IYGB - MMS PAPER F - QUESTION 1

a) USING A CALCULATOR IN STATISTICAL MODE

P.M.C.C $= r = 0.732$

b) THE HIGHER THE NUMBER OF PRIESTS, THE HIGHER THE NUMBER OF SHOPLIFTING INCIDENTS (POSITIVE CORRELATION)

c) UNCHANGED AS THE P.M.C.C. IS INDEPENDENT OF SCALING (UNITS)

$r = 0.732$

d) SETTING HYPOTHESES

- $H_0: \rho = 0$
- $H_1: \rho > 0$

WHERE ρ IS THE P.M.C.C IN GENERAL

THE CRITICAL VALUE FOR $n=7$ AT 5% SIGNIFICANCE IS 0.6694

AS $0.732 > 0.6694$ THERE IS EVIDENCE OF POSITIVE CORRELATION

SUFFICIENT EVIDENCE TO REJECT H_0

e) CORRELATION DOES NOT IMPLY CAUSATION AS THERE MAY BE A THIRD VARIABLE THAT CORRELATES WITH x & y

STATEMENT UNLIKELY TO BE TRUE

f) USING A STATISTICAL CALCULATOR

$$y = a + bx$$
$$y = 199 + 5.03x$$

IF $x = 220$, $y = k$

$$k = 199 + 5.03 \times 220$$
$$k = 310$$

g) RESIDUAL = ACTUAL − ESTIMATED

$$305 - (199 + 5.03 \times 23)$$
$$305 - 315$$

$\therefore$ RESIDUAL $= -10$

IYGB - MMS PAPER F - QUESTION 2

a) COPY DATA DIRECTLY INTO AN ORDERED STEM & LEAF DIAGRAM

0	4
1	1 2 3 4 5
2	0 2 7
3	0 2 3
4	4 5

2 | 7 = 27

b) MEAN $= \bar{x} = \frac{\Sigma x}{n} = \frac{322}{14} = 23$

STANDARD DEVIATION $= \sigma = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2} = \sqrt{\frac{9458}{14} - 23^2} \simeq 12.11$

c) $n = 14$, $n+1$ RULE APPLIES FOR THE UPPER & LOWER QUARTILE ONLY

- $Q_1 = \frac{1}{4}(14+1) = \frac{15}{4} = 3.75 = 4^{TH}$ OBS $\qquad Q_1 = 13$
- Q_2 IS HALF WAY BETWEEN 7^{TH} & 8^{TH} OBS $\qquad Q_2 = \frac{20+22}{2} = 21$
- $Q_3 = \frac{3}{4}(14+1) = 11.25 = 11^{TH}$ OBS. $\qquad Q_3 = 32$

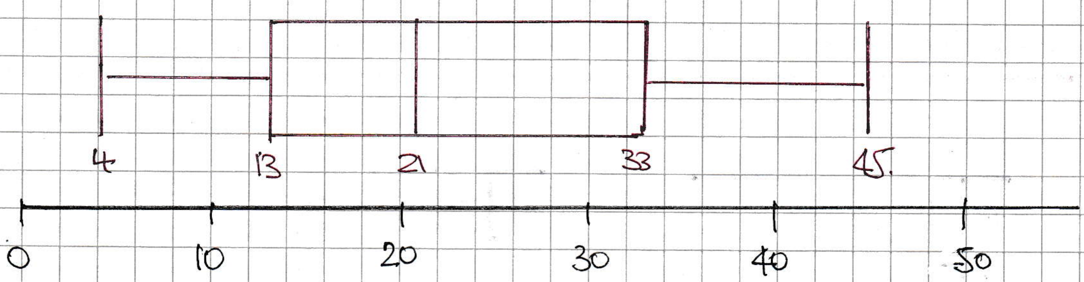
UPPER BOUND $= Q_3 + 1.5(Q_3 - Q_1) = 32 + 1.5(32-13) = 60.5$

LOWER BOUND $= Q_1 - 1.5(Q_3 - Q_1) = 13 - 1.5(32-13) = -15.5$

$\therefore$ NO OUTLIERS IN THE DATA.

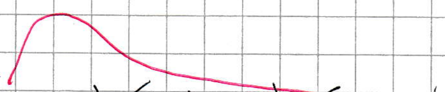
IYGB — MMS PAPER F — QUESTION 2

d)



e)

USING THE AVERAGES

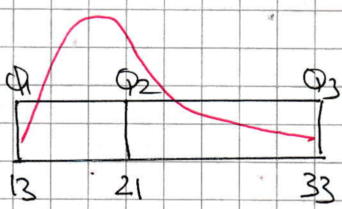


(MODE) < MEDIAN < MEAN

(21) (23)

POSITIVE SKEW

USING QUARTILES



POSITIVE SKEW SINCE $Q_2 - Q_1 < Q_3 - Q_2$

IYGB - MMS PAPER F - QUESTION 3

a) X = NUMBER OF DAYS WITH SEVERE FOG

$X \sim B(7, 0.06)$

I) $P(X=0) = \binom{7}{0}(0.06)^0(0.94)^7 = 0.6485$

II) $P(X \geq 3) = 1 - P(X \leq 2) = \ldots$ CALCULATOR $\ldots$

$= 1 - 0.9937\ldots$

$= 0.0063$

b) CONDITIONAL PROBABILITY NOW

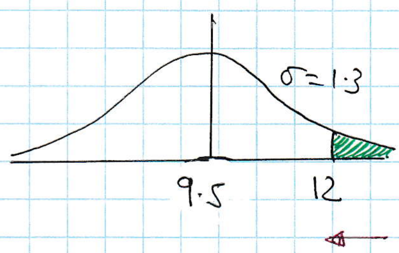
$P(X=3) = \binom{7}{3}(0.06)^3(0.94)^4 = 0.005902476213\ldots \simeq 0.0059$

WE REQUIRE

$$P(X=3 \mid X \geq 3) = \frac{0.0059024\ldots}{0.0063} \simeq 0.9378$$

IYGB - MMS PAPER F - QUESTION 4

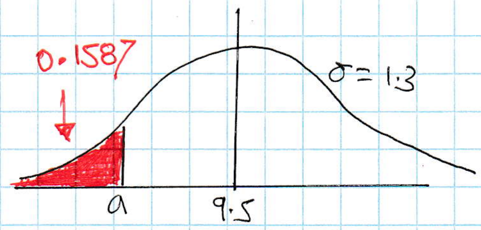
a) $X \sim N(9.5, 1.3^2)$



$$P(X>12) = 1 - P(X<12)$$
$$= 1 - P\left(Z < \frac{12-9.5}{1.3}\right)$$
$$= 1 - \Phi(1.923076..)$$
$$= 1 - 0.97276$$
$$= 0.02724$$

(CALCULATOR FIGURE)

b) LOOKING AT A NEW DIAGRAM



$$\Rightarrow P(X<a) = 0.1587$$
$$\Rightarrow P(X>a) = 0.8413$$
$$\Rightarrow P\left(Z > \frac{a-9.5}{1.3}\right) = 0.8413$$

INVERSION

$$\Rightarrow \frac{a-9.5}{1.3} = -\Phi^{-1}(0.8413)$$
$$\Rightarrow \frac{a-9.5}{1.3} = -1$$
$$\Rightarrow a - 9.5 = -1.3$$
$$\Rightarrow a = 8.2$$

IYGB - MMS PAPER F - QUESTION 5

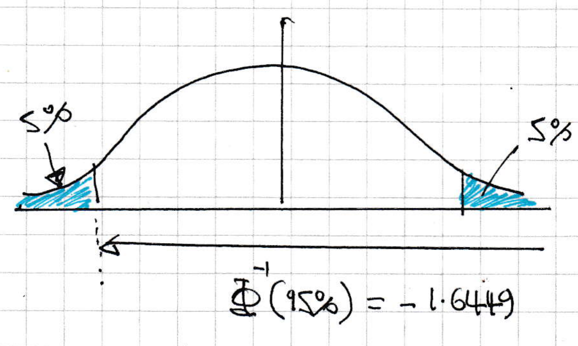
COLLECTING ALL THE "GIVENS" INCLUDING HYPOTHESES

$H_0: \mu = 6.6$

$H_1: \mu \neq 6.6$

WHERE μ IS THE POPULATION MEAN

HERE WE ALSO HAVE $n = 40$, $\sigma^2 = 3.9$, $\bar{x}_{40} = 6.1$, 10% SIGNIFICANCE



$$\text{Z-STAT} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{6.1 - 6.6}{\sqrt{\frac{3.9}{40}}}$$

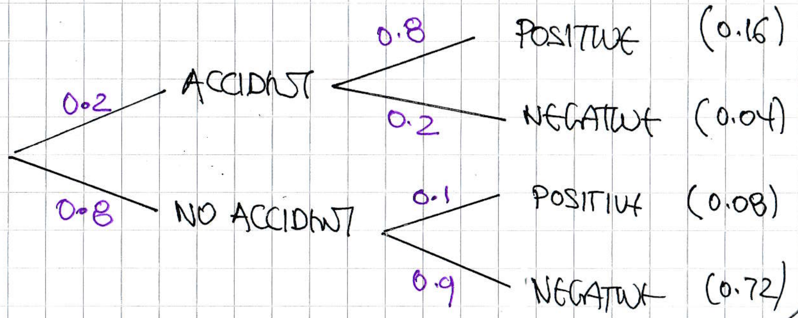
$$= -1.601\ldots$$

AS $-1.601 > -1.6449$ THERE IS NO SIGNIFICANT EVIDENCE THAT $\mu \neq 6.6$

CLAIM IS JUSTIFIED - INSUFFICIENT EVIDENCE TO REJECT H_0

IYGB - MMS PAPER F - QUESTION 6

a) DRAWING A TREE DIAGRAM



b) LOOKING AT THE DIAGRAM ABOVE

I) $P(\text{POSITIVE}) = (0.2 \times 0.8) + (0.8 \times 0.1) = 0.16 + 0.08 = 0.24$

II) $P(\text{CLASSED CORRECTLY}) = \underset{\text{ACCIDENT}|\text{POSITIVE}}{(0.2 \times 0.8)} + \underset{\text{NO ACCIDENT}|\text{NEGATIVE}}{(0.8 \times 0.9)} = 0.16 + 0.72 = 0.88$

c) $P(\text{ACCIDENT}|\text{POSITIVE}) = \dfrac{P(\text{ACCIDENT} \cap \text{POSITIVE})}{P(\text{POSITIVE})}$

$$= \frac{0.16}{0.24} = \frac{2}{3}$$

d) THE REQUIRED PROBABILITY IS

$P(\text{NO ACCIDENT} \cap \text{POSITIVE} \cap \text{MECHANICS CONCLUDED CORRECTLY})$

$= 0.8 \times 0.1 \times 0.9$

$= 0.072$

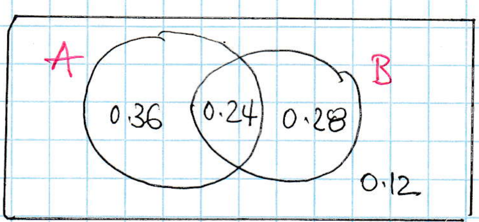
IYGB – MMS PAPER F – QUESTION 7

$P(A) = 0.6 \qquad P(B) = 0.52 \qquad P(A \cup B) = 0.88$

a) USING $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$\Longrightarrow \quad 0.88 = 0.6 + 0.52 - P(A \cap B)$$

$$\Longrightarrow \quad P(A \cap B) = 0.24$$



b) I) $P(B|A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{0.24}{0.6} = 0.4$

II) $P(A'|B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{0.12}{1-0.52} = \dfrac{0.12}{0.48} = 0.25$

c) I) NOT INDEPENDENT BECAUSE

$$P(B|A) = 0.4 \neq 0.52 = P(B)$$

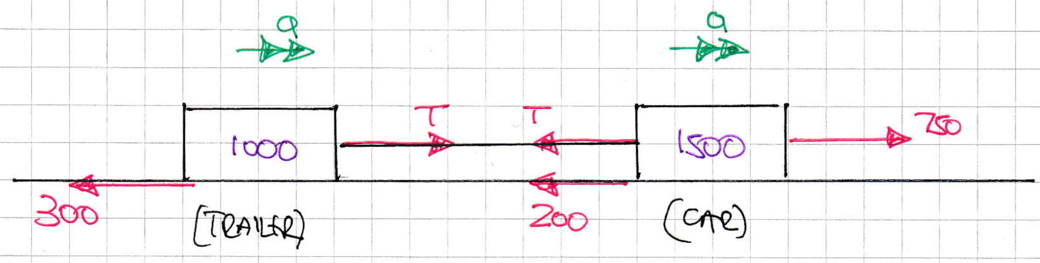
OR

$$P(A) \times P(B) = 0.6 \times 0.52 = 0.312 \neq 0.24 = P(A \cap B)$$

II) NOT MUTUALLY EXCLUSIVE BECAUSE $P(A \cap B) \neq 0$

IYGB - MMS PAPER F - QUESTION 8

a) LOOKING AT THE DIAGRAM — IGNORING "VERTICAL" FORCES



LOOKING AT THE CAR AND THE TRAILER SEPARATELY ("F = ma")

(CAR): $750 - T - 200 = 1500a$
(TRAILER): $T - 300 = 1000a$

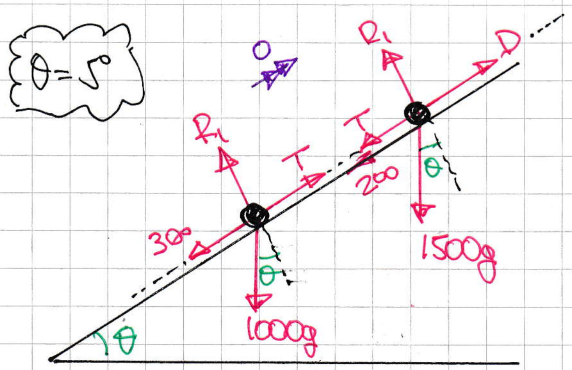
Adding $250 = 2500a$

$a = 0.1 \text{ ms}^{-1}$

& $T - 300 = 1000 \times 0.1$

$T = 400 \text{ N}$

b) REDRAW THE DIAGRAM ON AN INCLINE



- CONSTANT SPEED ⇒ EQUILIBRIUM
- LOOKING AT THE DIRECTION OF MOTION ONLY, FOR EACH OBJECT

TRAILER

$T = 300 + 1000g\sin\theta$

$T = 1154.126279...$

$T \approx 1154 \text{ N}$

CAR

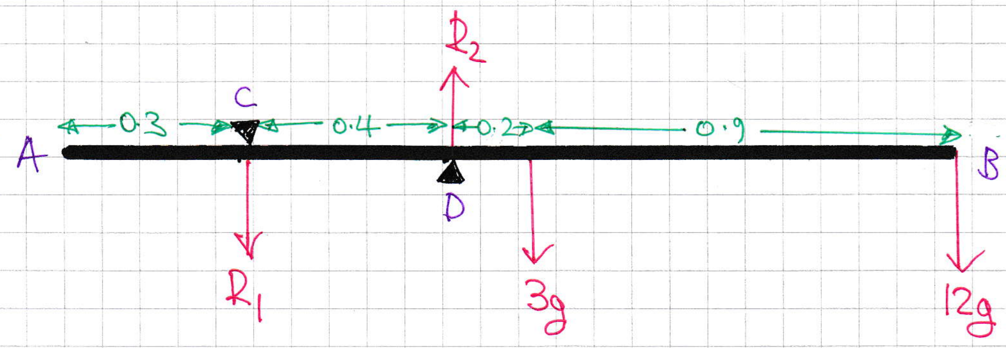
$D = T + 200 + 1500g\sin\theta$

$D = 2635.315697...$

$D \approx 2635 \text{ N}$

IYGB – MMS PAPER F – QUESTION 9

STARTING WITH A DIAGRAM



TAKING MOMENTS ABOUT C

$$\overset{\curvearrowright}{C}: \quad R_2 \times 0.4 = 3g \times 0.6 + 12g \times 1.5$$

$$0.4R_2 = 1.8g + 18g$$

$$0.4R_2 = 19.8g$$

$$\underline{R_2 = 485.1\text{ N}}$$

(REACTION AT D)

RESOLVING VERTICALLY

$$R_1 + 3g + 12g = R_2$$

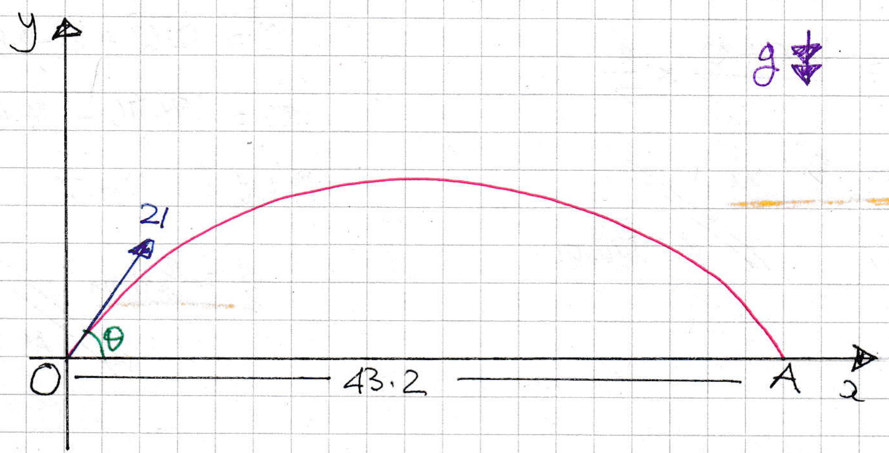
$$R_1 + 3g + 12g = 485.1$$

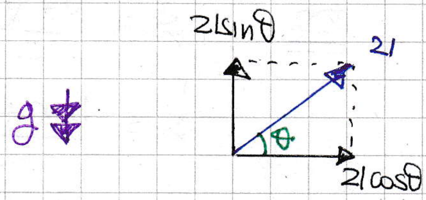
$$\underline{R_1 = 338.1\text{ N}}$$

(REACTION AT C)

IYGB — MMS PAPER F - QUESTION 10

a) START WITH A DIAGRAM





$\tan\theta = \frac{3}{4}$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$

LOOKING AT THE HORIZONTAL MOTION

$\Rightarrow$ " DISTANCE = SPEED × TIME "

$\Rightarrow \quad 43.2 = 21\cos\theta \times t$

$\Rightarrow \quad 43.2 = 21 \times 0.8 \times t$

$\Rightarrow \quad t = \frac{18}{7} \approx 2.57\,s$

ALTERNATIVE
(LOOKING VERTICALLY)

$u = 21\sin\theta = 12.6$
$a = -9.8$
$s = 0$
$t = ?$
v

$s = ut + \frac{1}{2}at^2$

$0 = 12.6t + \frac{1}{2}(-9.8)t^2$

$0 = 12.6t - 4.9t^2$

$0 = t(12.6 - 4.9t)$

$t = \begin{cases} 0 \\ \frac{12.6}{4.9} = \frac{18}{7} \end{cases}$

b) USING SYMMETRY SINCE WE HAVE THE FLIGHT TIME...

$u = 21\sin\theta = 12.6$
$a = -9.8$
$s =$
$t = 9/7$ ← HALF THE FLIGHT TIME
$v = 0$

$s = ut + \frac{1}{2}at^2$

$s = 12.6 \times \frac{9}{7} + \frac{1}{2}(-9.8) \times \left(\frac{9}{7}\right)^2$

$s = 8.1\,m$

IYGB – MMS PAPER F – QUESTION 10

OR

$$s = \frac{u+v}{2} \times t$$

$$s = \frac{12.6+0}{2} \times \frac{9}{7}$$

$$s = 8.1\text{ m}$$

AS BEFORE

OR

$$v^2 = u^2 + 2as$$

$$0^2 = 12.6^2 + 2(-9.8)s$$

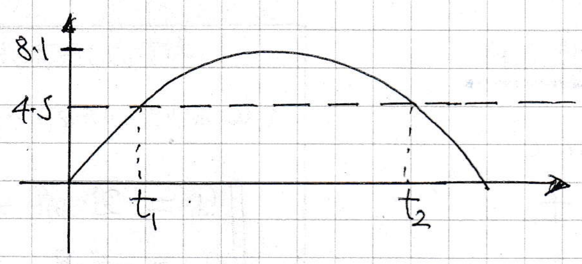
$$0 = 158.76 - 19.6s$$

$$19.6s = 158.76$$

$$s = 8.1\text{ m}$$

AS BEFORE

c) LOOKING AT THE DIAGRAM



u	=	12.6
a	=	−9.8
s	=	4.5
t	=	?
v	=	

$$\Rightarrow s = ut + \tfrac{1}{2}at^2$$

$$\Rightarrow 4.5 = 12.6t + \tfrac{1}{2}(-9.8)t^2$$

$$\Rightarrow 4.5 = 12.6t - 4.9t^2$$

$$\Rightarrow 45 = 126t - 49t^2$$

$$\Rightarrow 49t^2 - 126t + 45 = 0$$

QUADRATIC FORMULA OR FACTORIZATION

$$\Rightarrow (7t - 15)(7t - 3) = 0$$

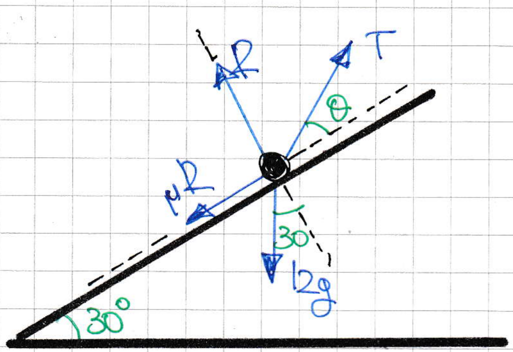
$$\Rightarrow t = \begin{cases} 3/7 \\ 15/7 \end{cases}$$

$\therefore$ REQUIRED TIME $= \frac{15}{7} - \frac{3}{7} = \frac{12}{7} = 1\frac{5}{7}$

AS REQUIRED

IYGB – MMS PAPER F – QUESTION 11

START WITH A DETAILED DIAGRAM

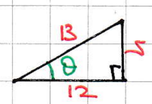


$\mu = \frac{1}{2}$

$\tan\theta = \frac{5}{12}$

$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$



RESOLVING PARALLEL & PERPENDICULAR TO THE PLANE

$$\left.\begin{aligned} (\parallel):&\ \mu R + 12g\sin 30 = T\cos\theta \\ (\perp):&\ R + T\sin\theta = 12g\cos 30 \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} \tfrac{1}{2}R + 6g &= \tfrac{12}{13}T \\ R + \tfrac{5}{13}T &= 6g\sqrt{3} \end{aligned}\right\} \Rightarrow$$

$$\left.\begin{aligned} R + 12g &= \tfrac{24}{13}T \\ R + \tfrac{5}{13}T &= 6g\sqrt{3} \end{aligned}\right\} \Rightarrow$$

SUBTRACT TO ELIMINATE R

$$\Rightarrow 12g - \tfrac{5}{13}T = \tfrac{24}{13}T - 6g\sqrt{3}$$

$$\Rightarrow 12g + 6g\sqrt{3} = \tfrac{29}{13}T$$

$$\Rightarrow \tfrac{29}{13}T = 6g(2+\sqrt{3})$$

$$\Rightarrow T = \tfrac{13}{29} \times 6g(2+\sqrt{3})$$

$$\Rightarrow T = \tfrac{78}{29}g(2+\sqrt{3})$$

$$\Rightarrow T = 98.371\ldots$$

$$\Rightarrow T \simeq 98.4\ \text{N}$$

IYGB - MMS PAPER F - QUESTION 12

a) LOOKING AT THE JOURNEY A TO B

$u = ?$
$a =$
$s = 180\text{m}$
$t = 12$
$v = 18\text{ ms}^{-1}$

USING $s = \frac{1}{2}(u+v)t$

$180 = \frac{1}{2}(u+18) \times 12$

$180 = 6(u+18)$

$30 = u + 18$

$u = 12\text{ ms}^{-1}$

b) FIRSTLY FIND THE ACCELERATION FROM PART (a)

$v = u + at$

$18 = 12 + a \times 12$

$6 = 12a$

$a = 0.5\text{ ms}^{-2}$

NOW THE JOURNEY FROM A TO THE MIDPOINT OF AB

$u = 12\text{ ms}^{-1}$
$a = 0.5\text{ ms}^{-2}$
$s = 90\text{m}$ ← HALF WAY
$t = ?$
$v =$

USING $s = ut + \frac{1}{2}at^2$

$90 = 12t + \frac{1}{2} \times 0.5t^2$

$90 = 12t + \frac{1}{4}t^2$

$360 = 48t + t^2$

$t^2 + 48t - 360 = 0$

$(t+24)^2 - 24^2 - 360 = 0$

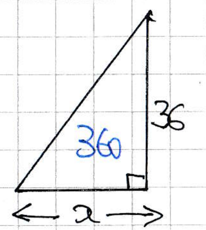
$(t+24)^2 = 936$

$t + 24 = \begin{cases} \sqrt{936} \\ -\sqrt{936} \end{cases}$

$t = \begin{cases} -24 + \sqrt{936} \approx 6.59\text{ s} \\ -24 - \sqrt{936} \approx -\cancel{54.6} \end{cases}$

IYGB – MMS PAPER F – QUESTION 13

a) ACCELERATION = GRADIENT & DISTANCE = AREA



- $\frac{1}{2} \times 36 \times x = 360$

  $18x = 360$

  $x = 20$

- $a = \frac{\Delta v}{\Delta t}$

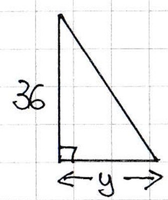
  $a = \frac{36}{20}$

  $a = 1.8 \text{ ms}^{-2}$

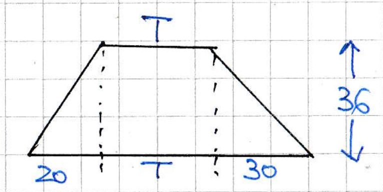
b) DECELERATION = GRADIENT

$a = \frac{\Delta v}{\Delta t} \implies -1.2 = \frac{-36}{y}$

$\implies y = 30$



NOW THE TOTAL AREA IS 6300 m



$= 6300$

$\implies \frac{1}{2}(20+T+30+T) \times 36 = 6300$

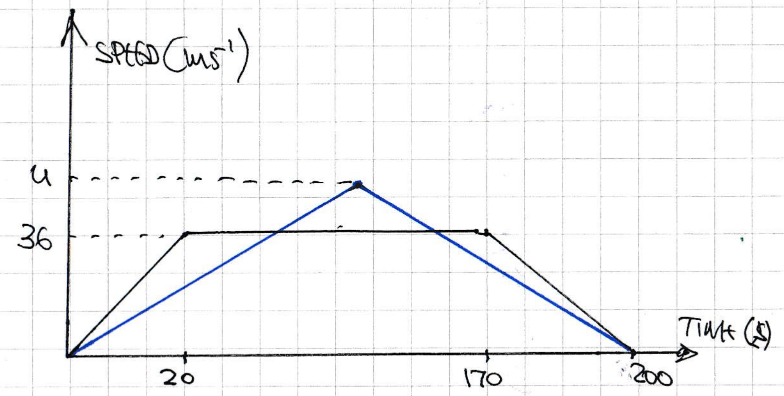
$\implies 18(2T+50) = 6300$

$\implies 36T + 900 = 6300$

$\implies 36T = 5400$

$\implies T = 150$

c) SKETCHING THE SPEED TIME GRAPH



AREA = 6300

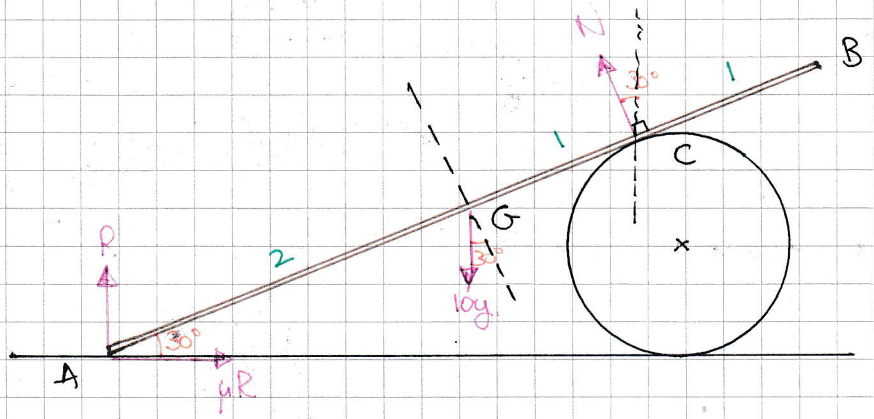
$\frac{1}{2} \times 200 \times u = 6300$

$100u = 6300$

$u = 63 \text{ ms}^{-1}$

IYGB - MMS PAPER F - QUESTION 14

STARTING WITH A DETAILED DIAGRAM



RESOLVING AND TAKING MOMENTS

$(\uparrow)$: $R + N\cos 30 = 10g$ (I)

$(\rightarrow)$: $\mu R = N\sin 30$ (II)

$\overset{\curvearrowright}{A}$: $(10g\cos 30) \times 2 = N \times 3$ (III)

SOLVING THE EQUATIONS, STARTING WITH (III)

$\Rightarrow 20g\cos 30 = 3N$

$\Rightarrow 10g\sqrt{3} = 3N$

$\Rightarrow N = \frac{98}{3}\sqrt{3}$

NOW (I) WILL FIND THE REQUIRED R

$\Rightarrow R + \frac{98}{3}\sqrt{3}\cos 30 = 10g$

$\Rightarrow R + 49 = 98$

$\Rightarrow R = 49\text{ N}$

FINALLY USING EQUATION (II) WITH $R = 49$ & $N = \frac{98}{3}\sqrt{3}$

$\mu \times 49 = \frac{98}{3}\sqrt{3} \times \sin 30$

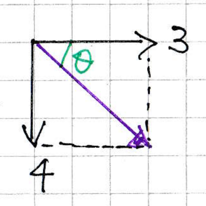
$49\mu = \frac{49}{3}\sqrt{3}$

$\mu = \frac{1}{3}\sqrt{3}$ AS REQUIRED

-1-

IYGB - MMS PAPER F - QUESTION 15

a) LOOKING AT A DIAGRAM



$\tan\theta = \frac{4}{3}$

$\theta = 53.13°$

$\therefore$ BEARING $= 90° + 53.13$

$= 143°$

b) SPEED IS THE MAGNITUDE OF $-2\underline{i} + 8\underline{j}$

SPEED $= \sqrt{(-2)^2 + 8^2} = \sqrt{4+64}$

$= \sqrt{68} = 8.25 \text{ km h}^{-1}$

c) USING $\underline{r} = \underline{r}_0 + \underline{v}t$ FOR EACH SHIP

$\underline{r}_A = (5\underline{i} - 7\underline{j}) + (3\underline{i} - 4\underline{j})t$

$\underline{r}_B = (3\underline{i} + 5\underline{j}) + (-2\underline{i} + 8\underline{j})t$

$\underline{r}_A = (3t+5)\underline{i} + (-4t-7)\underline{j}$

$\underline{r}_B = (3-2t)\underline{i} + (8t+5)\underline{j}$

OR AS "CO-ORDINATES"

$A(3t+5, -4t-7)$

$B(3-2t, 8t+5)$

USING THE DISTANCE FORMULA

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{[(3-2t)-(3t+5)]^2 + [(8t+5)-(-4t-7)]^2}$

$d = \sqrt{(-5t-2)^2 + (12t+12)^2}$

$d = \sqrt{25t^2 + 20t + 4 + 144t^2 + 288t + 144}$

$d = \sqrt{169t^2 + 308t + 148}$

AS REQUIRED

IYGB - MMS PAPER F - QUESTION 15

d) FOR COLLISION $d=0$

$$\Rightarrow 0 = \sqrt{169t^2 + 308t + 148}$$

$$\Rightarrow 0 = 169t^2 + 308t + 148$$

$$b^2 - 4ac = 308^2 - 4 \times 169 \times 148 = -5184 < 0$$

NO VALUE OF t, GIVES $d=0$, SO THEY NEVER COLLIDE

e) FINALLY $d=25$

$$\Rightarrow 25 = \sqrt{169t^2 + 308t + 148}$$

$$\Rightarrow 625 = 169t^2 + 308t + 148$$

$$\Rightarrow 0 = 169t^2 + 308t - 477$$

QUADRATIC FORMULA

$$t = \frac{-308 \pm \sqrt{308^2 - 4 \times 169(-477)}}{2 \times 169} = \begin{cases} 1 \\ \cancel{-2.82} \end{cases}$$

IE AT 13:00