C2, IYGB, PAPER M

1. a) BY INSPECTION OR LONG DIVISION

$$\begin{array}{r} 6x^2 - x - 2 \\ x-1 \overline{)\,6x^3 - 7x^2 - x + 2} \\ \underline{-6x^3 + 6x^2} \\ -x^2 - x + 2 \\ \underline{x^2 - x} \\ -2x + 2 \\ \underline{2x - 2} \\ 0 \end{array}$$

$\therefore$ $a = 6$, $b = -1$, $c = -2$

b) $6x^3 - 7x^2 - x + 2 = 0$

$(x-1)(6x^2 - x - 2) = 0$

$(x-1)(3x-2)(2x+1) = 0$

$\therefore x = 1,\ -\frac{1}{2},\ \frac{2}{3}$

2. a) b) $x^2 + y^2 - 8x + 6y = 0$

$x^2 - 8x + y^2 + 6y = 0$

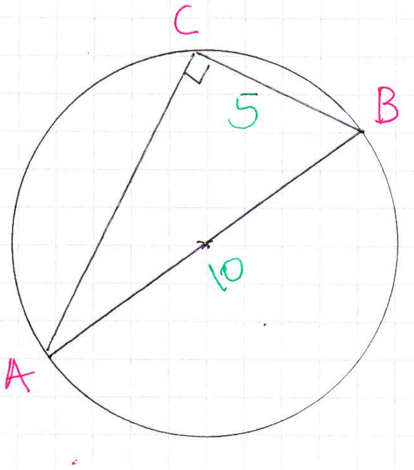
$(x-4)^2 - 16 + (y+3)^2 - 9 = 0$

$(x-4)^2 + (y+3)^2 = 25$

$\therefore$ CENTRE AT $(4,-3)$

RADIUS IS 5

c)



- EVIDENTLY $|AB|$ IS A DIAMETER SINCE $|AB| = 10 = 2 \times 5$
- BY PYTHAGORAS

$|BC|^2 + |CA|^2 = |AB|^2$

$5^2 + |CA|^2 = 10^2$

$25 + |AC|^2 = 100$

$|AC|^2 = 75$

$|AC| = \sqrt{75} = 5\sqrt{3}$

3.

$$\frac{\sin x - \cos x}{\cos x} = 2$$

$\Rightarrow \sin x - \cos x = 2\cos x$

$\Rightarrow \sin x = 3\cos x$

$\Rightarrow \frac{\sin x}{\cos x} = \frac{3\cos x}{\cancel{\cos x}}$

$\Rightarrow \tan x = 3$

$\arctan(3) = 71.57$

$x = 71.57 \pm 180n \quad n = 0,1,2,3 \ldots$

$x_1 = 71.57°$

$x_2 = 251.57°$

4.

- $y = x^3 - 6x^2 + 12x - 5$
- $\frac{dy}{dx} = 3x^2 - 12x + 12$

SOLVE FOR ZERO

$\Rightarrow 3x^2 - 12x + 12 = 0$

$\Rightarrow x^2 - 4x + 4 = 0$

$\Rightarrow (x-2)^2 = 0$

$\Rightarrow x = 2$

$\therefore y = 2^3 - 6\times2^2 + 12\times2 - 5$

$y = 8 - 24 + 24 - 5$

$y = 3$

$\therefore (2,3)$

$\frac{d^2y}{dx^2} = 6x - 12$

$\left.\frac{d^2y}{dx^2}\right|_{x=2} = 0$

POSSIBLE POINT OF INFLEXION

$\frac{d^3y}{dx^3} = 6$

$\left.\frac{d^3y}{dx^3}\right|_{x=2} = 6 \neq 0$

$\therefore (2,3)$ IS A STATIONARY POINT OF INFLEXION

5.

USING THE MEASUREMENTS PROVIDED

AREA $\approx \frac{\text{THICKNESS}}{2}\left[\text{FIRST} + \text{LAST} + 2\times\text{REST}\right]$

$\approx \frac{3}{2}\left[3.85 + 0 + 2(5.20 + 5.50 + 5.20 + 3.85 + 3)\right]$

≈ 74

6. a) $5^{2x-1} = 4^{300}$

$\Rightarrow \log 5^{2x-1} = \log 4^{300}$

$\Rightarrow (2x-1)\log 5 = 300\log 4$

$\Rightarrow 2x-1 = \dfrac{300\log 4}{\log 5}$

$\Rightarrow 2x-1 = 258.4059\ldots$

$\Rightarrow x \simeq 129.70$

$\Rightarrow x \simeq 130$ (3 sf)

b) $2^{y+1} = \dfrac{10}{2^y}$

$2^{y+1} \times 2^y = 10$

$2^{2y+1} = 10$

$\log 2^{2y+1} = \log 10$

$(2y+1)\log 2 = 1$

$2y+1 = \dfrac{1}{\log 2}$

$2y+1 = 3.32\ldots$

$y \simeq 1.16$ 3 sf.

ALTERNATIVE FOR (b)

$2^{y+1} = \dfrac{10}{2^y}$

$\Rightarrow \log\left(2^{y+1}\right) = \log\left(\dfrac{10}{2^y}\right)$

$\Rightarrow (y+1)\log 2 = \log 10 - \log 2^y$

$\Rightarrow (y+1)\log 2 = 1 - y\log 2$

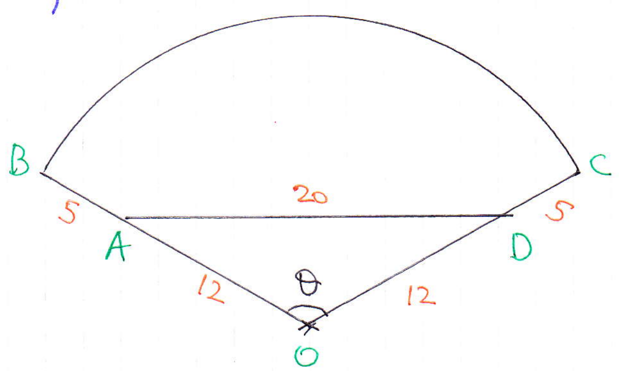
$\Rightarrow (y+1)\log 2 + y\log 2 = 1$

$\Rightarrow (2y+1)\log 2 = 1$

$\Rightarrow 2y+1 = \dfrac{1}{\log 2}$

ETC AS BEFORE

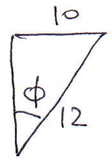
7. a)



• BY THE COSINE RULE ON $O\hat{A}D$

$$20^2 = 12^2 + 12^2 - 2\times12\times12\cos\theta$$
$$400 = 144 + 144 - 288\cos\theta$$
$$288\cos\theta = -112$$
$$\cos\theta = -\frac{7}{18}$$
$$\theta \simeq 1.97022\ldots$$
$$\theta \simeq 1.97^c$$

(CAN ALSO BE FOUND BY SPLITTING THE TRIANGLE INTO 2 RIGHT ANGLE TRIANGLES



$\sin\phi = \frac{10}{12}$

$\phi \simeq 0.9851\ldots$

$\theta = 2\phi = 1.97^c$)

b) AREA OF SECTOR $= \frac{1}{2}r^2\theta^c$

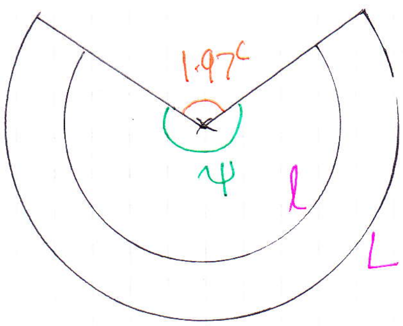
$$= \frac{1}{2}\times17^2\times1.97$$
$$\simeq 284.697\ldots$$

AREA OF TRIANGLE $= \frac{1}{2}\times12\times12\times\sin1.46^c$

$$\simeq 66.332\ldots$$

∴ AREA OF THE STAGE IS $284.697 - 66.332 \simeq 218\ m^2$ (OR $219\ m^2$)

c) $\boxed{\psi = 2\pi - 1.97 \simeq 4.31}$



• $l = r\psi = 12\times4.31 = 51.72$

$51.72 \div 0.82 \simeq 63$ SEATS

• $L = R\psi = 15.3\times4.31 = 65.943\ldots$

$65.943 \div 0.82 \simeq 80$ SEATS

$r = 12$
$R = 15.3$

∴ AN EXTRA 17 SEATS

8.

$$a + ar + ar^2 = 33500$$
$$a(1+r+r^2) = 33500$$
$$2000(1+r+r^2) = 33500$$
$$1+r+r^2 = \frac{67}{4}$$
$$4+4r+4r^2 = 67$$
$$4r^2+4r-63 = 0$$
$$(2r-7)(2r+9) = 0$$
$$r = \begin{cases} 7/2 \\ \cancel{-9/2} \end{cases}$$

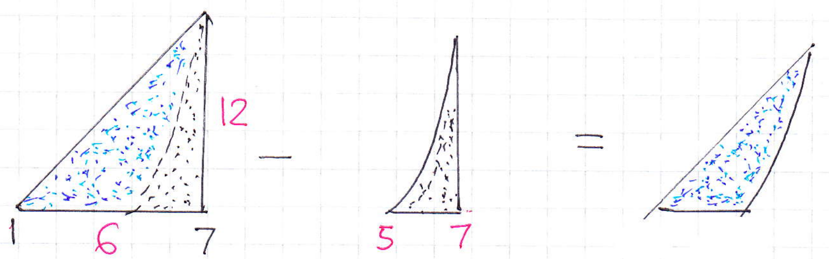
THUS LARGEST SHARE IS ar^2

$$2000 \times \left(\frac{7}{2}\right)^2 = 24500$$

i.e. £24500

9.

$$y = x^2 - 6x + 5$$
$$0 = (x-5)(x-1)$$
$$x = \begin{cases} 5 \\ 1 \end{cases} \quad \therefore \quad \begin{matrix} (5,0) \\ (1,0) \end{matrix}$$



$$\boxed{\tfrac{1}{2} \times 6 \times 12 = 36}$$

$$\int_5^7 x^2 - 6x + 5 \, dx = \left[\tfrac{1}{3}x^3 - 3x^2 + 5x\right]_5^7$$
$$= \left(\frac{343}{3} - 147 + 35\right) - \left(\frac{125}{3} - 75 + 25\right)$$
$$= \frac{7}{3} - \left(-\frac{25}{3}\right) = \frac{32}{3}$$

$\therefore$ REQUIRED AREA $= 36 - \frac{32}{3} = \frac{76}{3}$

10. $(1+ax)^n = 1 + \frac{n}{1}(ax)^1 + \frac{n(n-1)}{1\times 2}(ax)^2 + \frac{n(n-1)(n-2)}{1\times 2\times 3}(ax)^3 + \ldots$

$= 1 + \boxed{na}x + \boxed{\tfrac{1}{2}n(n-1)a^2}x^2 + \boxed{\tfrac{1}{6}n(n-1)(n-2)a^3}x^3 + \ldots$

$\uparrow$ -30 $\qquad$ $\uparrow$ 405 $\qquad$ $\uparrow$ b

$\left.\begin{array}{l} na = -30 \\ \frac{1}{2}n(n-1)a^2 = 405 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = -\frac{30}{n} \\ n(n-1)a^2 = 810 \end{array}\right\} \Rightarrow$

$\Rightarrow n(n-1)\left(-\frac{30}{n}\right)^2 = 810$

$\Rightarrow \cancel{n}(n-1) \times \frac{900}{n^{\cancel{2}}} = 810$

$\Rightarrow \frac{900(n-1)}{n} = 810$

$\Rightarrow 900n - 900 = 810n$

$\Rightarrow 90n = 900$

$n = 10$

$a = -\frac{30}{10}$

$a = -3$

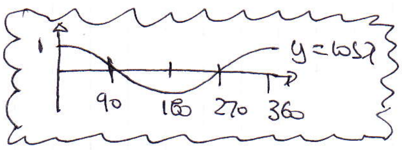
$b = \frac{1}{6}n(n-1)(n-2)a^3$

$b = \frac{1}{6}\times 10 \times 9 \times 8 \times (-3)^3$

$b = -3240$

11. a) $A = 5$ (SINCE MINIMUM HAS y VALUE -5)

$B = 40$ SINCE $(90,0)$ OF $y = \cos x$ NOW APPEARS AS $(130,0)$ (OR THE $180° \mapsto 220°$)



b) $C = 5$ MUST HAVE THE SAME AMPLITUDE (HEIGHT)

$D = 50$ $(180,0)$ OF $y = \sin x$ NOW APPEARS AS $(130,0)$