IYGB - FS2 PAPER N - QUESTION 1

a)

$\sum x = 235$	$\sum x^2 = 7853$	$\sum xy = 4904$
$\sum y = 152$	$\sum y^2 = 3214$	$n = 8$

CALCULATE THE VALUES OF S_{xx} S_{yy} S_{xy}

- $S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 7853 - \frac{235 \times 235}{8} = 949.875$
- $S_{yy} = \sum y^2 - \frac{\sum y \sum y}{n} = 3214 - \frac{152 \times 152}{8} = 326$
- $S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 4904 - \frac{235 \times 152}{8} = 439$

FIND THE P.M.C.C

$$r = \frac{S_{xy}}{\sqrt{S_{xx} S_{yy}}} = \frac{439}{\sqrt{949.875 \times 326}} = 0.7889009725\ldots$$

$$\simeq 0.789$$

b) THE P.M.C.C WILL BE UNCHANGED AT 0.789, AS IT IS NOT AFFECTED BY SCALING

IYGB — FS2 PAPER N — QUESTION 2

OBTAIN SUMMARY STATISTICS

- $n = 12$ • $\sum x = 1283$ • $\sum x^2 = 140415$

$$s^2 = \frac{1}{n-1} S_{xx} = \frac{1}{n-1}\left[\sum x^2 - \frac{\sum x \sum x}{n}\right] = \frac{1}{11}\left[140415 - \frac{1283 \times 1283}{12}\right]$$

$$= \frac{38891}{132} \simeq 294.628\ldots$$

SETTING HYPOTHESES

- $H_0 : \sigma = 250$
- $H_1 : \sigma > 250$

THE CRITICAL VALUE AT 10% SIGNIFICANCE & $\nu = 11$ IS 17.275

THE TEST STATISTIC IS $\frac{(n-1)s^2}{\sigma^2} = \frac{294.628\ldots \times 11}{250} = 12.96\ldots$

AS $12.96 < 17.275$ THERE IS NO SIGNIFICANT EVIDENCE THAT THE VARIANCE IS GREATER THAN 250 — INSUFFICIENT EVIDENCE TO REJECT H_0

IYGB - FS2 PAPER N - QUESTION 3

a) $P(X>0.5) = 1 - P(X<0.5) = 1 - F(0.5)$

$= 1 - \frac{1}{5}(0.5^2)(6-0.5^2) = 1 - \frac{23}{80}$

$= \frac{57}{80}$

b) USING $f(x) = \frac{d}{dx}[F(x)]$

$\frac{d}{dx}\left[\frac{1}{5}x^2(6-x^2)\right] = \frac{d}{dx}\left(\frac{6}{5}x^2 - \frac{1}{5}x^4\right) = \frac{12}{5}x - \frac{4}{5}x^3$

$= \frac{4}{5}x(3-x^2)$

$\therefore f(x) = \begin{cases} \frac{4}{5}x(3-x^2) & 1 \leq 0 \leq 1 \\ 0 & \text{OTHERWISE} \end{cases}$

c) $Var(X) = E(X^2) - [E(X)]^2$

$Var(X) = \frac{7}{15} - \left(\frac{16}{25}\right)^2 = \frac{107}{1875} = 0.057066...$

$\therefore$ STANDARD DEVIATION $= \sqrt{0.057066...} \simeq 0.238886...$

$\therefore$ MODE BY DIFFERENTIATION

$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(\frac{12}{5}x - \frac{4}{5}x^3\right) = \frac{12}{5} - \frac{12}{5}x^2$

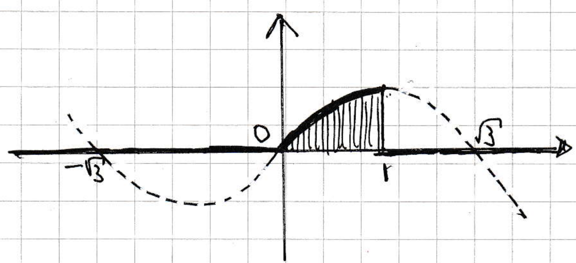
SET TO ZERO

$\frac{12}{5} - \frac{12}{5}x^2 = 0$

$\frac{12}{5} = \frac{12}{5}x^2$

$x^2 = 1$

$x = +1$



$\therefore$ MODE IS 1

IYGB - FS2 PAPER N - QUESTION 3

FINALLY

$$\frac{\text{MEAN} - \text{MODE}}{\text{STANDARD DEVIATION}} = \frac{0.64 - 1}{0.238886\ldots} \simeq -1.507$$

IYGB — FS2 PAPER N — QUESTION 4

SUBJECT	A	B	C	D	E	F	G	H	I
INITIAL WEIGHT	95.2	96.0	100.2	88.2	91.7	85.0	74.3	83.7	87.0
WEIGHT AFTER	93.1	95.1	98.1	90.7	90.6	87.2	71.3	80.1	89.1

SETTING UP A PAIRED VALUES t-TEST – LET d = "BEFORE – AFTER"

SUBJECT	A	B	C	D	E	F	G	H	I
DIFFERENCE (d)	2.1	0.9	2.1	−2.5	1.1	−2.2	3.0	3.6	−2.1

- $\sum d = 6.0$
- $\sum d^2 = 48.3$
- $n = 9$

$H_0 : \mu_d = 0$

$H_1 : \mu_d > 0$

OR

$H_0 : \mu_{BEF} = \mu_{AFTER}$

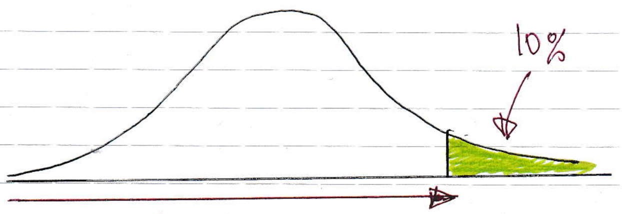
$H_1 : \mu_{BEF} > \mu_{AFTER}$

OBTAIN SUMMARY STATISTICS FOR THE SAMPLE DIFFERENCES

$$\bar{d} = \frac{\sum d}{n} = \frac{6}{9} = 0.666\ldots$$

$$s_d^2 = \sqrt{\frac{1}{n-1}\left[\sum d^2 - \frac{\sum d \sum d}{n}\right]} = \sqrt{\frac{1}{8}\left[48.3 - \frac{6^2}{9}\right]} = 2.353189\ldots$$

LOOKING FOR A CRITICAL VALUE AT 10%



$t_8(90\%) = 1.397$

IYGB – FS2 PAPER N – QUESTION 4

FINDING THE t – STAT

$$t \text{ stat} = \frac{\bar{d} - 0}{\frac{s}{\sqrt{n}}} = \frac{0.6666\ldots}{\frac{2.3531\ldots}{\sqrt{9}}} = 0.8499$$

AS $0.8499 < 1.397$ THERE IS NO SIGNIFICANT EVIDENCE THAT THE LITERATURE IS EFFECTIVE

NO SUFFICIENT EVIDENCE TO REJECT H_0.

IYGB – FS2 PAPER N – QUESTION 5

OBTAIN SUMMARY STATISTICS FOR A–G

$\sum x = 254$ $\quad \sum x^2 = 9906$ $\quad n = 7$

$\sum y = 209$ $\quad \sum xy = 7865$

TEST 1 → x
TEST 2 → y

CALCULATE S_{xx} & S_{xy}

$$S_{xx} = \sum x^2 - \frac{\sum x \sum x}{n} = 9906 - \frac{254 \times 254}{7} = \frac{4826}{7} \approx 689.43$$

$$S_{xy} = \sum xy - \frac{\sum x \sum y}{n} = 7865 - \frac{254 \times 209}{7} = \frac{1969}{7} \approx 281.29$$

OBTAIN THE GRADIENT

$$b = \frac{S_{xy}}{S_{xx}} = \frac{1969/7}{4826/7} = 0.407998...$$

OBTAIN THE EQUATION

$$\left.\begin{aligned} \bar{x} &= \frac{\sum x}{n} = \frac{254}{7} \\ \bar{y} &= \frac{\sum y}{n} = \frac{209}{7} \end{aligned}\right\}$$

$\Longrightarrow a = \bar{y} - b\bar{x}$

$\Longrightarrow a = \frac{209}{7} - 0.407998... \times \frac{254}{7}$

$\Longrightarrow a = \frac{286}{19} \approx 15.0526...$

$$\therefore \quad y = 0.408x + 15.05$$

FINALLY USING THE ABOVE EQUATION WITH $x = 40$

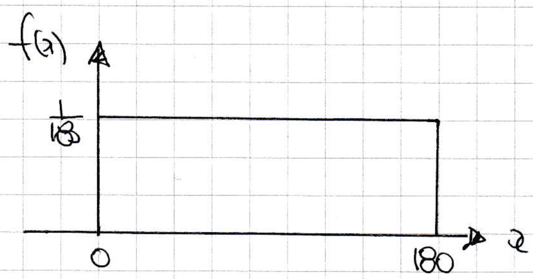
$y = 0.408 \times 40 + 15.05$

$y \approx 31.3726...$

$\therefore k = 31$

IYGB - FS PAPER N - QUESTION 6

a) MODELLING WITH A CONTINUOUS UNIFORM DISTRIBUTION (RECTANGULAR)



$$f(x) = \begin{cases} \frac{1}{180} & 0 \leqslant x \leqslant 180 \\ 0 & \text{OTHERWISE} \end{cases}$$

I) $P(X<70) = \frac{70}{180} = \frac{7}{18}$

II) USING STANDARD FORMULA $E(X) = \frac{a+b}{2}$

$$E(X) = \frac{0+180}{2} = 90$$

III) USING STANDARD FORMULA $\text{Var}(X) = \frac{(b-a)^2}{12}$

$$\text{Var}(X) = \frac{(180-0)^2}{12} = 2700$$

$\therefore$ STANDARD DEVIATION $= \sqrt{2700} \simeq 52.0$

b) MODEL AS FOLLOWS

$$P(\text{shorter piece is } \underline{\text{AT MOST}} \text{ 70 cm}) = P(X<70) + P(X>110)$$
$$= \frac{7}{18} + \frac{7}{18}$$
$$= \frac{7}{9}$$

IYGB – FS2 PAPER N – QUESTION 7

X = TIME TO CLEAN A CAR
$X \sim N(14, 4^2)$

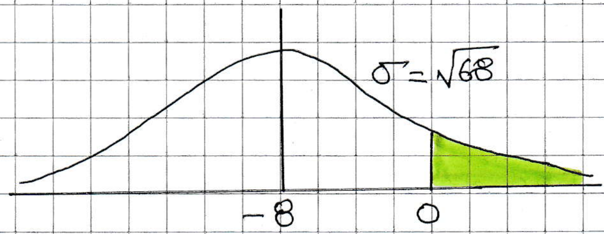
Y = TIME TO CLEAN A VAN
$Y = N(20, 6^2)$

a) DEFINE VARIABLE V AS:

- $V = Y - X_1 - X_2$
- $E(V) = 20 - 14 - 14 = -8$
- $\text{Var}(V) = 6^2 + 4^2 + 4^2 = 68$

$\therefore \; V \sim N(-8, 68)$

$$P(V > 0) = 1 - P(V < 0)$$
$$= 1 - P\left(Z < \frac{0-(-8)}{\sqrt{68}}\right)$$
$$= 1 - \Phi(0.9701)$$
$$= 1 - 0.8340$$
$$= \underline{0.1660}$$



b) DEFINE THE VARIABLE W AS:

$$W = Y - 2X$$
$$E(W) = E(Y - 2X) = E(Y) - 2E(X) = 20 - 2 \times 14 = -8$$
$$\text{Var}(W) = \text{Var}(Y - 2X) = \text{Var}(Y) + 2^2\text{Var}(X)$$
$$= \text{Var}(Y) + 4\text{Var}(X) = 6^2 + 4 \times 4^2 = 100$$

$\therefore \; W \sim N(-8, 10^2)$

IYGB – FS2 PAPER N – QUESTION 7

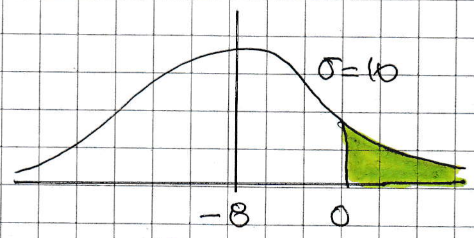
$$P(W>0) = 1 - P(W<0)$$

$$= 1 - P\left(Z < \frac{0-(-8)}{10}\right)$$

$$= 1 - \Phi(0.8)$$

$$= 1 - 0.7881$$

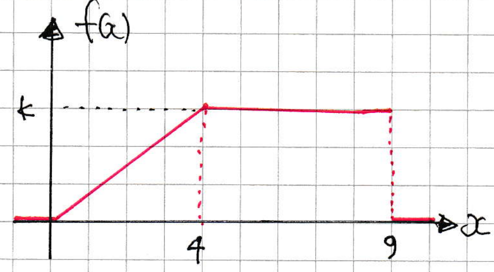
$$= \underline{0.2119}$$



IYGB - FS2 PAPER N - QUESTION 8

LOOKING AT THE DIAGRAM OPPOSITE

$$f(x) = \begin{cases} mx & 0 \leqslant x \leqslant 4 \\ k & 4 < x \leqslant 9 \\ 0 & \text{OTHERWISE} \end{cases}$$



USING "$y = mx$" WE OBTAIN

$$\Longrightarrow k = m \times 4$$
$$\Longrightarrow k = 4m$$
$$\Longrightarrow m = \tfrac{1}{4}k$$

ALSO WE HAVE LOOKING AT THE AREA

$$\left(\tfrac{1}{2} \times 4 \times k\right) + (5 \times k) = 1$$
$$2k + 5k = 1$$
$$7k = 1$$
$$k = \tfrac{1}{7} \qquad \therefore \quad m = \tfrac{1}{28}$$

FINALLY THE EXPECTATION CAN BE FOUND

$$E(X) = \int_a^b x f(x)\,dx = \int_0^4 x\left(\tfrac{1}{28}x\right)dx + \int_4^9 x\left(\tfrac{1}{7}\right)dx$$

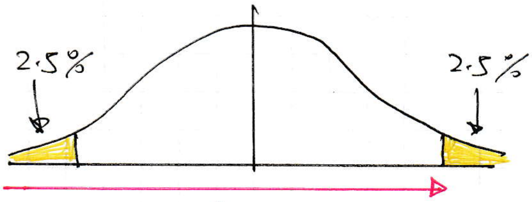
$$= \int_0^4 \tfrac{1}{28}x^2\,dx + \int_4^9 \tfrac{1}{7}x\,dx = \left[\tfrac{1}{84}x^3\right]_0^4 + \left[\tfrac{1}{14}x^2\right]_4^9$$

$$= \left(\tfrac{64}{84} - 0\right) + \left(\tfrac{81}{14} - \tfrac{16}{14}\right) = \tfrac{16}{21} + \tfrac{65}{14}$$

$$= \tfrac{227}{42} \approx 5.40$$

IYGB – FS2 PAPER N – QUESTION 9

$X \sim N(\mu, \sigma^2)$



$\Phi^{-1}(0.975) = 1.96$

- $$\frac{6.34 - 5.85}{2} = \frac{\sigma}{\sqrt{4}}\Phi^{-1}(0.975)$$

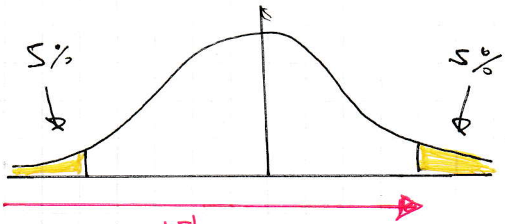
$$0.245 = \frac{\sigma}{\sqrt{4}} \times 1.96$$

$$\frac{\sigma}{\sqrt{4}} = 0.125$$

(STANDARD ERROR)

- $$\frac{6.34 + 5.85}{2} = 6.095 \longleftarrow \bar{x}_4$$

HENCE WE HAVE



$\Phi^{-1}(0.95) = 1.6449$

$$\mu = \bar{x}_4 \pm \frac{\sigma}{\sqrt{4}}\Phi^{-1}(0.95)$$

$$\mu = 6.095 \pm (0.125)(1.6449)$$

$$\mu = 6.095 \pm 0.2056\ldots$$

$\therefore$ C.I $= (5.89, 6.30)$