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1. $LHS = \cot x - \tan x = \dfrac{1}{\tan x} - \tan x = \dfrac{1-\tan^2 x}{\tan x}$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

$$\dots = 2\left[\frac{1-\tan^2 x}{2\tan x}\right] = 2 \times \frac{1}{\tan 2x} = 2\cot 2x = RHS$$

ALTERNATIVE

$$LHS = \cot x - \tan x = \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\cos^2 x - \sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos 2x}{\sin x \cos x} = \frac{2\cos 2x}{2\sin x \cos x} = \frac{2\cos 2x}{\sin 2x} = 2\cot 2x = RHS$$

2. $y = \left(2+e^{3x}\right)^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{3}{2}\left(2+e^{3x}\right)^{\frac{1}{2}} \times 3e^{3x} = \frac{9}{2}e^{3x}\left(2+e^{3x}\right)^{\frac{1}{2}}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{3}\ln 2} = \frac{9}{2} \times 2 \times (2+2)^{\frac{1}{2}} = 18$$

3. a) $f(x) = e^{-2x} + \dfrac{\ln 2}{x}$

$$f'(x) = -2e^{-2x} - \frac{\ln 2}{x^2} = -\left[2e^{-2x} + \frac{\ln 2}{x^2}\right] < 0$$

AS THE "BRACKET" IS ALWAYS POSITIVE

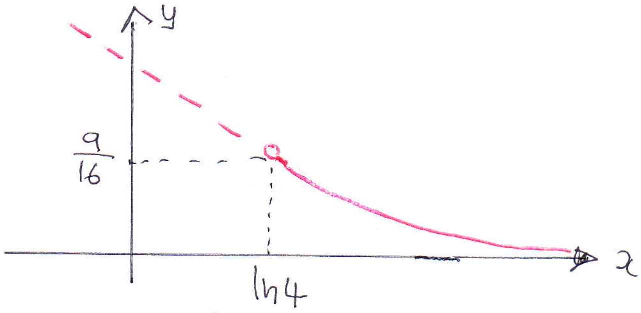
$\therefore f(x)$ IS A DECREASING FUNCTION

b) $f(\ln 4) = e^{-2\ln 4} + \frac{\ln 2}{\ln 4} = \frac{1}{16} + \frac{\ln 2}{2\ln 2} = \frac{9}{16}$

- SINCE $f(x)$ IS DECREASING
- AS $x \to \infty$ $\quad e^{-2x} \to 0$

  $\frac{\ln 2}{x} \to 0$

  SO $f(x) \to 0$



$\therefore$ RANGE: $0 < f(x) < \frac{9}{16}$

4. a) $y = \arcsin x \quad -1 \le x \le 1$

- $y = \arcsin x$

$\Rightarrow \sin y = x$

$\Rightarrow x = \sin y$

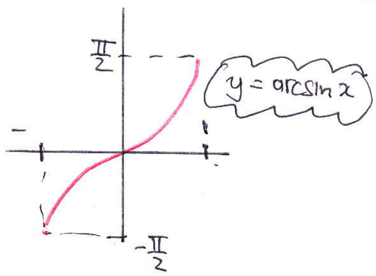
$\Rightarrow \frac{dx}{dy} = \cos y$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\cos y}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{+\sqrt{1-\sin^2 y}}$ (SEE OPPOSITE)

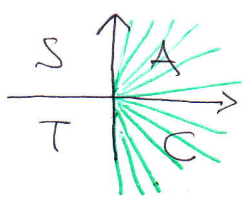
$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

AS REQUIRED



- $y = \arcsin x, \ -1 \le x \le 1$

  $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$



$\cos y$ CANNOT BE NEGATIVE

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b)

$$y = 3\arcsin x - 4x^{\frac{3}{2}} + 5$$

$$\frac{dy}{dx} = 3 \times \frac{1}{\sqrt{1-x^2}} - 6x^{\frac{1}{2}}$$

SOLVE FOR ZERO

$$\Rightarrow 0 = \frac{3}{(1-x^2)^{\frac{1}{2}}} - 6x^{\frac{1}{2}}$$

$$\Rightarrow 6x^{\frac{1}{2}} = \frac{3}{(1-x^2)^{\frac{1}{2}}}$$

$$\Rightarrow 2x^{\frac{1}{2}} = \frac{1}{(1-x^2)^{\frac{1}{2}}}$$

$$\Rightarrow 4x = \frac{1}{1-x^2}$$

$$\Rightarrow 4x - 4x^3 = 1$$

$$\Rightarrow 0 = 4x^3 + 4x + 1$$

$$\Rightarrow 4x^3 + 4x + 1 = 0$$

c) LET $f(x) = 4x^3 - 4x + 1$

$f(0) = 1 > 0$

$f(0.5) = -\frac{1}{2} < 0$

AS $f(x)$ IS CONTINUOUS & CHANGES SIGN IN THE INTERVAL, THERE MUST BE A SOLUTION IN THE INTERVAL

d)

$$x_{n+1} = x_n^3 + \frac{1}{4}$$

$x_0 = 0.5$

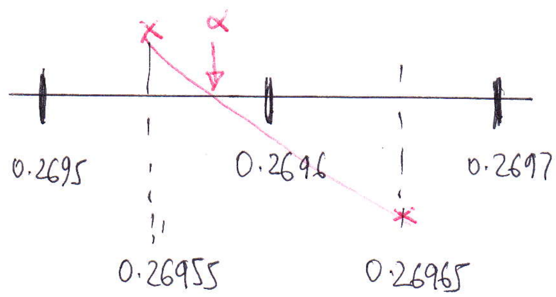
$x_1 = 0.375$

$x_2 = 0.303$

$x_3 = 0.278$

$x_4 = 0.271$

e)



CHANGE OF SIGN IMPLIES

$0.26955 < \alpha < 0.26965$

$\therefore \alpha = 0.2696$ (3 s.f.)

$f(0.26955) = 0.00014 > 0$

$f(0.26965) = -0.00017 < 0$

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5. a) $f(-x) = \dfrac{(-x)^2 - 4}{|-x| + 2} = \dfrac{x^2 - 4}{|x| + 2} = f(x)$

$\therefore f(x)$ is even

b) $f(x) = -\frac{1}{2}$

$\Rightarrow \dfrac{x^2 - 4}{|x| + 2} = -\dfrac{1}{2}$

$\Rightarrow 2x^2 - 8 = -|x| - 2$

$\Rightarrow |x| = 6 - 2x^2$

$\Rightarrow \begin{cases} x = 6 - 2x^2 \\ x = 2x^2 - 6 \end{cases}$

$\Rightarrow \begin{cases} 2x^2 + x - 6 = 0 \\ 2x^2 - x - 6 = 0 \end{cases}$

$\Rightarrow \begin{cases} (2x - 3)(x + 2) = 0 \\ (2x + 3)(x - 2) = 0 \end{cases}$

$x = \frac{3}{2}, \cancel{-2}, -\frac{3}{2}, \cancel{2}$

DO NOT SATISFY THE ORIGINAL

6. a) $\text{LHS} = 4\operatorname{cosec}^2 2\theta - \sec^2\theta$

$= \dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\cos^2\theta}$

$= \dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\cos^2\theta}$

$= \dfrac{\cancel{4}}{\cancel{4}\sin^2\theta\cos^2\theta} - \dfrac{1}{\cos^2\theta}$

$= \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta\sin^2\theta}$

$= \dfrac{1 - \sin^2\theta}{\sin^2\theta\cos^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta} = \dfrac{1}{\sin^2\theta}$

$= \operatorname{cosec}^2\theta = \text{RHS}$

b) $4(\operatorname{cosec}^2 2\theta - 2) = \sec^2\theta - 2\operatorname{cosec}\theta$

$4\operatorname{cosec}^2 2\theta - 8 = \sec^2\theta - 2\operatorname{cosec}\theta$

$4\operatorname{cosec}^2 2\theta - \sec^2\theta = 8 - 2\operatorname{cosec}\theta$

$\operatorname{cosec}^2\theta = 8 - 2\operatorname{cosec}\theta$

$\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta - 8 = 0$

$(\operatorname{cosec}\theta - 2)(\operatorname{cosec}\theta + 4) = 0$

$\operatorname{cosec}\theta = \begin{cases} 2 \\ -4 \end{cases}$

$\sin\theta = \begin{cases} 1/2 \\ -1/4 \end{cases}$ // AS REQUIRED

7. LET $\arcsin x = \arccos y = \theta$

$\Rightarrow \begin{cases} \arcsin x = \theta \\ \arccos y = \theta \end{cases}$

$\Rightarrow \begin{cases} \sin(\arcsin x) = \sin\theta \\ \cos(\arccos y) = \cos\theta \end{cases}$

$\Rightarrow \begin{cases} x = \sin\theta \\ y = \cos\theta \end{cases}$

THUS $x^2 + y^2 = \sin^2\theta + \cos^2\theta = 1$ //

8. $\boxed{y = e^{2x}(2\cos 3x - \sin 3x)}$

$$\frac{dy}{dx} = 2e^{2x}(2\cos 3x - \sin 3x) + e^{2x}(-6\sin 3x - 3\cos 3x)$$

$$\frac{dy}{dx} = e^{2x}\left[4\cos 3x - 2\sin 3x - 6\sin 3x - 3\cos 3x\right]$$

$$\frac{dy}{dx} = e^{2x}\left[\cos 3x - 8\sin 3x\right]$$

Diff again

$$\frac{d^2y}{dx^2} = 2e^{2x}\left[\cos 3x - 8\sin 3x\right] + e^{2x}\left[-3\sin 3x - 24\cos 3x\right]$$

$$\frac{d^2y}{dx^2} = e^{2x}\left[2\cos 3x - 16\sin 3x - 3\sin 3x - 24\cos 3x\right]$$

$$\frac{d^2y}{dx^2} = e^{2x}\left[-22\cos 3x - 19\sin 3x\right]$$

<u>Hence</u>

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 13y =$$

$$= e^{2x}\left[-22\cos 3x - 19\sin 3x\right] - 4e^{2x}\left[\cos 3x - 8\sin 3x\right] + 13 \times e^{2x}\left[2\cos 3x - \sin 3x\right]$$

$$= e^{2x}\left[\cancel{-22\cos 3x} - \cancel{19\sin 3x} - \cancel{4\cos 3x} + \cancel{32\sin 3x} + \cancel{26\cos 3x} - \cancel{13\sin 3x}\right]$$

$$= e^{2x} \times 0$$

$$= 0$$

AS REQUIRED

9.

$$\boxed{X = De^{-0.2t}}$$

a) $X = 20e^{-0.2t}$

$X = 20e^{-0.2}$

$X = 16.3746...$

$X \simeq 16.37$

b) $X = 20e^{-0.2t}$

$X = 20e^{-0.2\times 2}$

$X = 20e^{-0.4}$

$X = 13.41 > 12$

$\therefore$ STILL ASLEEP

c) $X_{TOTAL} = 20e^{-0.2\times 3} + 10e^{-0.2\times 1}$

$\Rightarrow X_{TOTAL} = 10.9762... + 8.1873...$

$\Rightarrow X_{TOTAL} = 19.164...$

$\Rightarrow X_{TOTAL} \simeq 19.16$ mg

d) $X_{TOTAL} = 20e^{-0.2T} + 10e^{-0.2(T-2)}$

WHERE T IS THE TIME SINCE THE START OF THE OPERATION

$\Rightarrow 12 = 20e^{-0.2T} + 10e^{-0.2(T-2)}$

$\Rightarrow 12 = 20e^{-0.2T} + 10e^{-0.2T} \times e^{0.4}$

$\Rightarrow 12 = e^{-0.2T}\left[20 + 10e^{0.4}\right]$

$\Rightarrow e^{-0.2T} = \dfrac{12}{20 + 10e^{0.4}}$

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$$\Rightarrow e^{0.2T} = \frac{20 + 10e^{0.4}}{12}$$

$$\Rightarrow 0.2T = \ln\left(\frac{10 + 5e^{0.4}}{6}\right)$$

$$\Rightarrow T = 5\ln\left(\frac{10 + 5e^{0.4}}{6}\right)$$

$$\Rightarrow T \approx 5.340514\ldots \text{ HOURS}$$

$\therefore 5.3405\ldots - 4 = 1.3405\ldots$ HOURS AFTER OP

$\downarrow \times 60$

$80.43\ldots$ MINUTES

AS REQUIRED