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IYGB GCE

Mathematics MP2

Advanced Level

Practice Paper X

Difficulty Rating: 4.4720/1.5707

Time: 2 hours 30 minutes

Candidates may use any calculator allowed by the regulations of this examination.

Information for Candidates

This practice paper follows closely the Pearson Edexcel Syllabus, suitable for first assessment Summer 2018.

The standard booklet "Mathematical Formulae and Statistical Tables" may be used.
Full marks may be obtained for answers to ALL questions.
The marks for the parts of questions are shown in round brackets, e.g. **(2)**.
There are 13 questions in this question paper.
The total mark for this paper is 125.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.
Non exact answers should be given to an appropriate degree of accuracy.
The examiner may refuse to mark any parts of questions if deemed not to be legible.

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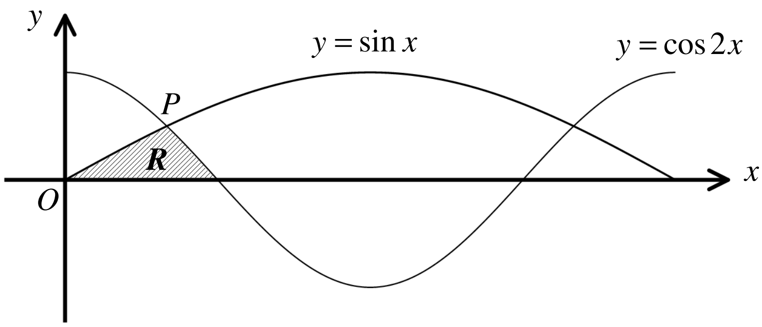
Question 1

Three numbers, A, B, C in that order, are in geometric progression with common ratio r.

Given further that A, $2B$, C in that order are in arithmetic progression, determine the possible values of r.

(5)

Question 2



The figure above shows the graphs of the curves with equations

$$y = \cos 2x \text{ and } y = \sin x, \text{ for } 0 \le x \le \pi .$$

The point P is the first intersection between the graphs for which $x > 0$.

a) Show clearly that the x coordinate of P is $\frac{\pi}{6}$. (5)

The finite region R, shown shaded in the figure, is bounded by the two curves and the x axis, and includes the point P on its boundary.

b) Determine the exact area of R. (6)

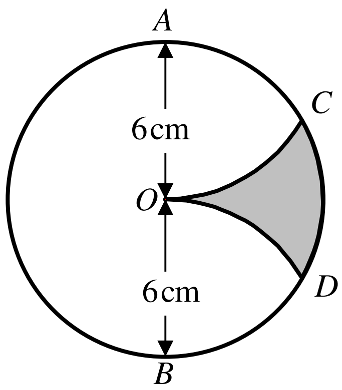
Question 3

Solve the following equation

$$x - 2x^2 + x^3 - 2x^4 + x^5 - 2x^6 + \ldots = -\frac{2}{5}.$$

You may assume that the left hand side of the equation converges. **(7)**

Question 4



The figure above shows a circle of radius 6 cm, centred at O.

An arc OC with centre at A and radius 6 cm is drawn inside the circle.

A second arc OD is drawn with centre at B and radius 6 cm.

Show clearly that the area of the shaded region OCD is

$$6\left(3\sqrt{3} - \pi\right) \text{ cm}^2.$$

(7)

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Question 5

$$y = \arcsin x, \ -1 \le x \le 1.$$

a) By writing $y = \arcsin x$ as $x = \sin y$ show that

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}. \qquad \textbf{(4)}$$

The curve C has equation

$$y = 2\arcsin x - 4x^{\frac{3}{2}}, \ 0 \le x \le 1.$$

b) Show that the x coordinates of the stationary points of C are the solutions of the equation

$$9x^3 - 9x + 1 = 0. \qquad \textbf{(4)}$$

c) Show further that one of the roots, α, of the equation of part **(b)** is 0.9390, correct to 4 decimal places. **(3)**

It is further given that the equation of part **(b)** has 2 more real roots, $\beta \approx -1.0515$, and γ.

d) Determine the value of γ, correct to 3 places. **(4)**

Question 6

The mass of a radioactive isotope decays at a rate proportional to the mass of the isotope present.

The half life of the isotope is 80 years.

Determine the percentage of the original amount which remains after 50 years. **(11)**

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Question 7

$$y = \ln(1 + \sin x), \ \sin x \neq -1.$$

Show clearly that $\frac{d^2y}{dx^2} = f(y)$, where $f(y)$ is a function to be found. **(7)**

Question 8

Solve the trigonometric equation

$$\sin x \cos x \cos 2x \cos 4x = \frac{\sqrt{2}}{16}, \quad 0 \leq x \leq \frac{\pi}{2},$$

giving the answers in terms of π. **(7)**

Question 9

The function f is defined as

$$f(x) \equiv \frac{a(2-3x)}{(1-2x)(2+x)}, \ x \in \mathbb{R}, \ |x| < \tfrac{1}{2}, \ x \neq 0.$$

where a is a non zero constant.

a) Show that for all values of the constant a, the coefficient of x in the binomial series expansion of $f(x)$, is zero. **(6)**

b) Find the value of a, given that the coefficient of x^2 in the binomial series expansion of $f(x)$, is 10. **(3)**

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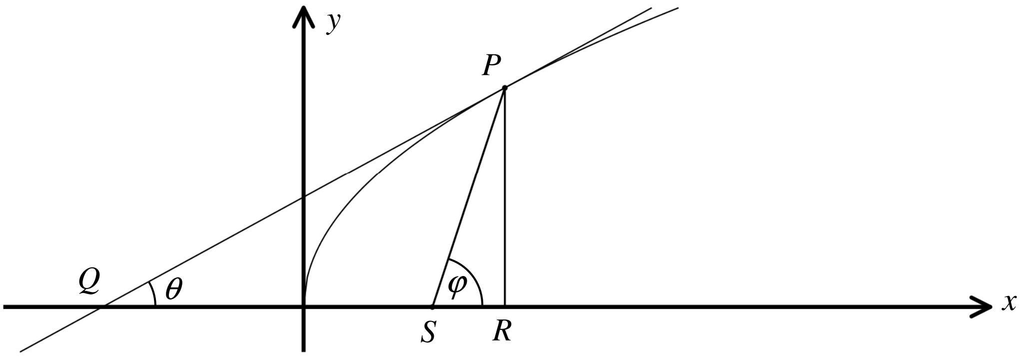
Question 10



The figure above shows the curve C with parametric equations

$$x = t^2, \; y = 2t, \; t \in \mathbb{R}, \; t \geq 0.$$

The point P lies on C, where $t = p$. The point R lies on the x axis so that PR is parallel to the y axis. The tangent to C at the point P meets the x axis at the point Q, so that the angle $\measuredangle PQR = \theta$.

a) Find the coordinates of Q in terms of p. **(4)**

b) By considering the triangle PQR, show $\tan\theta = \dfrac{1}{p}$. **(2)**

The point S has coordinates $(1,0)$ and $\measuredangle PSR = \varphi$.

c) Find an expression for $\tan\varphi$ in terms of p and hence show that $\varphi = 2\theta$. **(5)**

d) Deduce that $|SP| = |SQ|$. **(3)**

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Question 11

The functions f and g are defined by

$$f(x) = 2x + 1, \ x \in \mathbb{R}, \ x \le 5$$

$$g(x) = \sqrt{x-1}, \ \ x \in \mathbb{R}, \ \ x \ge 10.$$

a) Find an expression for the composite function $fg(x)$, further stating its domain and range. **(7)**

The domain of $g(x)$ is next changed to $x > a$.

b) Given that now $gf(x)$ **cannot** be formed, determine the smallest possible value of the constant a. **(4)**

Question 12

Use trigonometric identities to find the value of

$$\int_0^{\frac{\pi}{3}} 32 \sin x \sin 2x \sin 3x \ dx.$$ **(9)**

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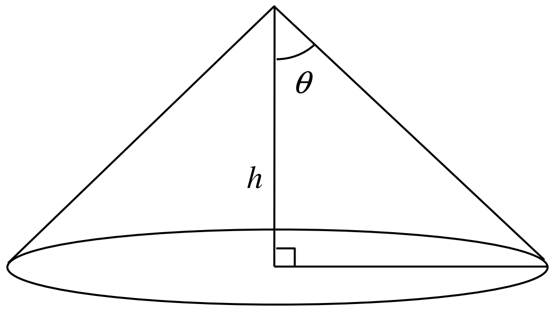
Question 13



Fine sand starts falling onto a horizontal floor at the constant rate of $50 \text{ cm}^3\text{s}^{-1}$.

A heap is formed in the shape of a right circular cone of height h cm.

The angle θ, where $\tan\theta = 3$, is formed between the vertical height and the slant height of the cone, as shown in the figure above.

Show that when $t = 60$

$$\frac{dh}{dt} = \frac{1}{18}\pi^{-\frac{1}{3}}.$$

where t is the time in seconds since the sand started falling.

$\left[\text{volume of a cone of radius } r \text{ and height } h \text{ is given by } \tfrac{1}{3}\pi r^2 h\right]$ **(12)**