IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper V

Difficulty Rating: 4.4120/1.5113

Time: 2 hours 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 14 questions in this question paper.
The total mark for this paper is 125.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

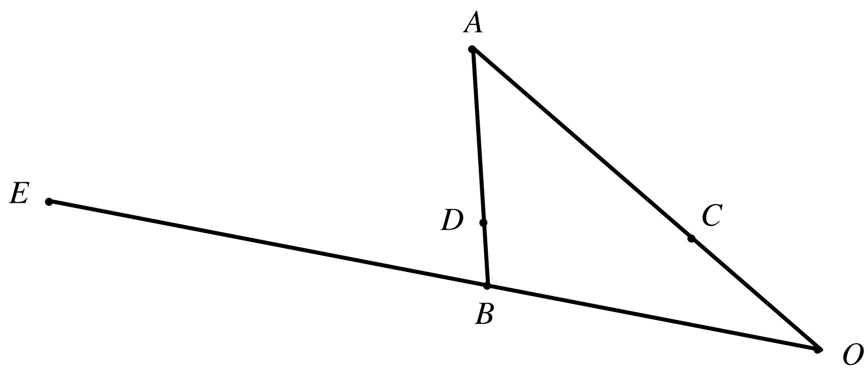
Question 1

Evaluate the following sum

$$\sum_{r=13}^{30}\left[(-2)^r - 4r - 78\right].$$

Detailed workings must support the answer. **(7)**

Question 2



The figure above shows the triangle OAB, where O is the origin and the position vectors of A and B relative to O, are $-6\mathbf{i}+27\mathbf{j}-9\mathbf{k}$ and $4\mathbf{i}+6\mathbf{j}-6\mathbf{k}$, respectively.

The point E is such so that O, B and E are collinear with $OB:BE=1:2$

The point C is such so that O, C and A are collinear with $OC:CA=1:2$

The point D is such so that B, D and A are collinear with $BD:DA=1:3$

a) Determine the coordinates of C, D and E, relative to O. **(3)**

b) Show that the points C, D and E are collinear, and find the ratio $CD:DE$. **(3)**

c) Show further that BC is parallel to EA, and find the ratio $BC:EA$. **(3)**

Question 3

Solve the following trigonometric equation

$$2\arctan\left(\frac{3}{x}\right)=\arcsin\left(\frac{6x}{25}\right).$$ **(7)**

Question 4

$$f(x)\equiv\left|4e^{2x}-28\right|,\ x\in\mathbb{R}.$$

a) Sketch the graph of $f(x)$.

The sketch must include the coordinates of any intersections with the coordinate axes and the equations of any asymptotes. **(3)**

The equation $f(x)-40=3x$ has a negative root α.

b) Show that $\alpha=-4.00045$, correct to 5 decimal places. **(5)**

The equation $f(x)-40=3x$ also has a positive root β.

c) Use a numerical method, based on an iterative method, to determine the value of β correct to 5 decimal places. **(6)**

Question 5

In the convergent expansion of

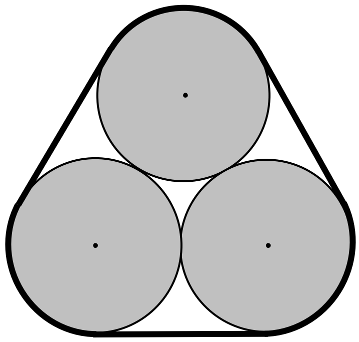
$$(1+kx)^n,\ |kx|<1,$$

where k and n are non zero constants, the coefficient of x^2 is 12 and the coefficient of x^3 is 32.

Given the coefficient of x is negative determine the values of k and n. **(8)**

Question 6

The figure below shows the plan of three identical circular cylinders of radius 6 cm, held together by an elastic band.



Show that the exact length of the stretched elastic band is $12(\pi+3) \text{ cm}$. **(4)**

Question 7

A curve has equation

$$y = \ln\left[\tan\left(x+\frac{\pi}{4}\right)\right], \text{ where } \tan\left(x+\frac{\pi}{4}\right) > 0.$$

Show that

$$\frac{dy}{dx} = 2\sec 2x \ . \qquad \textbf{(6)}$$

Question 8

The variables y, x and t are related by the equations

$$x^2 + 2xy + 2y^2 = 10 \qquad \text{and} \qquad y^2 = 4t, \ \ t \geq 0.$$

Find the possible values of $\frac{dx}{dt}$, when $t = \frac{1}{4}$. **(10)**

Question 9

Given that

$$\sec^2 x - \left(1+\sqrt{3}\right)\tan x + \sqrt{3} = 1,$$

show that either $\tan x = 1$ or $\tan x = \sqrt{3}$.

Detailed workings must support the answer in this question. **(6)**

Question 10

A curve is given parametrically by the equations

$$x = 4\sin\theta, \ \ y = \cos 2\theta, \ \ 0 \le \theta < \pi.$$

The tangent to the curve at the point P meets the x axis at the point $(3,0)$.

Determine the possible coordinates of P. **(8)**

Question 11

Mould is spreading on a wall of area 20 m^2 and when it was first noticed 2 m^2 of the wall was already covered by this mould.

Let A, in m^2, represent the area of the wall covered by the mould, after time t weeks.

The rate at which A is changing is proportional to the product of the area covered by the mould and the area of the wall not yet covered by the mould.

After a further period of 2 weeks the area of the wall covered by the mould is 4 m^2.

By forming and solving a suitable differential equation, show that

$$A = \frac{20}{1+9\left(\frac{2}{3}\right)^t}.$$

(12)

Question 12

The curve C has equation

$$y = \sqrt{e^{2x}+1}, \ x \in \mathbb{R}.$$

The tangent to the curve at the point P, where $x = p$, passes through the origin.

a) Show that $x = p$ is a solution of the equation

$$(x-1)e^{2x} = 1. \qquad \textbf{(6)}$$

b) Show further that the equation of part **(a)** has a root between 1 and 2. **(1)**

c) By using the Newton Raphson method once, starting with $x = 1$, find an approximation for this root, correct to 1 decimal place. **(4)**

It is further given that the Newton Raphson method fails on this occasion.

d) Use an appropriate method to verify that the root of the equation of part **(a)** is 1.10886 correct to 5 decimal places. **(2)**

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Question 13

The function f is defined as

$$f(x) = 3 - \ln 4x, \quad x \in \mathbb{R},\ x > 0$$

a) Determine, in exact form, the coordinates of the point where the graph of f crosses the x axis. **(1)**

Consider the following sequence of transformations T_1, T_2 and T_3.

$$\ln x \xrightarrow{T_1} \ln 4x \xrightarrow{T_2} -\ln 4x \xrightarrow{T_3} 3 - \ln 4x$$

b) Describe geometrically each of the transformations T_1, T_2 and T_3, and hence sketch the graph of $f(x)$.
Indicate clearly any intersections with the coordinate axes. **(3)**

The function g is defined by

$$g(x) = \mathrm{e}^{5-x}, \quad x \in \mathbb{R}.$$

c) Show that

$$fg(x) = x - k - k \ln k \ ,$$

where k is a positive integer. **(4)**

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Question 14

It is given that

$$\sqrt{5-4x-x^2} = (1-x)u \, , \; x \neq 1 \, , \; x \neq -5 \, .$$

a) Show clearly that …

i. … $x = \dfrac{u^2-5}{u^2+1}$. **(3)**

ii. … $dx = \dfrac{12u}{\left(u^2+1\right)^2}\,du$. **(3)**

b) Hence show further that

$$\int \frac{x}{\left(5-4x-x^2\right)^{\frac{3}{2}}}\,dx = \int \frac{u^2-5}{18u^2}\,du \, .$$ **(4)**

c) Find a simplified expression for

$$\int \frac{x}{\left(5-4x-x^2\right)^{\frac{3}{2}}}\,dx \, .$$ **(3)**