

C3 1YGB, PAPER B

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1. BY LONG DIVISION

$$x^2+1 \overline{) 2x^3+0x^2+x-2}$$

$$\underline{-2x^3 \quad -2x}$$

$$-2x-2$$

$$\therefore \frac{2x^3+x-2}{x^2+1} = 2x + \frac{-2x-2}{x^2+1} = 2x - \frac{x+2}{x^2+1}$$

$$A=2, B=0, C=-1, D=-2$$

2. (a) Let $y = \frac{x}{x+3}$

$$\Rightarrow yx+3y=x$$

$$\Rightarrow 3y=x-yx$$

$$\Rightarrow 3y=x(1-y)$$

$$\Rightarrow x = \frac{3y}{1-y}$$

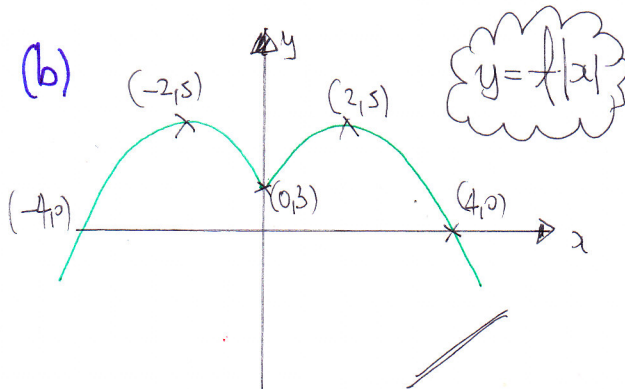
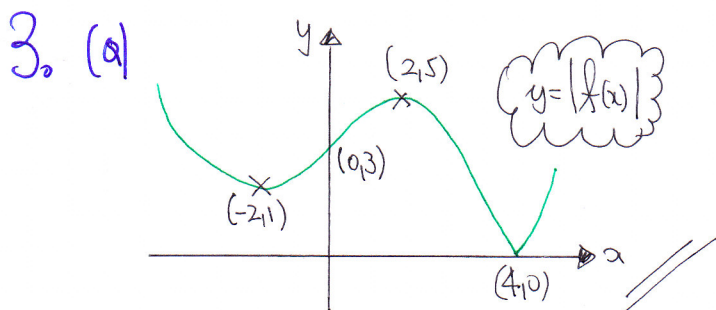
$$\therefore f(x) = \frac{3x}{1-x}$$

(b) $f(g(\frac{2}{3})) = f(\frac{-2}{3})$

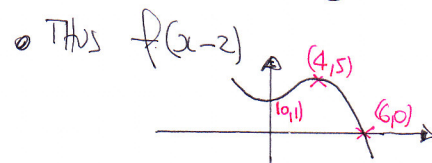
$$= f(3)$$

$$= \frac{3}{3+3}$$

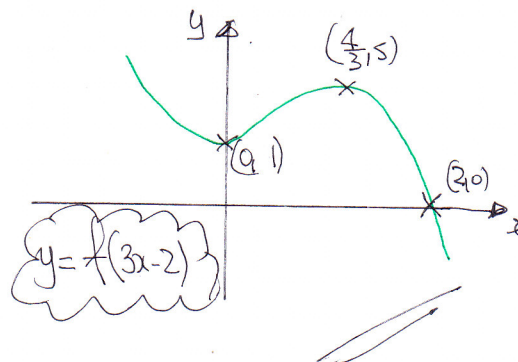
$$= \frac{1}{2}$$



(c) • THIS IS TRANSLATION, RIGHT, BY 2
 FOLLOWED BY HORIZONTAL STRETCH
 BY SCALE FACTOR $\frac{1}{3}$



Thus



$$\begin{aligned}
 4. (a) \quad \sqrt{2}\cos\theta - \sqrt{6}\sin\theta &\equiv R\cos(\theta+\alpha) \\
 &\equiv R\cos\theta\cos\alpha - R\sin\theta\sin\alpha \\
 &\equiv (R\cos\alpha)\cos\theta - (R\sin\alpha)\sin\theta
 \end{aligned}$$

$$\begin{aligned}
 R\cos\alpha &= \sqrt{2} \\
 R\sin\alpha &= \sqrt{6}
 \end{aligned}$$

• SQUARE & ADD

$$R = \sqrt{(\sqrt{2})^2 + (\sqrt{6})^2}$$

$$R = \sqrt{2+6}$$

$$R = \sqrt{8}$$

• DIVIDE EQUATIONS

$$\frac{R\sin\alpha}{R\cos\alpha} = \frac{\sqrt{6}}{\sqrt{2}}$$

$$\tan\alpha = \sqrt{3}$$

$$\alpha = \frac{\pi}{3} = 60^\circ$$

$$\therefore y = \sqrt{8}\cos(\theta+60^\circ)$$

$$(b) \quad y = 2$$

$$\Rightarrow \sqrt{8}\cos(\theta+60^\circ) = 2$$

$$\Rightarrow \cos(\theta+60^\circ) = \frac{2}{\sqrt{8}}$$

$$\arccos\left(\frac{2}{\sqrt{8}}\right) = 45^\circ$$

$$\begin{aligned}
 (\theta+60^\circ) &= 45^\circ \pm 360n \\
 (\theta+60^\circ) &= 315^\circ \pm 360n
 \end{aligned}$$

$$n = 0, 1, 2, 3, \dots$$

$$\begin{aligned}
 (\theta) &= -15^\circ \pm 360n \\
 (\theta) &= 255^\circ \pm 360n
 \end{aligned}$$

$$\text{BUT } 0 < \theta < 360^\circ$$

$$\therefore \theta = 255^\circ, 345^\circ$$

(c) (i) LOOKING AT

$$y = \sqrt{8}\cos(\theta+60^\circ)$$

$$y^2 = 8\cos^2(\theta+60^\circ)$$

MIN OCCURS WHEN $\cos(\theta+60^\circ) = 0$

\therefore MINIMUM IS 0

$$(ii) \quad \frac{1}{y^2} = \frac{1}{8\cos^2(\theta+60^\circ)}$$

MIN OCCURS WHEN THE DENOMINATOR IS LARGEST

\therefore MINIMUM VALUE IS $\frac{1}{8}$

(P.T.O)

5. (a) $N = Ae^{-kt}$

• With $t=0$, $N=12000$

$$12000 = Ae^0$$

$$12000 = A$$

$$\therefore N = 12000e^{-kt}$$

• With $t=24$, $N=2000$

$$2000 = 12000e^{-k \times 24}$$

$$\frac{1}{6} = e^{-24k}$$

$$e^{24k} = 6$$

$$24k = \ln 6$$

$$k = \frac{1}{24} \ln 6 \approx 0.07466$$

As required

(b) $N = 12000e^{-0.07466t}$

$$1000 = 12000e^{-0.07466t}$$

$$\Rightarrow \frac{1}{12} = e^{-0.07466t}$$

$$\Rightarrow 12 = e^{0.07466t}$$

$$\Rightarrow 0.07466t = \ln 12$$

$$\Rightarrow t = \frac{\ln 12}{0.07466}$$

$$\Rightarrow t \approx 33.2829 \dots$$

$$\Rightarrow t \approx 33.3$$

6. $y = \frac{x}{y^2 + \ln y}$

$$\Rightarrow y^3 + y \ln y = x$$

$$\Rightarrow x = y^3 + y \ln y$$

$$\Rightarrow \frac{dx}{dy} = 3y^2 + 1 \times \ln y + y \times \frac{1}{y}$$

$$\Rightarrow \frac{dx}{dy} = 3y^2 + \ln y + 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{3y^2 + \ln y + 1}$$

$$\Rightarrow \left. \frac{dy}{dx} \right|_{y=1} = \frac{1}{3+0+1} = \frac{1}{4}$$

NORMAL GRADIENT IS -4

$$y - y_0 = m(x - x_0)$$

$$y - 1 = -4(x - 1)$$

$$y - 1 = -4x + 4$$

$$y + 4x = 5$$

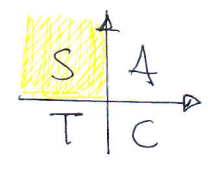
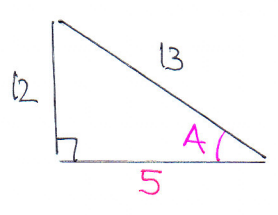
$$4x + y = 5$$

As required

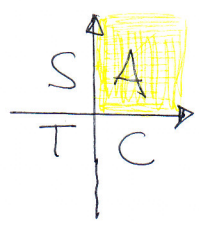
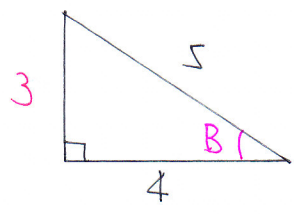
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7.



$$\cos A = -\frac{5}{13}$$



$$\sin B = \frac{3}{5}$$

$$\begin{aligned} \sin(A+B) &= \sin A \cos B + \cos A \sin B \\ &= \frac{12}{13} \times \frac{4}{5} + \left(-\frac{5}{13}\right) \left(\frac{3}{5}\right) \\ &= \frac{48}{65} - \frac{15}{65} \\ &= \frac{33}{65} \end{aligned}$$

~~As required~~

8. (a) $y = e^{2x} - 4e^x - 16$

$$\frac{dy}{dx} = 2e^{2x} - 4e^x - 16$$

Set for zero

$$2e^{2x} - 4e^x - 16 = 0$$

$$e^{2x} - 2e^x - 8 = 0$$

(b) $e^{2x} - 2e^x - 8 = 0$

$$(e^x + 2)(e^x - 4) = 0$$

$$e^x = \begin{matrix} \nearrow 2 \\ \searrow 4 \end{matrix}$$

$$x = \ln 4$$

$$x = 2 \ln 2$$

Now $y = e^{2(2 \ln 2)} - 4e^{2 \ln 2} - 16(2 \ln 2)$

$$y = 16 - 4 \times 4 - 32 \ln 2$$

$$y = -32 \ln 2$$

$$\therefore (2 \ln 2, -32 \ln 2)$$

9. a) $y = \frac{3x+1}{x^3-x^2+5}$

$$\frac{dy}{dx} = \frac{(x^3-x^2+5)(3) - (3x+1)(3x^2-2x)}{(x^3-x^2+5)^2} = \frac{3x^3-3x^2+15 - (9x^3-6x^2+3x^2-2x)}{(x^3-x^2+5)^2}$$

$$= \frac{3x^3-3x^2-15-9x^3+3x^2+2x}{(x^3-x^2+5)^2} = \frac{-6x^3+2x+15}{(x^3-x^2+5)^2}$$

SO WE GET ZERO

$$\frac{-6x^3+2x+15}{(x^3-x^2+5)^2} = 0$$

$$\Rightarrow -6x^3+2x+15=0$$

$$\Rightarrow 6x^3-2x-15=0$$

$$\Rightarrow 6x^3=2x+15$$

$$\Rightarrow x^3 = \frac{1}{3}x + \frac{5}{2}$$

$$\Rightarrow x = \sqrt[3]{\frac{1}{3}x + \frac{5}{2}}$$

As required

b) $x_{n+1} = \sqrt[3]{\frac{1}{3}x_n + \frac{5}{2}}$

START AT $x_1 = 1.4$

$$x_2 = 1.43689\dots$$

$$x_3 = 1.43887\dots$$

$$x_4 = 1.43898\dots$$

$$x_5 = 1.43898\dots$$

Now $y = \frac{3(1.43898) + 1}{(1.43898)^3 - (1.43898)^2 + 5}$

$$y = 0.899806\dots$$

$$\therefore M(1.439, 0.900)$$

3 d.p