Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

AS and A level Further Mathematics
Decision Mathematics 1

Practice Paper
Prim and Kruskal

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 68.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

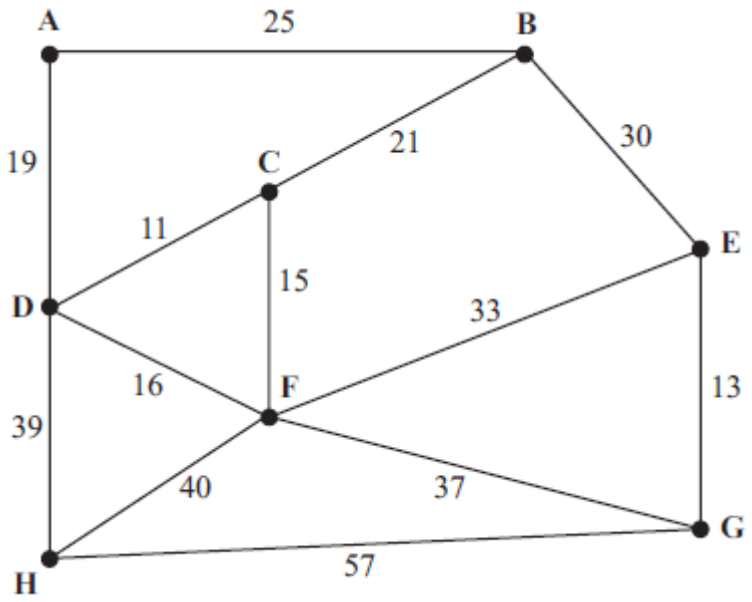

Figure 1

Figure 1 represents the distances, in km, between eight vertices **A**, **B**, **C**, **D**, **E**, **F**, **G** and **H** in a network.

(*a*) Use **Kruskal's** algorithm to find the minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(*b*) Starting at A, use **Prim's** algorithm to find the minimum spanning tree. You must clearly state the order in which you selected the arcs of your tree.

(3)

(*c*) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

(*d*) State the weight of the tree.

(1)

(Total 8 marks)

2.

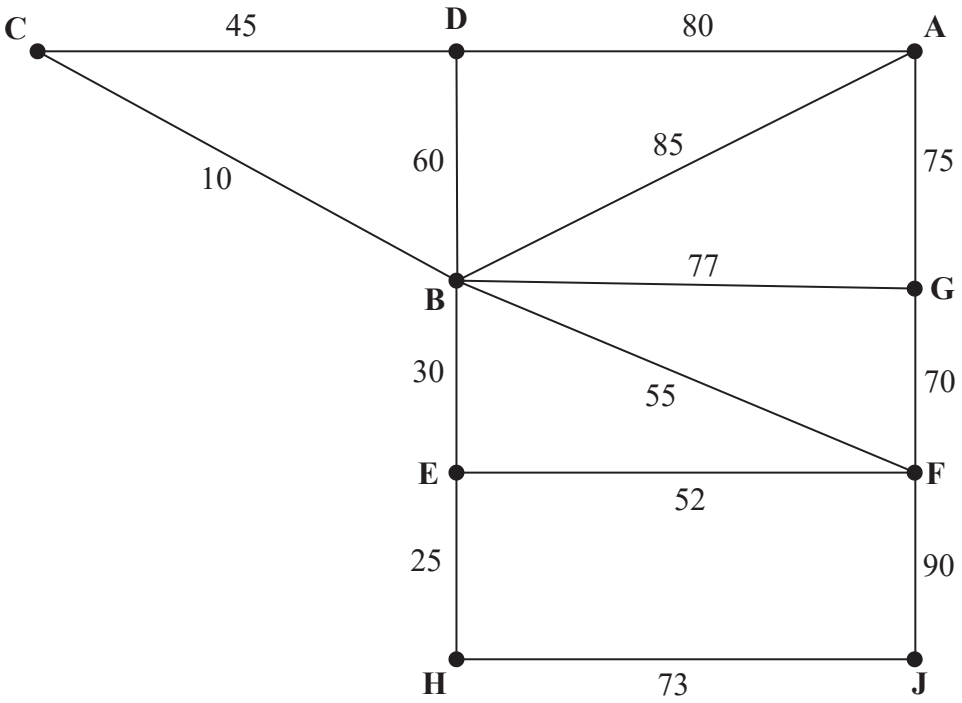

Figure 2

Figure 2 represents nine computer terminals, A, B, C, D, E, F, G, H and J, at Pearsonby School. The school wishes to connect them to form a single computer network. The number on each arc represents the cost, in pounds, of connecting the corresponding computer terminals.

(*a*) Use Prim's algorithm, starting at B, to find the minimum spanning tree for the computer network. You must clearly state the order in which you select the arcs of your tree.

(3)

(*b*) State the minimum cost of connecting the nine computer terminals.

(1)

It is discovered that some computer terminals are already connected. There are already direct connections along BD and FJ, as shown in bold in Diagram 1 in the answer book. It is decided to use these connections.

(*c*) Use Kruskal's algorithm to find the minimum spanning tree that includes arcs BD and FJ. You must list the arcs in the order that you consider them. In each case, state whether or not you are adding the arcs to your spanning tree.

(3)

(Total 7 marks)

3.

	A	B	C	D	E	F
A	–	85	110	160	225	195
B	85	–	100	135	180	150
C	110	100	–	215	200	165
D	160	135	215	–	235	215
E	225	180	200	235	–	140
F	195	150	165	215	140	–

The table shows the average journey time, in minutes, between six towns, A, B, C, D, E and F.

(*a*) Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network. You must list the arcs that form your tree in the order in which you selected them.

(3)

(*b*) Draw your tree using the vertices given in Diagram 1 in the answer book.

(1)

(*c*) Find the weight of your minimum spanning tree.

(1)

Kruskal's algorithm may also be used to find a minimum spanning tree.

(*d*) State three differences between Prim's algorithm and Kruskal's algorithm.

(3)

(Total 8 marks)

4.

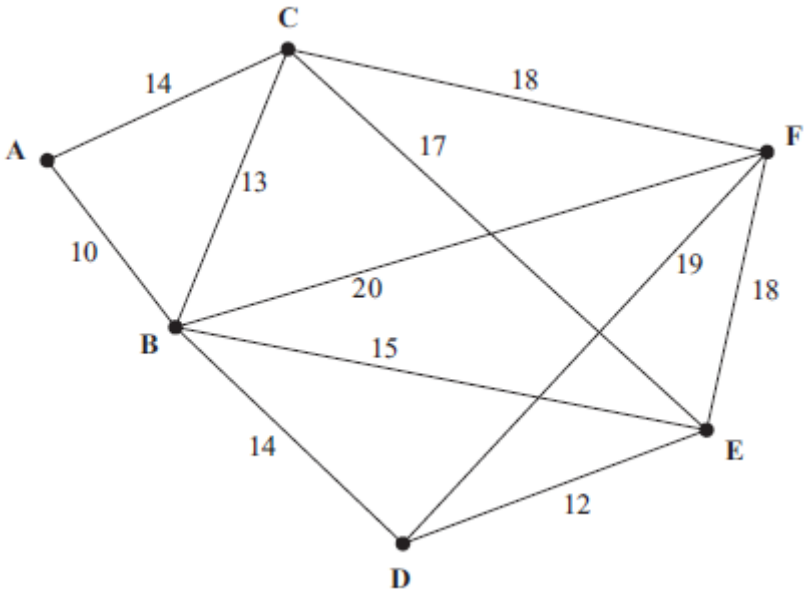

Figure 3

(*a*) Define the terms

(i) tree,

(ii) minimum spanning tree.

(3)

(*b*) Use Kruskal's algorithm to find a minimum spanning tree for the network shown in Figure 3.

You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(*c*) Draw your minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

(*d*) State whether your minimum spanning tree is unique. Justify your answer.

(1)

(Total 8 marks)

5.

	A	B	C	D	E	F
A	–	15	6	9	–	–
B	15	–	12	–	14	–
C	6	12	–	7	10	–
D	9	–	7	–	11	17
E	–	14	10	11	–	5
F	–	–	–	17	5	–

The table shows the times, in days, needed to repair the network of roads between six towns, A, B, C, D, E and F, following a flood.

(*a*) Use **Prim's** algorithm, starting at A, to find the minimum connector for this network. You must list the **arcs** that form your tree in the order that you selected them.

(3)

(*b*) Draw your minimum connector using the vertices given in Diagram 1 in the answer book.

(1)

(*c*) Add arcs from D, E and F to Diagram 2 in the answer book, so that it shows the network of roads shown by the table.

(2)

(*d*) Use **Kruskal's** algorithm to find the minimum connector. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum connector.

(3)

(*e*) State the minimum time needed, in days, to reconnect the six towns.

(1)

(Total 10 marks)

6.

	A	B	C	D	E	F	G
A	–	15	19	–	22	24	–
B	15	–	–	8	13	–	–
C	19	–	–	12	–	16	–
D	-	8	12	–	10	–	18
E	22	13	–	10	–	15	16
F	24	–	16	–	15	–	17
G	–	–	–	18	16	17	–

The table shows the lengths, in km, of a network of roads between seven villages, A, B, C, D, E, F and G.

(*a*) Complete the drawing of the network in Diagram 1 of the answer book by adding the necessary arcs from vertex D together with their weights.

(2)

(*b*) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(*c*) Draw the minimum spanning tree using the vertices provided in Diagram 2 in the answer book.

(1)

(*d*) State the weight of the minimum spanning tree.

(1)

(Total 7 marks)

7.

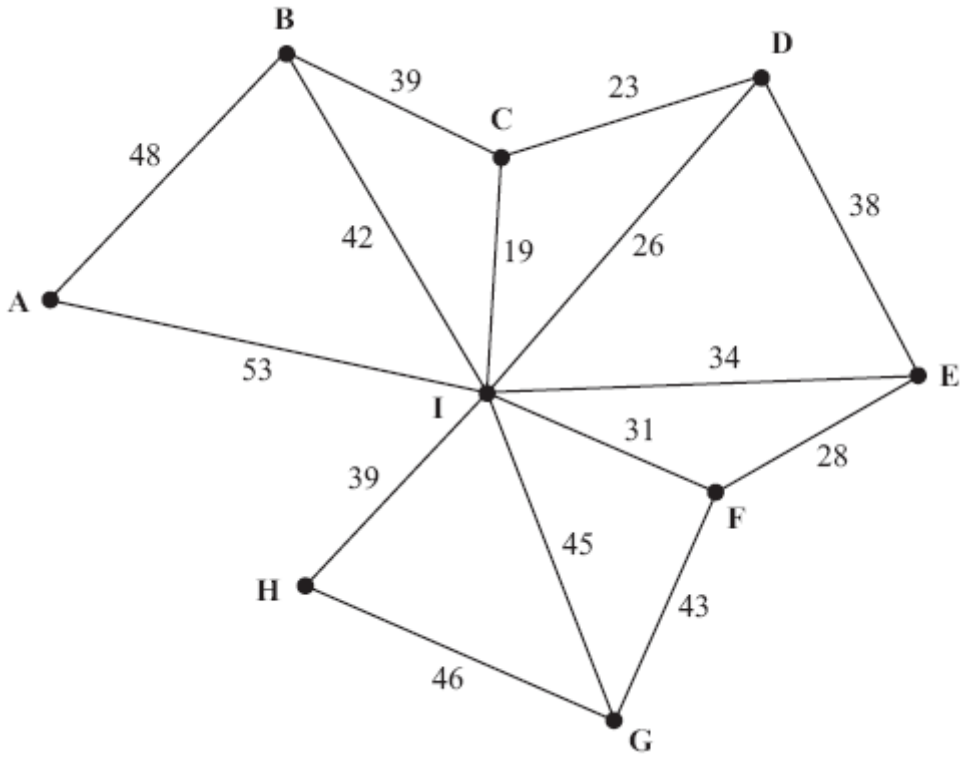

Figure 4

(*a*) Use Kruskal's algorithm to find a minimum spanning tree for the network shown in Figure 4. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(*b*) Starting at **A**, use Prim's algorithm to find a minimum spanning tree for the network in Figure 4. You must clearly state the order in which you include the arcs in your tree.

(3)

(*c*) Draw a minimum spanning tree for the network in Figure 4 using the vertices given in Diagram 1 of the answer book. State the weight of the minimum spanning tree.

(2)

A new spanning tree is required which includes the arcs **DI** and **HG**, and which has the lowest possible total weight.

(*d*) Explain which algorithm you would choose to complete the tree, and how the algorithm should be adapted. (You do not need to find the tree.)

(2)

(Total 10 marks)

8.

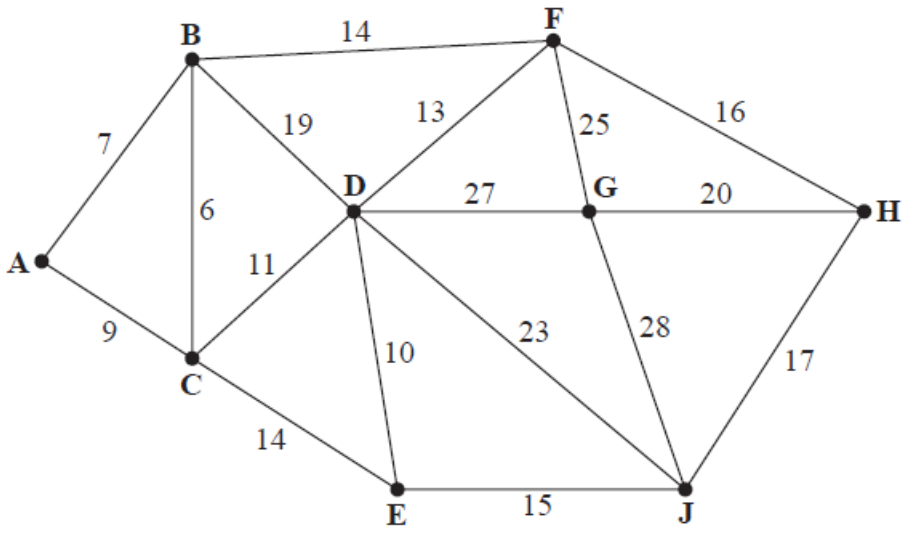

Figure 5

The numbers on the 17 arcs in the network shown in Figure 5 represent the distances, in km, between nine nodes, **A**, **B**, **C**, **D**, **E**, **F**, **G**, **H** and **J**.

(*a*) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(*b*) Starting at **G**, use Prim's algorithm to find a minimum spanning tree. You must clearly state the order in which you select the arcs of your tree.

(3)

(*c*) Find the weight of the minimum spanning tree.

(1)

A connected graph V has n nodes. The sum of the degrees of all the nodes in V is m. The graph T is a minimum spanning tree of V.

(*d*) (i) Write down, in terms of m, the number of arcs in V.

(ii) Write down, in terms of n, the number of arcs in T.

(iii) Hence, write down an inequality, in terms of m and n, comparing the number of arcs in T and V.

(3)

(Total 10 marks)

TOTAL FOR PAPER: 68 MARKS