Pearson Edexcel

Mark Scheme (Result)

November 2021

Pearson Edexcel GCE Further Mathematics Advanced Subsidiary Level in Mathematics Paper 1 8MA0/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\surd$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5, x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
			(3 marks)

Notes

M1: Attempts to find the critical values using an algebraic method. Condone slips but an allowable method should be used and two critical values should be found

M1: Chooses the outside region for their critical values. This may appear in incorrect inequalities such as $5 < x < -4$

A1: Presents in set notation as required $\{x : x < -4\} \cup \{x : x > 5\}$ Accept $\{x < -4 \cup x > 5\}$.

Do not accept $\{x < -4,\ x > 5\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.

Question	Scheme	Marks	AOs
2	$\dfrac{9^{x-1}}{3^{y+2}}=81\Rightarrow\dfrac{3^{2x-2}}{3^{y+2}}=3^4$ or $\dfrac{9^{x-1}}{3^{y+2}}=81\Rightarrow\dfrac{9^{x-1}}{9^{\frac{1}{2}(y+2)}}=9^2$	M1	1.1b
	$\Rightarrow 2x-2-y-2=4\Rightarrow y=$ or $\Rightarrow x-1-\dfrac{1}{2}y-1=2\Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
		(3)	
Alt	Eg. $\log_3\left(\dfrac{9^{x-1}}{3^{y+2}}\right)=\log_3 81$	M1	1.1b
	$\Rightarrow (x-1)\log_3\left(9^{x-1}\right)-(y+2)\log_3\left(3^{y+2}\right)=4$ $\Rightarrow 2(x-1)-y-2=4\Rightarrow y=$	dM1	1.1b
	$\Rightarrow y=2x-8$	A1	1.1b
			(3 marks)

Notes

M1: Attempts to set 9^{x-1} and 81 as powers of 3. Condone $9^{x-1}=3^{2x-1}$ and $9^{x-1}=3^{3x-3}$.
Alternatively attempts to write each term as a logarithm of base 3 or 9. You must see the base written to award this mark.

dM1: Attempts to use the addition (or subtraction) index law, or laws or logarithms, correctly and rearranges the equation to reach y in terms of x. Condone slips in their rearrangement.

A1: $y=2x-8$

Question	Scheme	Marks	
3	$\int \frac{3x^4-4}{2x^3}\,dx = \int \frac{3}{2}x - 2x^{-3}\,dx$	M1 A1	1.1b 1.1b
	$= \frac{3}{2}\times\frac{x^2}{2} - 2\times\frac{x^{-2}}{-2} \quad (+c)$	dM1	3.1a
	$= \frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e	A1	1.1b
		(4)	
		(4 marks)	

Notes:

(i)

M1: Attempts to divide to form a sum of terms. Implied by two terms with one correct index.

$\int \frac{3x^4}{2x^3} - \frac{4}{2x^3}\,dx$ scores this mark.

A1: $\int \frac{3}{2}x - 2x^{-3}\,dx$ o.e such as $\frac{1}{2}\int\left(3x - 4x^{-3}\right)\,dx$. The indices must have been processed on both terms. Condone spurious notation or lack of the integral sign for this mark.

dM1: For the full strategy to integrate the expression. It requires two terms with one correct index.

Look for $= ax^p + bx^q$ where $p = 2$ or $q = -2$

A1: Correct answer $\frac{3}{4}x^2 + \frac{1}{x^2} + c$ o.e. such as $\frac{3}{4}x^2 + x^{-2} + c$

Question	Scheme	Marks	AO
4(a)	Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ E.g. $(-24\mathbf{i}-10\mathbf{j}) = -2\times(12\mathbf{i}+5\mathbf{j})$	M1	1.1b
	Explains that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ (and the stone is travelling in a straight line) the stone passes through the point *O*.	A1	2.4
		(2)	
(b)	Attempts distance $AB = \sqrt{(12+24)^2+(10+5)^2}$	M1	1.1b
	Attempts speed = $\dfrac{\sqrt{(12+24)^2+(10+5)^2}}{4}$	dM1	3.1a
	Speed = 9.75 ms^{-1}	A1	3.2a
		(3)	
			(5 marks)
Alt(a)	Attempts to find the equation of the line which passes through *A* and *B* E.g. $y-5=\dfrac{5+10}{12+24}(x-12)$ $\left(y=\dfrac{5}{12}x\right)$	M1	1.1b
	Shows that when $x=0,\ y=0$ and concludes the stone passes through the point *O*.	A1	2.4

Notes

(a)

M1: Attempts to compare the two position vectors. Allow an attempt using two of $\overrightarrow{AO}, \overrightarrow{OB}$ or $\overrightarrow{AB}$ either way around.
E.g. States that $(-24\mathbf{i}-10\mathbf{j}) = -2\times(12\mathbf{i}+5\mathbf{j})$
Alternatively, allow an attempt finding the gradient using any two of *AO*, *OB* or *AB*

Alternatively attempts to find the equation of the line through *A* and *B* proceeding as far as $y=\ldots x$ Condone sign slips.

A1: States that as $\overrightarrow{AO}$ is parallel to $\overrightarrow{OB}$ or as *AO* is parallel to *OB* (and the stone is travelling in a straight line) the stone passes through the point *O*. Alternatively, shows that the point $(0,0)$ is on the line and concludes (the stone) passes through the point *O*.

(b)

M1: Attempts to find the distance *AB* using a correct method.
Condone slips but expect to see an attempt at $\sqrt{a^2+b^2}$ where *a* or *b* is correct

dM1: Dependent upon the previous mark. Look for an attempt at $\dfrac{\text{distance } AB}{4}$

A1: 9.75 ms^{-1} Requires units

Question	Scheme	Marks	AO
5(a)	Attempts to find the value of $\frac{dy}{dx}$ at $x=2$	M1	1.1b
	$\frac{dy}{dx}=6x \Rightarrow$ gradient of tangent at P is 12	A1	1.1b
		(2)	
(b)	Gradient $PQ = \frac{3(2+h)^2-2-10}{(2+h)-2}$ oe	B1	1.1b
	$=\frac{3(2+h)^2-12}{(2+h)-2}=\frac{12h+3h^2}{h}$	M1	1.1b
	$=12+3h$	A1	2.1
		(3)	
(c)	Explains that as $h \to 0,\ 12+3h \to 12$ and states that the gradient of the chord tends to the gradient of (the tangent to) the curve	B1	2.4
		(1)	
			(6 marks)

Notes

(a)

M1: Attempts to differentiate, allow $3x^2-2 \to \ldots x$ and substitutes $x=2$ into their answer

A1: cso $\frac{dy}{dx}=6x \Rightarrow$ gradient of tangent at P is 12

(b)

B1: Correct expression for the gradient of the chord seen or implied.

M1: Attempts $\frac{\delta y}{\delta x}$, condoning slips, and attempts to simplify the numerator. The denominator must be h

A1: cso $12+3h$

(c)

B1: Explains that as $h \to 0,\ 12+3h \to 12$ and states that the gradient of the chord tends to the gradient of the curve

Question	Scheme	Marks	AO
6 (a)	$3x^3 - 17x^2 - 6x = 0 \Rightarrow x(3x^2 - 17x - 6) = 0$	M1	1.1a
	$\Rightarrow x(3x+1)(x-6) = 0$	dM1	1.1b
	$\Rightarrow x = 0, -\frac{1}{3}, 6$	A1	1.1b
		(3)	
(b)	Attempts to solve $(y-2)^2 = n$ where n is any solution $\ldots 0$ to (a)	M1	2.2a
	Two of $2, 2 \pm \sqrt{6}$	A1ft	1.1b
	All three of $2, 2 \pm \sqrt{6}$	A1	2.1
		(3)	
			(6 marks)

Notes

(a)

M1: Factorises out or cancels by x to form a quadratic equation.

dM1: Scored for an attempt to find x. May be awarded for factorisation of the quadratic or use of the quadratic formula.

A1: $x = 0, -\frac{1}{3}, 6$ and no extras

(b)

M1: Attempts to solve $(y-2)^2 = n$ where n is any solution $\ldots 0$ to (a). At least one stage of working must be seen to award this mark. Eg $(y-2)^2 = 0 \Rightarrow y = 2$

A1ft: Two of $2, 2 \pm \sqrt{6}$ but follow through on $(y-2)^2 = n \Rightarrow y = 2 \pm \sqrt{n}$ where n is a positive solution to part (a). (Provided M1 has been scored)

A1: All three of $2, 2 \pm \sqrt{6}$ and no extra solutions. (Provided M1A1 has been scored)

Question	Scheme	Marks	AOs
7 (a)	Sets $50 = 7\times14\sin(SPQ)$ oe	B1	1.2
	Finds $180° - \arcsin\left("\frac{50}{98}"\right)$	M1	1.1b
	$= 149.32°$	A1	1.1b
		(3)	
(b)	Method of finding SQ $SQ^2 = 14^2 + 7^2 - 2\times14\times7\cos"149.32"$	M1	1.1b
	$= 20.3$ cm	A1	1.1b
		(2)	
			(5 marks)
Alt(a)	States or uses $14h = 50$ or $7h_1 = 50$	B1	1.2
	Full method to find obtuse $\angle SPQ$. In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$	M1	1.1b
	awrt $149.32°$	A1	1.1b

Notes

(a)

B1: Sets $50 = 7\times14\sin(SPQ)$ oe

M1: Attempts the correct method of finding obtuse $\angle SPQ$. See scheme.

A1: awrt $149.32°$

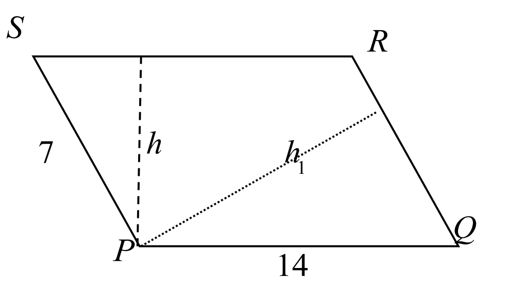
(b)

M1: A correct method of finding SQ using their $\angle SPQ$.

$SQ^2 = 14^2 + 7^2 - 2\times14\times7\cos"149.32"$ scores this mark.

A1: awrt 20.3 cm (condone lack of units)

Alt(a)



B1: States or uses $14h = 50$ or $7h_1 = 50$

M1: Full method to find obtuse $\angle SPQ$.

In this case it is $90° + \arccos\left(\frac{h}{7}\right)$ or $90° + \arccos\left(\frac{h_1}{14}\right)$

A1: awrt $149.32°$

Question	Scheme	Marks	AOs

8 (a)	$(2+ax)^8$ Attempts the term in $x^5 = {}^8C_5 2^3 (ax)^5 = 448a^5x^5$	M1 A1	1.1a 1.1b
	Sets $448a^5 = 3402 \Rightarrow a^5 = \frac{243}{32}$	M1	1.1b
	$\Rightarrow a = \frac{3}{2}$	A1	1.1b
		(4)	
(b)	Attempts either term. So allow for 2^8 or ${}^8C_4 2^4 a^4$	M1	1.1b
	Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$	dM1	2.1
	$= 256 + 5670 = 5926$	A1	1.1b
		(3)	
			(7 marks)

Notes

(a)

M1: An attempt at selecting the correct term of the binomial expansion. If all terms are given then the correct term must be used. Allow with a missing bracket ${}^8C_5 2^3 ax^5$ and left without the binomial coefficient expanded

A1: $448a^5x^5$ Allow unsimplified but ${}^8C_5$ must be "numerical"

M1: Sets their $448a^5 = 3402$ and proceeds to $\Rightarrow a^k = ...$ where $k \in \mathbb{N} \quad k \neq 1$

A1: Correct work leading to $a = \frac{3}{2}$

(b)

M1: Finds either term required. So allow for 2^8 or ${}^8C_4 2^4 a^4$ (even allowing with *a*)

dM1: Attempts the sum of both terms $2^8 + {}^8C_4 2^4 a^4$

A1: cso 5926

Question	Scheme	Marks	
9	$\int_k^9 \frac{6}{\sqrt{x}}\,dx = \left[ax^{\frac{1}{2}}\right]_k^9 = 20 \Rightarrow 36 - 12\sqrt{k} = 20$	M1 A1	1.1b 1.1b
	Correct method of solving Eg. $36 - 12\sqrt{k} = 20 \Rightarrow k =$	dM1	3.1a
	$\Rightarrow k = \frac{16}{9}$ oe	A1	1.1b
		(4)	
			(4 marks)

Notes:

M1: For setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$

A1: A correct equation involving *p* Eg. $36 - 12\sqrt{k} = 20$

dM1: For a whole strategy to find *k*. In the scheme it is awarded for setting $\left[ax^{\frac{1}{2}}\right]_k^9 = 20$, using both limits and proceeding using correct index work to find *k*. It cannot be scored if $k^{\frac{1}{2}} < 0$

A1: $k = \frac{16}{9}$

Question	Scheme	Marks	
10(a)	Selects a correct strategy. E.g uses an odd number is $2k \pm 1$	B1	3.1a
	Attempts to simplify $(2k \pm 1)^3 - (2k \pm 1) = ...$	M1	2.1
	……….and factorise $8k^3 \pm 12k^2 \pm 4k = 4k\left(2k^2 \pm 3k \pm 1\right) =$	dM1	1.1b
	Correct work with statement $4\times..$ is a multiple of 4	A1	2.4
		(4)	
(b)	Any counter example with correct statement. Eg. $2^3 - 2 = 6$ which is not a multiple of 4	B1	2.4
		(1)	
			(5 marks)
Alt (a)	Selects a correct strategy. Factorises $k^3 - k = k(k-1)(k+1)$	B1	3.1a
	States that if k is odd then both $k-1$ and $k+1$ are even	M1	2.1
	States that $k-1$ multiplied by $k+1$ is therefore a multiple of 4	dM1	1.1b
	Concludes that $k^3 - k$ is a multiple of 4 as it is odd $\times$ multiple of 4	A1	2.4
		(4)	

Notes:

(a)
Note: May be in any variable (condone use of n)

B1: Selects a correct strategy. E.g uses an odd number is $2k \pm 1$

M1: Attempts $(2k \pm 1)^3 - (2k \pm 1) = ...$ Condone errors in multiplying out the brackets and invisible brackets for this mark. Either the coefficient of the k term or the constant of $(2k \pm 1)^3$ must have changed from attempting to simplify.

dM1: Attempts to take a factor of 4 or $4k$ from their cubic

A1: Correct work with statement $4\times..$ is a multiple of 4

(b)

B1: Any counter example with correct statement.

Question	Scheme	Marks	AO
11 (a)	35 (km^2)	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45e^{14c} \Rightarrow 45e^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \frac{1}{14}\ln\left(\frac{20}{45}\right) = \cdots - 0.0579$	dM1	3.1b
	$A = 80 - 45e^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
			(6 marks)

Notes

(a)

B1: Uses the equation of the model to find that 35 (km^2) of the reserve was covered on 1st January 2005. Do not accept eg. 35 m^2

(b)

M1: Sets their $60 = 80 - 45e^{14c} \Rightarrow Ae^{14c} = B$

A1: $45e^{14c} = 20$ or equivalent.

dM1: A full and careful method using precise algebra, correct log laws and a knowledge that e^x and $\ln x$ are inverse functions and proceeds to a value for c.

A1: Gives a complete equation for the model $A = 80 - 45e^{-0.0579t}$

(c)

B1: Gives a suitable interpretation (See scheme)

Question	Scheme	Marks	AO
12 (i)	Uses $\cos^2\theta = 1-\sin^2\theta$ $5\cos^2\theta = 6\sin\theta \Rightarrow 5\sin^2\theta + 6\sin\theta - 5 = 0$	M1 A1	1.2 1.1b
	$\Rightarrow \sin\theta = \dfrac{-3+\sqrt{34}}{5} \Rightarrow \theta = ...$	dM1	3.1a
	$\Rightarrow \theta = 34.5°, 145.5°, 394.5°$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	One of • They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$) • They do not find all the solutions of $\cos x = \frac{3}{5}$ (in the given range) or they missed the solution $x = -53.1°$	B1	2.3
	Both of the above	B1	2.3
		(2)	
(ii) (b)	Sets $5\alpha + 40° = 720° - 53.1°$	M1	3.1a
	$\alpha = 125°$	A1	1.1b
		(2)	
			(9 marks)

Notes

(i)

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to form a 3TQ in $\sin\theta$

A1: Correct 3TQ=0 $5\sin^2\theta + 6\sin\theta - 5 = 0$

dM1: Solves their 3TQ in $\sin\theta$ to produce one value for θ. It is dependent upon having used $\cos^2\theta = \pm 1 \pm \sin^2\theta$

A1: Two of awrt $\theta = 34.5°, 145.5°, 394.5°$ (or if in radians two of awrt 0.60, 2.54, 6.89)

A1: All three of awrt $\theta = 34.5°, 145.5°, 394.5°$ and no other values

(i) (a)

See scheme

(ii)(b)

M1: Sets $5\alpha + 40° = 666.9°$ o.e.

A1: awrt $\alpha = 125°$

Question	Scheme		Marks	AOs
13 (a)	$\log_{10} h = 2.25 - 0.235\log_{10} m$ $\Rightarrow h = 10^{2.25-0.235\log_{10} m}$ $\Rightarrow h = 10^{2.25} \times m^{-0.235}$	$h = pm^q$ $\Rightarrow \log_{10} h = \log_{10} p + \log_{10} m^q$ $\Rightarrow \log_{10} h = \log_{10} p + q\log_{10} m$	M1	1.1b
	Either one of $p = 10^{2.25}$ $q = -0.235$	Or either one of $\log_{10} p = 2.25$ $q = -0.235$	A1	1.1b
	$\Rightarrow p = 178$ and $q = -0.235$		A1	2.2a
			(3)	
(b)	$h = "178" \times 5^{"-0.235"}$	$\log_{10} h = "2.25" - "0.235"\log_{10} 5$	M1	3.1b
	$h = 122$	$h = 122$	A1	1.1b
	Reasonably accurate (to 2 sf) so suitable		A1ft	3.2b
			(3)	
(c)	"p" would be the (resting) heart rate (in bpm) of a mammal with a mass of 1 kg		B1	3.4
			(1)	
				(7 marks)

Notes

(a)

M1: Establishes a link between $h = pm^q$ and $\log_{10} h = 2.25 - 0.235\log_{10} m$.
May be implied by a correct equation in p or q

A1: For a correct equation in p or q

A1: $p = 178$ and $q = -0.235$

(b)

M1: Uses either model to set up an equation in h (or m)

A1: h = awrt 122. Condone h = awrt 122 bpm

A1ft: Comments on the suitability of the model. Follow through on their answer.

Requires a comment consistent with their answer from using the model.

E.g. It is a suitable model as it is only "3" bpm away from the real value ✓
Do not allow an argument stating that it should be the same.
It is an unsuitable model as "122" bpm is not equal to 119 bpm ✗

(c)

B1: "p" would be the (resting) heart rate of a mammal with a mass of 1 kg

Question	Scheme	Marks	
14 (a)	$f(x) = -3x^2 + 12x + 8 = -3(x \pm 2)^2 + \ldots$	M1	1.1b
	$= -3(x-2)^2 + \ldots$	A1	1.1b
	$= -3(x-2)^2 + 20$	A1	1.1b
		(3)	
(b)	Coordinates of $M = (2, 20)$	B1ft B1ft	1.1b 2.2a
		(2)	
(c)	$\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x$	M1 A1	1.1b 1.1b
	Method to find $R =$ their $2 \times 20 - \int_0^2 \left(-3x^2 + 12x + 8\right) dx$	M1	3.1a
	$R = 40 - \left[-2^3 + 24 + 16\right]$	dM1	1.1b
	$= 8$	A1	1.1b
		(5)	
			(10 marks)
Alt(c)	$\int 3x^2 - 12x + 12 \, dx = x^3 - 6x^2 + 12x$	M1 A1	1.1b 1.1b
	Method to find $R = \int_0^2 3x^2 - 12x + 12 \, dx$	M1	3.1a
	$R = 2^3 - 6 \times 2^2 + 12 \times 2$	dM1	1.1b
	$= 8$	A1	1.1b

Notes:

(a)

M1: Attempts to take out a common factor and complete the square. Award for $-3(x \pm 2)^2 + \ldots$

Alternatively attempt to compare $-3x^2 + 12x + 8$ to $ax^2 + 2abx + ab^2 + c$ to find values of a and b

A1: Proceeds to a form $-3(x-2)^2 + \ldots$ or via comparison finds $a = -3, b = -2$

A1: $-3(x-2)^2 + 20$

(b)

B1ft: One correct coordinate

B1ft: Correct coordinates. Allow as $x = ..., y = ...$
Follow through on their $(-b, c)$

(c)

M1: Attempts to integrate. Award for any correct index

A1: $\int -3x^2 + 12x + 8 \, dx = -x^3 + 6x^2 + 8x \ (+c)$ (which may be unsimplified)

M1: Method to find area of R. Look for their $2 \times "20" - \int_0^{"2"} f(x) \, dx$

dM1: Correct application of limits on their integrated function. Their 2 must be used

A1: Shows that area of $R = 8$

Question	Scheme	Marks	AO
15 (a)	Deduces the line has gradient "-3" and point $(7,4)$ Eg $y-4=-3(x-7)$	M1	2.2a
	$y=-3x+25$	A1	1.1b
		(2)	
(b)	Solves $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously	M1	3.1a
	$P=\left(\frac{15}{2},\frac{5}{2}\right)$ oe	A1	1.1b
	Length $PN=\sqrt{\left(\frac{15}{2}-7\right)^2+\left(4-\frac{5}{2}\right)^2}=\left(\sqrt{\frac{5}{2}}\right)$	M1	1.1b
	Equation of C is $(x-7)^2+(y-4)^2=\frac{5}{2}$ o.e.	A1	1.1b
		(4)	
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C using vectors Eg: $\begin{pmatrix}7.5\\2.5\end{pmatrix}+2\times\begin{pmatrix}-0.5\\1.5\end{pmatrix}$	M1	3.1a
	Substitutes their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$ to find k	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	
			(9 marks)
(c)	Attempts to find where $y=\frac{1}{3}x+k$ meets C via simultaneous equations proceeding to a 3TQ in x (or y) FYI $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$	M1	3.1a
	Uses $b^2-4ac=0$ oe and proceeds to $k=...$	M1	2.1
	$k=\frac{10}{3}$	A1	1.1b
		(3)	

Notes:

(a)

M1: Uses the idea of perpendicular gradients to deduce that gradient of PN is -3 with point $(7,4)$ to find the equation of line PN

So sight of $y-4=-3(x-7)$ would score this mark

If the form $y=mx+c$ is used expect the candidates to proceed as far as $c=$ ... to score this mark.

A1: Achieves $y=-3x+25$

(b)

M1: Awarded for an attempt at the key step of finding the coordinates of point *P*. ie for an attempt at solving their $y=-3x+25$ and $y=\frac{1}{3}x$ simultaneously. Allow any methods (including use of a calculator) but it must be a valid attempt to find both coordinates.

A1: $P=\left(\frac{15}{2},\frac{5}{2}\right)$

M1: Uses Pythagoras' Theorem to find the radius or radius ^{2} using their $P=\left(\frac{15}{2},\frac{5}{2}\right)$ and $(7,4)$. There must be an attempt to find the difference between the coordinates in the use of Pythagoras

A1: Full and careful work leading to a correct equation. Eg $(x-7)^2+(y-4)^2=\frac{5}{2}$ or its expanded form. Do not accept $(x-7)^2+(y-4)^2=\left(\sqrt{\frac{5}{2}}\right)^2$

(c)

M1: Attempts to find where $y=\frac{1}{3}x+k$ meets *C* using a vector approach

M1: For a full method leading to k . Scored for substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ in $y=\frac{1}{3}x+k$

A1: $k=\frac{10}{3}$ only

Alternative I

M1: For solving $y=\frac{1}{3}x+k$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$ and creating a quadratic eqn of the form $ax^2+bx+c=0$ **where both *b* and *c* are dependent upon *k***. The terms in x^2 and *x* must be collected together or implied to have been collected by their correct use in "b^2-4ac"

FYI the correct quadratic is $\frac{10}{9}x^2+\left(\frac{2}{3}k-\frac{50}{3}\right)x+k^2-8k+\frac{125}{2}=0$ oe

M1: For using the discriminant condition $b^2-4ac=0$ to find k. It is not dependent upon the previous M and may be awarded from only one term in *k*.
Award if you see use of correct formula but it would be implied by ± correct roots

A1: $k=\frac{10}{3}$ only

Alternative II

M1: For solving $y=-3x+25$ with their $(x-7)^2+(y-4)^2=\frac{5}{2}$, creating a 3TQ and solving.

M1: For substituting their $\left(\frac{13}{2},\frac{11}{2}\right)$ into $y=\frac{1}{3}x+k$ and finding k

A1: $k=\frac{10}{3}$ only

Question	Scheme	Marks	AO
16 (a) (i)	Uses $\frac{\mathrm{d}y}{\mathrm{d}x}=-3$ at $x=2 \Rightarrow 12a+60-39=-3$	M1	1.1b
	Solves a correct equation and shows one correct intermediate step $12a+60-39=-3 \Rightarrow 12a=-24 \Rightarrow a=-2$ *	A1*	2.1
(a) (ii)	Uses the fact that $(2,10)$ lies on C $10=8a+60-78+b$	M1	3.1a
	Subs $a=-2$ into $10=8a+60-78+b \Rightarrow b=44$	A1	1.1b
		(4)	
(b)	$\mathrm{f}(x)=-2x^3+15x^2-39x+44 \Rightarrow \mathrm{f}'(x)=-6x^2+30x-39$	B1	1.1b
	Attempts to show that $-6x^2+30x-39$ has no roots Eg. calculates $b^2-4ac=30^2-4\times-6\times-39=-36$	M1	3.1a
	States that as $\mathrm{f}'(x)\neq 0 \Rightarrow$ hence $\mathrm{f}(x)$ has no turning points *	A1*	2.4
		(3)	
(c)	$-2x^3+15x^2-39x+44 \equiv (x-4)\left(-2x^2+7x-11\right)$	M1 A1	1.1b 1.1b
		(2)	
(d)	Deduces either intercept. $(0,44)$ or $(20,0)$	B1 ft	1.1b
	Deduces both intercepts $(0,44)$ and $(20,0)$	B1 ft	2.2a
		(2)	
		(11 marks)	

Notes

(a)(i)

M1: Attempts to use $\frac{\mathrm{d}y}{\mathrm{d}x}=-3$ at $x=2$ to form an equation in a. Condone slips but expect to see two of the powers reduced correctly

A1*: Correct differentiation with one correct intermediate step before $a=-2$

(a)(ii)

M1: Attempts to use the fact that $(2,10)$ lies on C by setting up an equation in a and b with $a=-2$ leading to $b=\ldots$

A1: $b=44$

(b)

B1: $\mathrm{f}'(x)=-6x^2+30x-39$ oe

M1: Correct attempt to show that "$-6x^2+30x-39$" has no roots.
This could involve an attempt at

- finding the numerical value of b^2-4ac
- finding the roots of $-6x^2+30x-39$ using the quadratic formula (or their calculator)
- completing the square for $-6x^2+30x-39$

A1*: A fully correct method with reason and conclusion. Eg as $b^2-4ac=-36<0, \mathrm{f}'(x)\neq 0$ meaning that no stationary points exist

(c)

M1: For an attempt at division (seen or implied) Eg $-2x^3+15x^2-39x+b \equiv (x-4)\left(-2x^2 ... \pm \frac{b}{4}\right)$

A1: $(x-4)(-2x^2+7x-11)$ Sight of the quadratic with no incorrect working seen can score both marks.

(d)

See scheme. You can follow through on their value for b