IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper U

Difficulty Rating: 4.2600/1.3793

Time: 2 hours 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 16 questions in this question paper.
The total mark for this paper is 125.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

Question 1

$$\frac{\cos\left(\frac{1}{2}x\right)}{1+\sin x} = 0.925 .$$

Given that the above equation has a solution that is numerically small, find this solution by using a quadratic approximation.

No credit will be given for solving a trigonometric equation. (7)

Question 2

Solve the following equation

$$2^x + 4^x + 8^x + 16^x + 32^x + \ldots = 1 .$$

You may assume that the left hand side of the equation converges. (5)

Question 3

$$f(x) = 2 + \sec\left(x - \frac{\pi}{3}\right), \quad x \in \mathbb{R}, \; 0 \le x \le 2\pi .$$

a) Solve the equation $f(x) = 0$. (4)

b) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any stationary points, the coordinates of any x or y intercepts and equations of the vertical asymptotes. (4)

Question 4

Relative to a fixed origin O, the positions vectors of the points A, B and C are defined below.

$$\overrightarrow{OA} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}, \qquad \overrightarrow{OB} = 2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, \qquad \overrightarrow{OC} = 4\mathbf{i} + 12\mathbf{k}.$$

If $\overrightarrow{OD} = \frac{1}{3}\overrightarrow{OC}$ prove that the point D lies on the straight line AB. **(5)**

Question 5

The function f is defined by

$$f(x) = \begin{cases} x+1, & x \in \mathbb{R},\ x \le 2 \\ (x-2)^2 + 3, & x \in \mathbb{R},\ x > 2 \end{cases}$$

a) Sketch the graph of $f(x)$. **(3)**

b) Find an expression for $f^{-1}(x)$, fully specifying its domain. **(6)**

Question 6

The function f is defined below.

$$f(x) \equiv 4\cos x - 3\sin\left(\tfrac{1}{2}x\right), \quad x \in \mathbb{R}.$$

Show that if θ satisfies the equation

$$4\sin\left(\tfrac{1}{2}\theta\right) + \sqrt{3} = 0,$$

then $f(\theta) = \frac{1}{4}\left(a + b\sqrt{3}\right)$, where a and b are integers to be found. **(7)**

Question 7

A curve has equation

$$y = x\sqrt{\ln x}\,, \qquad x \in \mathbb{R}\,, \qquad x > 0\,.$$

Find the exact coordinates of the two points on the curve which have gradient $\frac{3}{2}$. **(9)**

Question 8

It is given that a and b are positive odd integers, with $a > b$.

Use **proof by contradiction** to show that if $a+b$ is a multiple of 4, then $a-b$ **cannot** be a multiple of 4. **(5)**

Question 9

It is required to find the single real root α of the following equation

$$x^2 = \frac{2}{\sqrt{x}} + \frac{3}{x^2}\,, \quad x > 0\,.$$

a) Show that the α lies between 1 and 2. **(2)**

b) Use the Newton Raphson method to show that α can be found by the iterative formula

$$x_{n+1} = \frac{x_n^5 + 3x_n^{\frac{5}{2}} + 9x_n}{2x_n^4 + x_n^{\frac{3}{2}} + 6}\,,$$

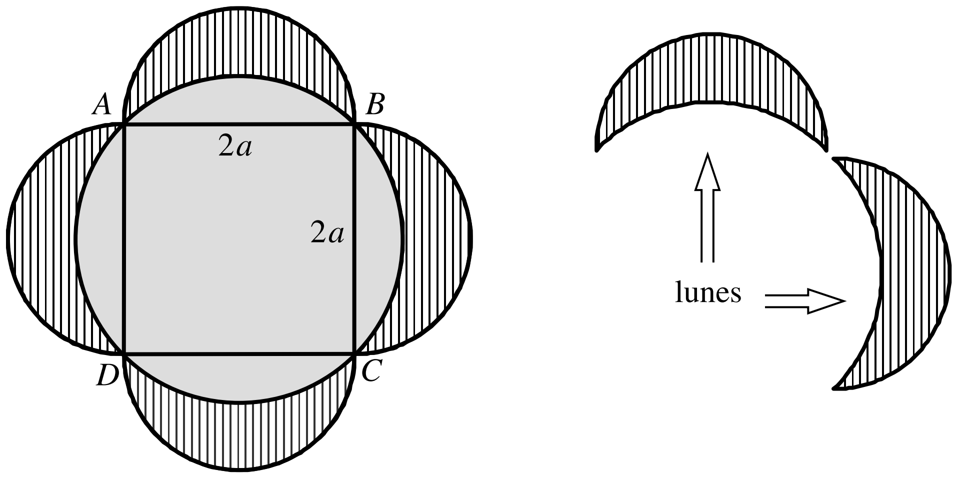
starting with a suitable value for x_1. **(5)**

c) Hence find the value of α, correct to 8 decimal places. **(1)**

Question 10

The figure below shows a square $ABCD$ of side length $2a$ cm, circumscribed by a circle.

Four semicircles are then drawn outside the square having each of the sides of the square as a diameter.



Each of the four regions bounded by a semicircle and the circumscribing circle is known by the mathematical name of a "lune", i.e. moon shaped.

Show that the area of the four lunes is equal to the area of the square $ABCD$. **(8)**

Question 11

$$\frac{A+Bx}{(2-x)^3} \equiv \frac{1}{4} + Cx^2 + Dx^3 + \ldots,$$

where A, B, C and D are constants, and $|x| < 2$

Determine the value of A, B, C and D. **(8)**

Question 12

A sequence t_1, t_2, t_3, t_4, t_5, ... is given by

$$t_{n+1} = at_n + 3n + 2, \quad t \in \mathbb{N}, \quad t_1 = -2,$$

where a is a non zero constant.

a) Given that $\displaystyle\sum_{r=1}^{3}\left(r^3 + t_r\right) = 12$, determine the possible values of a. **(7)**

b) Evaluate $\displaystyle\sum_{r=8}^{31}\left(t_{r+1} - at_r\right)$. **(5)**

Question 13

In a cold winter morning when the temperature of the air is $10°\text{C}$, Ben the builder pours a cup of coffee out of his flask.

Let x be the temperature of the coffee, in $°\text{C}$, t minutes after it was poured.

The rate at which the temperature of the coffee is decreasing is proportional to the square of the difference between the temperature of the coffee and the air temperature.

The initial temperature of the coffee is $80°\text{C}$ and ten minutes later the temperature of the coffee has dropped to $40°\text{C}$.

By forming and solving a suitable differential equation show that

$$x = \frac{20t + 1200}{2t + 15},$$

and hence find after how many minutes the coffee will have a temperature of $20°\text{C}$.

(10)

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Question 14

$$f(u) \equiv \frac{1}{u^2+5u+6}.$$

a) Express $f(u)$ into partial fractions. **(1)**

$$I = \int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}} \frac{1}{(\sin x + 2\cos x)(\sin x + 3\cos x)} \, dx.$$

b) Express I in the form

$$I = \int_{\arcsin\frac{3}{5}}^{\arccos\frac{3}{5}} \frac{\sec^2 x}{g(\tan x)} \, dx,$$

where g is a function to be found. **(2)**

c) Hence show that

$$I = \ln\left(\frac{a}{b}\right),$$

where a and b are positive integers to be found. **(7)**

Question 15

The point P lies on the curve given parametrically as

$$x = t^2, \quad y = t^2 - t, \quad t \in \mathbb{R}.$$

The tangent to the curve at P passes through the point with coordinates $\left(4, \frac{3}{2}\right)$.

Determine the possible coordinates of P. **(9)**

Question 16

A curve has equation

$$y = 2^{3e^{2x}}, \quad x \in \mathbb{R}.$$

Express $\dfrac{dy}{dx}$ in terms of y. **(5)**