1)\right)\mathbf{a} + \left(\frac{1}{2} + \frac{1}{2}(\lambda - 1)\right)\mathbf{b} \Rightarrow \overrightarrow{ON} = \left(2 - \frac{3}{2}\lambda\right)\mathbf{a} + \frac{1}{2}\lambda\mathbf{b}$
(c)	**Way 1, Way 2 and Way 3**
M1:	Deduces that $\left(2 - \frac{3}{2}\lambda\right) = 0$ and attempts to find the value of λ

A1*:	Correct proof
(c)	**Way 4**
M1:	Complete attempt to find the value of μ
A1*:	Correct proof

Notes for Question 8 Continued	
Note:	Part (b) and part (c) can be marked together.
(a) Special Case	**<u>Special Case where the point *C* is believed to be below the origin *O*</u>** 
	Give Special Case M1 A0 in part (a) for $\left\{\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} \Rightarrow\right\}$ $\overrightarrow{CM} = 3\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$
	$\left\{\text{which leads to } \overrightarrow{CM} = \frac{5}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}\right\}$

<u>Question 9</u>

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2 + 2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2 + 2$ **cannot be divided by 4 to give an integer.**
- Students who write $n^2 + 2 = 4k \Rightarrow k = \frac{1}{4}n^2 + \frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

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All $n \in \mathbb{N}$ mod 4	0	1	2	3
All $n^2 \in \mathbb{N}$ mod 4	0	1	0	1
All $n^2+2 \in \mathbb{N}$ mod 4	2	3	2	3

Hence for all n, n^2+2 is not divisible by 4.

Question 9 (i)	Scheme	Marks	AOs

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up

(i)

M1: Awarded for setting up the proof for either the even or odd numbers.

A1: Concludes correctly with a reason why n^2+2 cannot be divisible by 4 for either n odd or even.

dM1: Awarded for setting up the proof for both even and odd numbers

A1: Fully correct proof with valid explanation and conclusion for all n

Example of an algebraic proof

For $n=2m$, $n^2+2=4m^2+2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n=2m+1$, $n^2+2=(2m+1)^2+2=\ldots$ FYI $\left(4m^2+4m+3\right)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4\left(m^2+m\right)+3$ ....AND states ......hence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n=2m$, $\dfrac{4m^2+2}{4}=m^2+\dfrac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n=2m+1$, $\dfrac{n^2+2}{4}=\dfrac{4m^2+4m+3}{4}=m^2+m+\dfrac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\dfrac{3}{4}$ ...AND states ...... hence for all n, n^2+2 is not divisible by 4	A1*	2.4
	(4)	

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Example of a proof via logic

When n is odd, "odd $\times$odd" = odd	M1	2.1
so n^2+2 is odd , so (when n is odd) n^2+2 cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$even" is a multiple of 4	dM1	2.1
Concludes that when n is even n^2+2 cannot be divisible by 4 because n^2 is divisible by 4…..AND STATES ……..trues for all n.	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction 'Assume that n^2+2 is divisible by 4 $\Rightarrow n^2+2=4k$ '	M1	2.1
$\Rightarrow n^2=4k-2=2(2k-1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then n^2+2 cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n.	A1*	2.4
	(4)	

A similar proof exists via contradiction where
A1: $n^2=2(2k-1)\Rightarrow n=\sqrt{2}\times\sqrt{2k-1}$
dM1: States that $2k-1$ is odd, so does not have a factor of 2, meaning that n is irrational

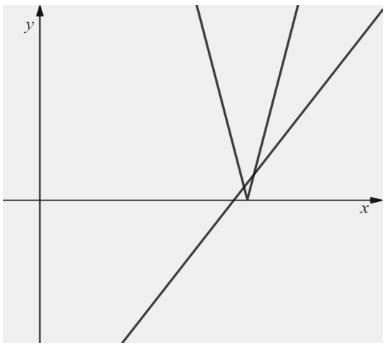
Question 9 (ii)	Scheme	Marks	AOs

(ii)
M1: States or implies 'sometimes true' or 'not always true' and gives an example where it is not true.
A1: and gives an example where it is true,
Proof using numerical values

SOMETIMES TRUE and chooses any number $x: 9.25<x<9.5$ and shows false Eg $x=9.4$ $\lvert 3x-28\rvert=0.2$ and $x-9=0.4$ ×	M1	2.3
Then chooses a number where it is true Eg $x=12$ $\lvert 3x-28\rvert=8$ $x-9=3$ ✓	A1	2.4

	(2)	

Graphical Proof

 States or implies ''sometimes true'' Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x -axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $\lvert 3x-28\rvert < x-9$ and points where $\lvert 3x-28\rvert > x-9$ oe in words like 'above' and 'below' or 'dips below at one point'	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x-28<x-9$ and $-3x+28<x-9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25<x<9.5$ or $9.25<x<9.\dot{3}$ or $9.\dot{3}<x<9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x-28\geqslant x-9$ and $-3x+28\geqslant x-9$	M1	2.3
States that it is false when $9.25<x<9.5$ or $9.25<x<9.\dot{3}$ or $9.\dot{3}<x<9.5$	A1	2.4
	(2)	

Question	Scheme	Marks	AOs
10	States $\left\{\lim\limits_{\delta x\to 0}\sum\limits_{x=4}^{9}\sqrt{x}\,\delta x \text{ is}\right\}\int_4^9 \sqrt{x}\,\mathrm{d}x$	B1	1.2
	$=\left[\frac{2}{3}x^{\frac{3}{2}}\right]_4^9$	M1	1.1b

	$= \frac{2}{3}\times 9^{\frac{3}{2}} - \frac{2}{3}\times 4^{\frac{3}{2}} = \frac{54}{3} - \frac{16}{3}$		
	$= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7	A1	1.1b
		(3)	
			(3 marks)

Notes for Question 10

B1:	States $\int_4^9 \sqrt{x}\,\mathrm{d}x$ with or without the '$\mathrm{d}x$'
M1:	Integrates $\sqrt{x}$ to give $\lambda x^{\frac{3}{2}}$; $\lambda \neq 0$
A1:	See scheme
Note:	You can imply B1 for $\left[\lambda x^{\frac{3}{2}}\right]_4^9$ or for $\lambda \times 9^{\frac{3}{2}} - \lambda \times 4^{\frac{3}{2}}$
Note:	Give B0 for $\int_1^9 \sqrt{x}\,\mathrm{d}x - \int_1^3 \sqrt{x}\,\mathrm{d}x$ or for $\int_3^9 \sqrt{x}\,\mathrm{d}x$ without reference to a correct $\int_4^9 \sqrt{x}\,\mathrm{d}x$
Note:	Give B1 M1 A1 for no working leading to a correct $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\int_4^9 \sqrt{x}\,\mathrm{d}x = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for $\left[\frac{2}{3}x^{\frac{3}{2}} + c\right]_4^9 = \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give B1 M1 A1 for no working followed by an answer $\frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7
Note:	Give M0 A0 for use of a trapezium rule method to give an answer of awrt 12.7, but allow B1 if $\int_4^9 \sqrt{x}\,\mathrm{d}x$ is seen in a trapezium rule method
Note:	Otherwise, give B0 M0 A0 for using the trapezium rule to give an answer of awrt 12.7

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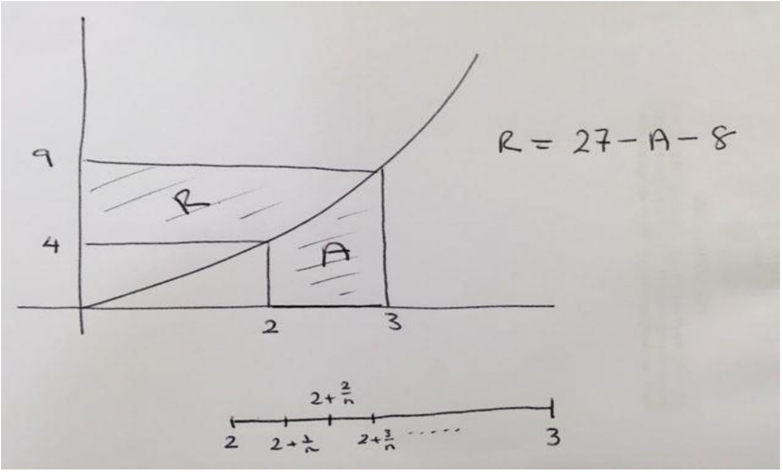
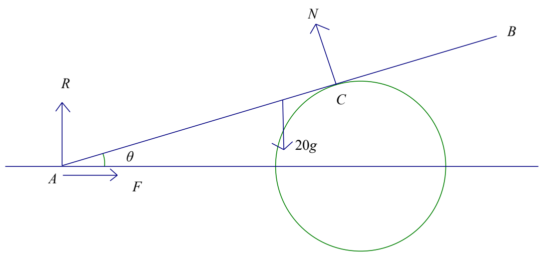
Notes for Question 10 Continued	
Alt	The following method is correct:  Area $(A) = \lim_{n\to\infty}\sum_{i=1}^{n}(x_i - x_{i-1})\mathrm{f}(x_i) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(2+\frac{i}{n}\right)^2$ $= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{4i}{n}\right) + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i^2}{n^2}\right)\right]$ $= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n}4 + \frac{4}{n^2}\sum_{i=1}^{n}i + \frac{1}{n^3}\sum_{i=1}^{n}i^2\right]$ $= \lim_{n\to\infty}\left[\frac{4n}{n} + \frac{4}{n^2}\left(\frac{1}{2}n(n+1)\right) + \frac{1}{n^3}\left(\frac{1}{6}n(n+1)(2n+1)\right)\right]$ $= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{4n^2+4n}{2n^2} + \frac{2n^3+3n^2+n}{6n^3}\right]$ $= \lim_{n\to\infty}\left[4 + 2 + \frac{2}{n} + \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right]$ $= 4 + 2 + \frac{1}{3} = \frac{19}{3}$ So, $\lim_{\delta x\to 0}\sum_{x=4}^{9}\sqrt{x}\,\delta x = \mathrm{Area}(R) = (3\times 9) - (2\times 4) - \frac{19}{3}$ $= \frac{38}{3}$ or $12\frac{2}{3}$ or awrt 12.7

Question	Scheme	Marks	AO
11(a)	Drum **smooth**, or no friction, (therefore reaction is perpendicular to the ramp)	B1	2.4
		(1)	

(b)	**N.B.** In (b), for a moments equation, if there is an extra $\sin\theta$ or $\cos\theta$ on a length, give M0 for the equation e.g. $\text{M}(A)$: $20g\times 4\cos\theta = 5N\sin\theta$ would be given M0A0		
	 		
	Possible equns $(\nearrow): F\cos\theta + R\sin\theta = 20g\sin\theta$ $(\nwarrow): N + R\cos\theta = 20g\cos\theta + F\sin\theta$ $(\uparrow) R + N\cos\theta = 20g$ $(\rightarrow): F = N\sin\theta$ $\text{M}(A)$: $20g\times 4\cos\theta = 5N$ $\text{M}(B)$: $3N + R\times 8\cos\theta = F\times 8\sin\theta + 20g\times 4\cos\theta$ $\text{M}(C)$: $R\times 5\cos\theta = F\times 5\sin\theta + 20g\times\cos\theta$ $\text{M}(G)$: $R\times 4\cos\theta = F\times 4\sin\theta + N$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $F = 42.14784$; $R = 51.49312$)		
	Alternative 1: using cpts along ramp (X) and perp to ramp(Y) Possible equations: $(\nearrow): X = 20g\sin\theta$ $(\nwarrow): Y + N = 20g\cos\theta$ $(\uparrow): X\sin\theta + Y\cos\theta + N\cos\theta = 20g$ $(\rightarrow): X\cos\theta = Y\sin\theta + N\sin\theta$ $\text{M}(A): 20g\times 4\cos\theta = 5N$ $\text{M}(B): 20g\times 4\cos\theta = 8Y + 3N$ $\text{M}(C): 20g\times\cos\theta = 5Y$ $\text{M}(G): 4Y = N\times 1$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $X = 54.88$; $Y = 37.632$)		

	Alternative 2: using horizontal cpt (H) and cpt perp to ramp (S) $(\nearrow): H\cos\theta = 20g\sin\theta$ $(\nwarrow): S + N = H\sin\theta + 20g\cos\theta$ $(\uparrow): S\cos\theta + N\cos\theta = 20g$ $(\rightarrow): H = S\sin\theta + N\sin\theta$ $\mathrm{M}(A): 20g \times 4\cos\theta = 5N$ $\mathrm{M}(B): 20g \times 4\cos\theta + H \times 8\sin\theta = 8S + 3N$ $\mathrm{M}(C): 20g \times \cos\theta + H \times 5\sin\theta = 5S$ $\mathrm{M}(G): 4S = N \times 1 + H \times 4\sin\theta$	M1 A1 M1 A1 M1 A1	3.3 1.1b 3.4 1.1b 3.4 1.1b
	(The values of the 3 unknowns are: $N = 150.528$; $H = 57.1666\ldots$; $S = 53.638666\ldots$)		
	Solve their 3 equations for F and R **OR** X and Y **OR** H and S	M1	1.1b
	$\lvert\text{Force}\rvert = \sqrt{R^2 + F^2}$ Main scheme **OR** $= \sqrt{X^2 + Y^2}$ Alternative 1 **OR** $= \sqrt{(H^2 + S^2 - 2HS\cos(90^o - \theta)}$ Alternative 2	M1	3.1b
	Magnitude = 67 or 66.5 (N)	A1	2.2a
		(9)	
(c)	Magnitude of the normal reaction (at C) will **decrease**.	B1	3.5a
		(1)	
		(11)	

Marks		Notes
11a	B1	Ignore any extra incorrect comments.
		Generally 3 independent equations required so **at least one moments equation.**: M1A1M1A1M1A1. More than 3 equations, give marks for the best 3. For each: M1 All terms required. Must be dimensionally correct so if a length is missing from a moments equation it's M0 Condone sin/cos confusion. A1 For a correct equation (trig ratios do not need to be substituted and allow e.g. cos(24/25) if they recover Enter marks on ePEN in order in which equations appear. **N.B.** If reaction at C is not perpendicular to the ramp, can only score marks for M(C) Allow use of (μR) for F
11b	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required, dim correct, condone sin/cos confusion
	A1	Correct unsimplified equation
		N.B. They can find F and R using only TWO equations, the 1st and 7th in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 1	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation

		N.B. They can find X and Y using only TWO equations, the 1^{st} and 7^{th} in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
Alt 2	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct. Condone sin/cos confusion.
	A1	Correct unsimplified equation
	M1	All terms required. Must be dimensionally correct.
	A1	Correct unsimplified equation
		N.B. They can find H and S using only TWO equations, the 1^{st} and 7^{th} in the list. Mark the better equation as M2A2 (-1 each error). Mark the second equation as M1A1
	M1	Substitute for trig and solve for their two cpts. This is an independent mark but must use 3 equations (unless it's the special case when 2 is sufficient)
	M1	Use Pythagoras to find magnitude (this is an independent M mark but must have found a value for F (or X) and a value for R (or Y)) **OR** a complete method to find magnitude e.g. cosine rule but must have found a value for H and a value for S
	A1	Correct answer only
	B1	Ignore reasons