IYGB GCE

Mathematics FP1

Advanced Level

Practice Paper V

Difficulty Rating: 4.2133/2.2388

Time: 1 hour 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 9 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

The vectors $\mathbf{p}$ and $\mathbf{q}$ are defined as

$$\mathbf{p} = \mathbf{a} + 2\mathbf{b} \quad \text{and} \quad \mathbf{q} = 5\mathbf{a} - 4\mathbf{b}$$

where $\mathbf{a}$ and $\mathbf{b}$ are unit vectors.

Given that $\mathbf{p}$ and $\mathbf{q}$ are perpendicular, determine the acute angle between $\mathbf{a}$ and $\mathbf{b}$. **(5)**

Question 2

The complex number $17 + k\text{i}$, where k is a real constant, satisfies the locus

$$\arg(z - 1 - \text{i}) = \theta,$$

where $\theta = \arctan\frac{3}{4}$.

a) Determine the value of k. **(2)**

b) Find the complex number z which satisfies the locus $\arg(z - 1 - \text{i}) = \theta$ so that $|z - 22 + 2\text{i}|$ is least. **(6)**

Question 3

Prove by induction that for all natural numbers n,

$$2^n + 6^n$$

is divisible by 8. **(8)**

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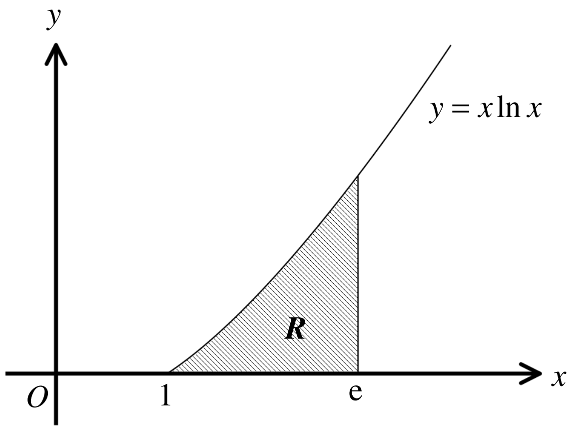
Question 4



The figure above shows the graph of the curve C with equation

$$y = x \ln x, \quad x \geq 1.$$

The shaded region R is bounded by the curve, the x axis and the vertical line $x = \mathrm{e}$.

The region R is rotated by 2π radians in the x axis forming a solid of revolution S.

Find an exact value for the volume of S. **(8)**

Question 5

$$2x^3 - 4x + 1 = 0.$$

The cubic equation shown above has three roots, denoted by α, β and γ.

Determine, as an exact simplified fraction, the value of

$$\frac{1}{\alpha - 2} + \frac{1}{\beta - 2} + \frac{1}{\gamma - 2}.$$ **(9)**

Question 6

The point $P(1,3,8)$ lies on the plane Π_1.

The straight line L, whose Cartesian equation is given below also lies on Π_1.

$$x-4=\frac{y-3}{3}=\frac{2-z}{4}.$$

a) Find a Cartesian equation of Π_1. **(8)**

*You may not use the vector product (cross product) in part **(a)**.*

The point $R(-2,-2,k)$, where k is a constant, lies on another plane Π_2, which is parallel to Π_1.

b) Given that the distance between Π_1 and Π_2 is 3 units determine, in exact fractional form, the possible values of k. **(7)**

*You may not use the standard formula which finds the distance between two parallel planes in part **(b)**.*

Question 7

$$\sum_{r=1}^{n}(r+a)(r+b)\equiv\frac{1}{3}n(n-1)(n+4),$$

where a and b are integer constants.

Use a clear algebraic method to determine the value of a and the value of b. **(10)**

Question 8

Under the transformation represented by the 2×2 matrix

$$\mathbf{A}=\begin{pmatrix}1 & 2\\ 4 & -7\end{pmatrix},$$

the straight line with equation $y=mx$ is reflected about the x axis.

Find the possible values of m. **(6)**

Question 9

$$z=(a+b\mathrm{i})^{4n}+(b+a\mathrm{i})^{4n},\ a\in\mathbb{R},\ b\in\mathbb{R},\ n\in\mathbb{N}.$$

Show that z is a real number. **(6)**

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