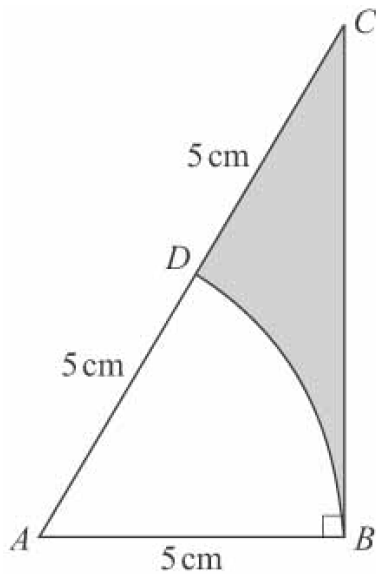


Radians Mixed exercise

1



a In the right-angled triangle ABC :

$$\cos \angle BAC = \frac{BA}{AC} = \frac{5}{10} = \frac{1}{2}$$

$$\text{so } \angle BAC = \frac{\pi}{3}$$

b Area of triangle ABC

$$\begin{aligned} &= \frac{1}{2} \times AB \times AC \times \sin \angle BAC \\ &= \frac{1}{2} \times 5 \times 10 \times \sin \frac{\pi}{3} = 21.650\dots \text{cm}^2 \end{aligned}$$

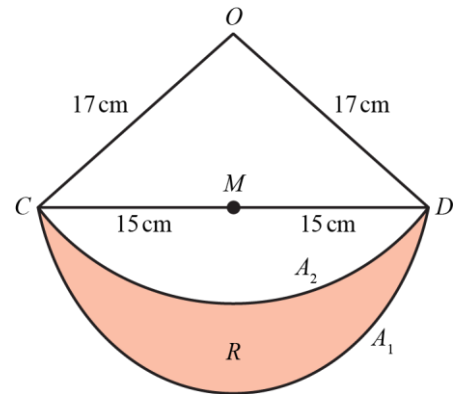
Area of sector DAB

$$= \frac{1}{2} \times 5^2 \times \frac{\pi}{3} = 13.089\dots \text{cm}^2$$

Area of shaded region

$$\begin{aligned} &= \text{area of } \triangle ABC - \text{area of sector } DAB \\ &= 21.650\dots - 13.089\dots = 8.56 \text{ cm}^2 \quad (3 \text{ s.f.}) \end{aligned}$$

2



a Using Pythagoras' theorem to find OM :

$$OM^2 = 17^2 - 15^2 = 64 \Rightarrow OM = 8 \text{ cm}$$

$$\begin{aligned} \text{Area of } \triangle OCD &= \frac{1}{2} \times CD \times OM \\ &= \frac{1}{2} \times 30 \times 8 = 120 \text{ cm}^2 \end{aligned}$$

b Area of shaded region R

$$\begin{aligned} &= \text{area of semicircle } CDA_1 \\ &\quad - \text{area of segment } CDA_2 \end{aligned}$$

Area of semicircle CDA_1

$$= \frac{1}{2} \times \pi \times 15^2 = 353.429\dots \text{cm}^2$$

Area of segment CDA_2

$$\begin{aligned} &= \text{area of sector } OCD \\ &\quad - \text{area of triangle } OCD \end{aligned}$$

$$= \frac{1}{2} \times 17^2 \times \angle COD - 120$$

In right-angled triangle COM :

$$\sin \angle COM = \frac{CM}{OC} = \frac{15}{17}$$

$$\text{so } \angle COM = 1.0808\dots$$

$$\text{hence } \angle COD = 2.1616\dots$$

So area of segment CDA_2

$$= \frac{1}{2} \times 17^2 \times 2.1616\dots - 120$$

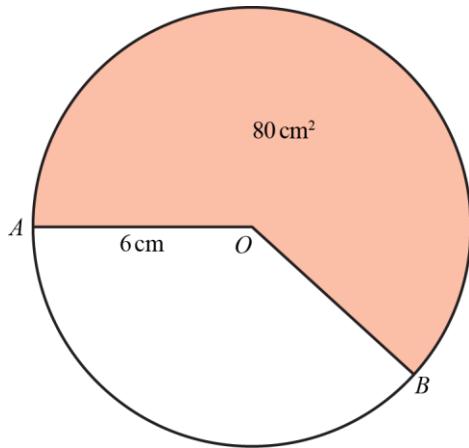
$$= 192.362\dots \text{cm}^2$$

So area of shaded region R

$$= 353.429\dots - 192.362\dots$$

$$= 161.07 \text{ cm}^2 \quad (2 \text{ d.p.})$$

3



- a** Reflex angle $AOB = (2\pi - \theta)$ rad
Area of shaded sector

$$= \frac{1}{2} \times 6^2 \times (2\pi - \theta)$$

$$= (36\pi - 18\theta) \text{ cm}^2$$

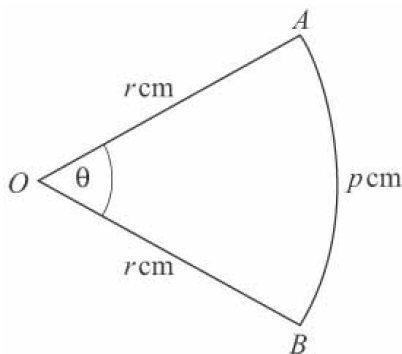
$$\text{So } 80 = 36\pi - 18\theta$$

$$\Rightarrow 18\theta = 36\pi - 80$$

$$\Rightarrow \theta = \frac{36\pi - 80}{18} = 1.839 \text{ (3 d.p.)}$$

- b** Length of minor arc AB
 $= 6\theta = 6 \times 1.8387\dots = 11.03 \text{ cm (2 d.p.)}$

4



- a** Using $l = r\theta$:

$$p = r\theta \Rightarrow \theta = \frac{p}{r}$$

- b** Area of sector

$$= \frac{1}{2} r^2 \theta = \frac{1}{2} r^2 \times \frac{p}{r} = \frac{1}{2} pr \text{ cm}^2$$

- c** $4.65 \leq r < 4.75$, $5.25 \leq p < 5.35$
Least possible value for area of sector
 $= \frac{1}{2} \times 5.25 \times 4.65 = 12.207 \text{ cm}^2 \text{ (3 d.p.)}$
(Note: Least possible value is 12.20625, so 12.207 should be given, not 12.206)

- d** Maximum possible value of θ

$$= \frac{\max p}{\min r} = \frac{5.35}{4.65} = 1.1505\dots$$

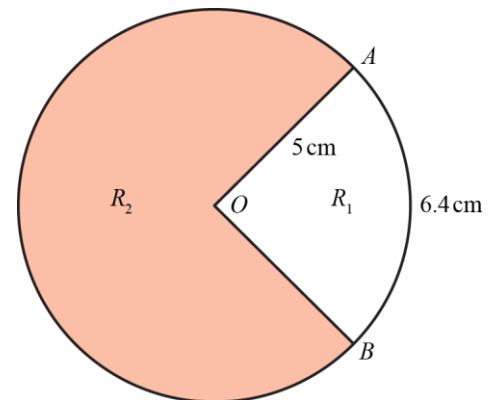
So give 1.150 (3 d.p.)

Minimum possible value of θ

$$= \frac{\min p}{\max r} = \frac{5.25}{4.75} = 1.1052\dots$$

So give 1.106 (3 d.p.)

5



- a** Using $l = r\theta$:

$$6.4 = 5\theta \Rightarrow \theta = \frac{6.4}{5} = 1.28 \text{ rad}$$

- b** Using area of sector $= \frac{1}{2} r^2 \theta$:

$$R_1 = \frac{1}{2} \times 5^2 \times 1.28 = 16$$

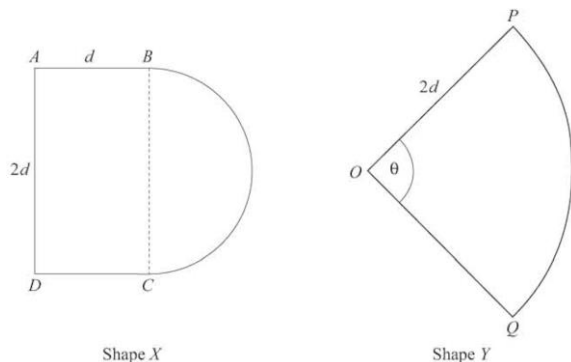
- c** $R_2 = \text{area of circle} - R_1$

$$= \pi \times 5^2 - 16 = 62.5398\dots$$

$$\text{So } \frac{R_1}{R_2} = \frac{16}{62.5398\dots} = \frac{1}{3.908\dots} = \frac{1}{p}$$

$$\Rightarrow p = 3.91 \text{ (3 s.f.)}$$

6



- a** Area of shape X
 = area of rectangle + area of semicircle
 = $(2d^2 + \frac{1}{2}\pi d^2)$ cm²

$$\text{Area of shape } Y = \frac{1}{2}(2d)^2\theta = 2d^2\theta \text{ cm}^2$$

Since $X = Y$:

$$2d^2 + \frac{1}{2}\pi d^2 = 2d^2\theta$$

Divide by $2d^2$:

$$1 + \frac{\pi}{4} = \theta$$

- b** Perimeter of shape X
 = $(d + 2d + d + \pi d)$ cm with $d = 3$
 = $(3\pi + 12)$ cm

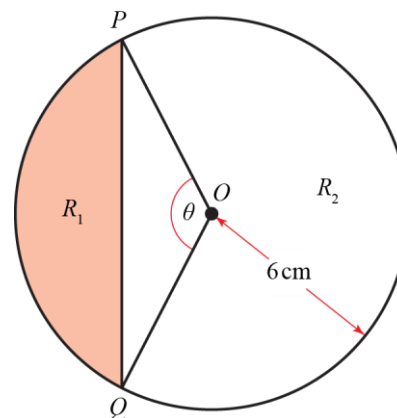
- c** Perimeter of shape Y
 = $(2d + 2d + 2d\theta)$ cm
 with $d = 3$ and $\theta = 1 + \frac{\pi}{4}$

$$= 12 + 6\left(1 + \frac{\pi}{4}\right)$$

$$= \left(18 + \frac{3\pi}{2}\right) \text{ cm}$$

- d** Difference
 = $\left(18 + \frac{3\pi}{2}\right) - (3\pi + 12)$
 = $6 - \frac{3\pi}{2}$
 = 1.287... cm
 = 12.9 mm (3 s.f.)

7



- a** Area of segment R_1
 = area of sector OPQ
 – area of triangle OPQ
 $\Rightarrow A_1 = \frac{1}{2} \times 6^2 \times \theta - \frac{1}{2} \times 6^2 \times \sin \theta$
 $\Rightarrow A_1 = 18(\theta - \sin \theta)$

- b** $A_2 =$ area of circle – A_1
 = $\pi \times 6^2 - 18(\theta - \sin \theta)$
 = $36\pi - 18(\theta - \sin \theta)$

Since $A_2 = 3A_1$:

$$36\pi - 18(\theta - \sin \theta) = 3 \times 18(\theta - \sin \theta)$$

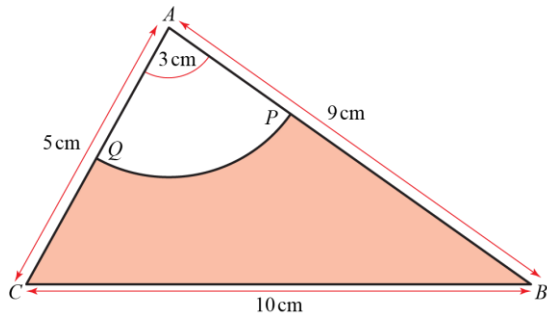
$$36\pi - 18(\theta - \sin \theta) = 54(\theta - \sin \theta)$$

$$36\pi = 72(\theta - \sin \theta)$$

$$\pi = \theta - \sin \theta$$

$$\sin \theta = \theta - \frac{\pi}{2}$$

8



a Using the cosine rule in $\triangle ABC$:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos \angle BAC = \frac{5^2 + 9^2 - 10^2}{2 \times 5 \times 9} = 0.0\dot{6}$$

$$\Rightarrow \angle BAC = 1.50408\dots$$

$$= 1.504 \text{ rad (3 d.p.)}$$

b i Using the sector area formula:

$$\text{area of sector} = \frac{1}{2} r^2 \theta$$

$$\Rightarrow \text{area of sector } APQ$$

$$= \frac{1}{2} \times 3^2 \times 1.504 = 6.77 \text{ cm}^2 \text{ (3 s.f.)}$$

ii Area of shaded region $BPQC$

$$= \text{area of } \triangle ABC - \text{area of sector } APQ$$

$$= \frac{1}{2} \times 5 \times 9 \times \sin 1.504 - \frac{1}{2} \times 3^2 \times 1.504$$

$$= 15.681\dots$$

$$= 15.7 \text{ cm}^2 \text{ (3 s.f.)}$$

iii Perimeter of shaded region $BPQC$

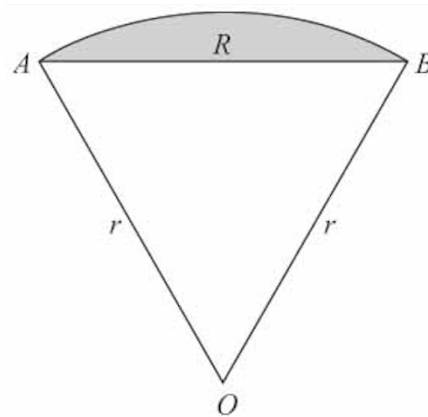
$$= QC + CB + BP + \text{arc length } PQ$$

$$= 2 + 10 + 6 + (3 \times 1.504)$$

$$= 22.51\dots$$

$$= 22.5 \text{ cm (3 s.f.)}$$

9



a Area of sector $= \frac{1}{2} r^2 \theta = \frac{1}{2} r^2 \times 1.5 \text{ cm}^2$

$$\text{So } \frac{3}{4} r^2 = 15$$

$$\Rightarrow r^2 = \frac{60}{3} = 20$$

$$\Rightarrow r = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

b Arc length $AB = r(1.5) = 3\sqrt{5} \text{ cm}$

Perimeter of sector

$$= AO + OB + \text{arc length } AB$$

$$= 2\sqrt{5} + 2\sqrt{5} + 3\sqrt{5}$$

$$= 7\sqrt{5}$$

$$= 15.7 \text{ cm (3 s.f.)}$$

c Area of segment R

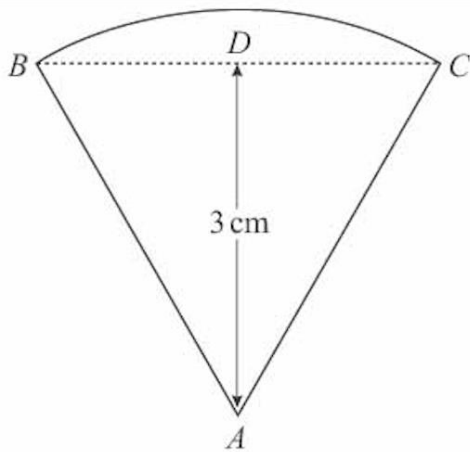
$$= \text{area of sector} - \text{area of } \triangle AOB$$

$$= 15 - \frac{1}{2} r^2 \sin 1.5$$

$$= 15 - 10 \sin 1.5$$

$$= 5.025 \text{ cm}^2 \text{ (3 d.p.)}$$

10



a Using the right-angled $\triangle ABD$, with

$$\angle ABD = \frac{\pi}{3} :$$

$$\sin \frac{\pi}{3} = \frac{3}{AB}$$

$$\Rightarrow AB = \frac{3}{\sin \frac{\pi}{3}} = \frac{3}{\frac{\sqrt{3}}{2}}$$

$$= 3 \times \frac{2}{\sqrt{3}} = 2\sqrt{3} \text{ cm}$$

b Area of badge = area of sector

$$= \frac{1}{2} \times (2\sqrt{3})^2 \theta \text{ where } \theta = \frac{\pi}{3}$$

$$= \frac{1}{2} \times 4 \times 3 \times \frac{\pi}{3}$$

$$= 2\pi \text{ cm}^2$$

c Perimeter of badge

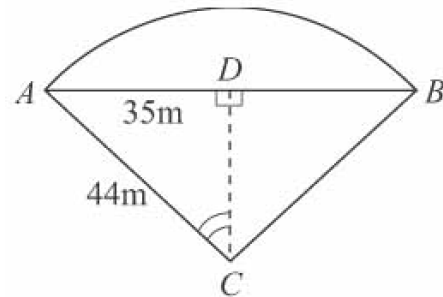
$$= AB + AC + \text{arc length } BC$$

$$= 2\sqrt{3} + 2\sqrt{3} + 2\sqrt{3} \times \frac{\pi}{3}$$

$$= 2\sqrt{3} \left(2 + \frac{\pi}{3} \right)$$

$$= \frac{2\sqrt{3}}{3} (6 + \pi) \text{ cm}$$

11



a Using the right-angled $\triangle ADC$:

$$\sin \angle ACD = \frac{35}{44}$$

$$\text{So } \angle ACD = \sin^{-1} \frac{35}{44}$$

$$\text{and } \angle ACB = 2 \sin^{-1} \frac{35}{44}$$

$$\Rightarrow \angle ACB = 1.8395\dots$$

$$= 1.84 \text{ rad (2 d.p.)}$$

b i Length of railway track

$$= \text{length of arc } AB$$

$$= 44 \times 1.8395\dots$$

$$= 80.9 \text{ m (3 s.f.)}$$

ii Shortest distance from C to AB is DC .

Using Pythagoras' theorem:

$$DC^2 = 44^2 - 35^2$$

$$DC = \sqrt{44^2 - 35^2} = 26.7 \text{ m (3 s.f.)}$$

iii Area of region

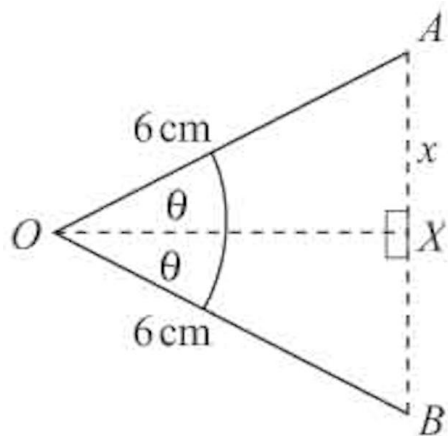
$$= \text{area of segment}$$

$$= \text{area of sector } ABC - \text{area of } \triangle ABC$$

$$= \frac{1}{2} \times 44^2 \times 1.8395\dots - \frac{1}{2} \times 70 \times DC$$

$$= 847 \text{ m}^2 \text{ (3 s.f.)}$$

12



a In right-angled $\triangle OAX$ (see diagram):

$$\frac{x}{6} = \sin \theta$$

$$\Rightarrow x = 6 \sin \theta$$

$$\text{So } AB = 2x = 12 \sin \theta \text{ (} AB = DC \text{)}$$

Perimeter of the cross-section

$$= \text{arc length } AB + AD + DC + BC$$

$$= 6 \times 2\theta + 4 + 12 \sin \theta + 4 \text{ cm}$$

$$= (8 + 12\theta + 12 \sin \theta) \text{ cm}$$

$$\text{So } 2(7 + \pi) = 8 + 12\theta + 12 \sin \theta$$

$$\Rightarrow 14 + 2\pi = 8 + 12\theta + 12 \sin \theta$$

$$\Rightarrow 12\theta + 12 \sin \theta - 6 = 2\pi$$

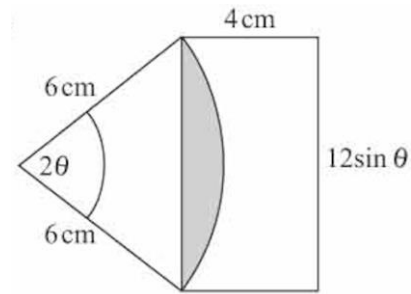
Divide by 6:

$$2\theta + 2 \sin \theta - 1 = \frac{\pi}{3}$$

b When $\theta = \frac{\pi}{6}$,

$$\begin{aligned} 2\theta + 2 \sin \theta - 1 &= \frac{\pi}{3} + \left(2 \times \frac{1}{2}\right) - 1 \\ &= \frac{\pi}{3} \end{aligned}$$

c



Area of the cross-section
= area of rectangle $ABCD$

– area of shaded segment

$$\text{Area of rectangle} = 4 \times 12 \times \sin \frac{\pi}{6}$$

$$= 24 \text{ cm}^2$$

Area of shaded segment

= area of sector – area of triangle

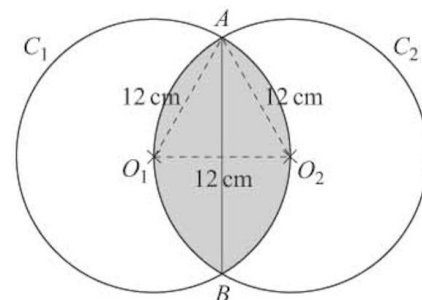
$$= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} - \frac{1}{2} \times 6^2 \times \sin \frac{\pi}{3}$$

$$= 3.261 \dots \text{ cm}^2$$

So area of cross-section

$$= 20.7 \text{ cm}^2 \text{ (3 s.f.)}$$

13



a $O_1A = O_2A = 12$, as they are radii of their respective circles.

$O_1O_2 = 12$, as O_2 is on the circumference of C_1 and hence is a radius (and vice versa).

Therefore $\triangle AO_1O_2$ is equilateral.

$$\text{So } \angle AO_1O_2 = \frac{\pi}{3}$$

$$\text{and } \angle AO_1B = 2 \times \angle AO_1O_2 = \frac{2\pi}{3}$$

13 b Consider arc AO_2B of circle C_1 .

Using arc length = $r\theta$:

$$\text{arc length } AO_2B = 12 \times \frac{2\pi}{3} = 8\pi \text{ cm}$$

Perimeter of R

$$= \text{arc length } AO_2B + \text{arc length } AO_1B$$

$$= 2 \times 8\pi = 16\pi \text{ cm}$$

c Consider the segment AO_2B in circle C_1 .

Area of segment AO_2B

$$= \text{area of sector } O_1AB - \text{area of } \triangle O_1AB$$

$$= \frac{1}{2} \times 12^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 12^2 \times \sin \frac{2\pi}{3}$$

$$= 88.442... \text{ cm}^2$$

Area of region R

$$= \text{area of segment } AO_2B$$

$$+ \text{area of segment } AO_1B$$

$$= 2 \times 88.442...$$

$$= 177 \text{ cm}^2 \text{ (3 s.f.)}$$

14 a The student has used an angle measured in degrees – it needs to be measured in radians to use that formula.

b $50^\circ = \frac{50}{180} \times \pi \text{ rad}$

$$\frac{1}{2} r^2 \theta = \frac{1}{2} \times 3^2 \times \frac{5}{18} \pi$$

$$= \frac{5}{4} \pi \text{ cm}^2$$

15 a
$$\frac{\cos \theta - 1}{\theta \tan 2\theta} \approx \frac{\left(1 - \frac{\theta^2}{2}\right) - 1}{\theta \times 2\theta}$$

$$= \frac{-\frac{\theta^2}{2}}{2\theta^2}$$

$$= \frac{-\theta^2}{4\theta^2}$$

$$= -\frac{1}{4}$$

b

$$\frac{2(1 - \cos \theta) - 1}{\tan \theta - 1} \approx \frac{2\left(1 - \left(1 - \frac{\theta^2}{2}\right)\right) - 1}{\theta - 1}$$

$$= \frac{2 \times \frac{\theta^2}{2} - 1}{\theta - 1}$$

$$= \frac{\theta^2 - 1}{\theta - 1}$$

$$= \frac{(\theta - 1)(\theta + 1)}{\theta - 1}$$

$$= \theta + 1$$

16 a
$$\frac{7 + 2 \cos 2\theta}{\tan 2\theta + 3} \approx \frac{7 + 2\left(1 - \frac{(2\theta)^2}{2}\right)}{2\theta + 3}$$

$$= \frac{7 + 2\left(1 - \frac{4\theta^2}{2}\right)}{2\theta + 3}$$

$$= \frac{9 - 4\theta^2}{2\theta + 3}$$

$$= \frac{(3 + 2\theta)(3 - 2\theta)}{2\theta + 3}$$

$$= 3 - 2\theta$$

b 3

17 a When θ is small:

$$\text{LHS} = 32 \cos 5\theta + 203 \tan 10\theta$$

$$\approx 32\left(1 - \frac{(5\theta)^2}{2}\right) + 203(10\theta)$$

$$= 32 - 16(25\theta^2) + 2010\theta$$

$$\text{So } 32 - 400\theta^2 - 2030\theta = 182$$

$$400\theta^2 + 2030\theta + 150 = 0$$

$$40\theta^2 + 203\theta + 15 = 0$$

b $40\theta^2 + 203\theta + 15 = 0$

$$(40\theta - 3)(\theta - 5) = 0$$

$$\theta = \frac{3}{40}, 5$$

17 c $\theta = 5$ is not a valid solution, as 5 is not 'small'. $\frac{3}{40}$ is 'small', so this solution is valid.

18

$$\begin{aligned} \cos^4 \theta - \sin^4 \theta &= (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta) \\ &= \cos^2 \theta - \sin^2 \theta \\ &= 1 - \sin^2 \theta - \sin^2 \theta \\ &= 1 - 2\sin^2 \theta \\ &\approx 1 - 2\theta^2 \end{aligned}$$

19 a $3 \sin \theta = 2, 0 \leq \theta \leq \pi$

$$\begin{aligned} \sin \theta &= \frac{2}{3} \\ \theta &= 0.730, 2.41 \end{aligned}$$

b $\sin \theta = -\cos \theta, -\pi \leq \theta \leq \pi$

$$\begin{aligned} \tan \theta &= -1 \\ \theta &= -\frac{\pi}{4}, \frac{3\pi}{4} \end{aligned}$$

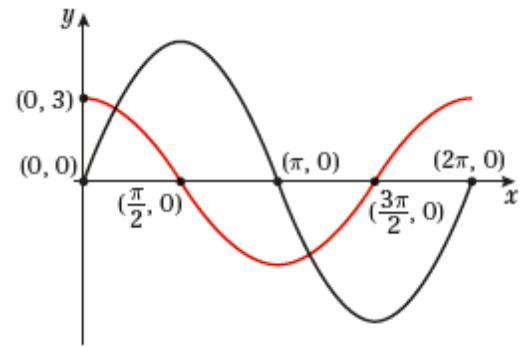
c

$$\begin{aligned} \tan \theta + \frac{1}{\tan \theta} &= 2, 0 \leq \theta \leq 2\pi \\ \tan^2 \theta + 1 &= 2 \tan \theta \\ \tan^2 \theta - 2 \tan \theta + 1 &= 0 \\ (\tan \theta - 1)^2 &= 0 \\ \tan \theta &= 1 \\ \theta &= \frac{\pi}{4}, \frac{5\pi}{4} \end{aligned}$$

d $2 \sin^2 \theta - \sin \theta - 1 = \sin^2 \theta, -\pi \leq \theta \leq \pi$

$$\begin{aligned} \sin^2 \theta - \sin \theta - 1 &= 0 \\ \sin \theta &= \frac{1 \pm \sqrt{5}}{2} \\ \sin \theta &= 1.618 \text{ (no solutions)} \\ \text{or } \sin \theta &= -0.618 \\ \Rightarrow \theta &= -0.666, -2.48 \end{aligned}$$

20 a



b The curves intersect twice in the given range, so the equation has two solutions.

c $5 \sin x = 3 \cos x$

$$\begin{aligned} \tan x &= \frac{3}{5} \\ x &= 0.540, 3.68 \end{aligned}$$

21 a $4 \sin \theta - \cos\left(\frac{\pi}{2} - \theta\right) = 4 \sin \theta - \sin \theta$

$$= 3 \sin \theta$$

b $4 \sin \theta - \cos\left(\frac{\pi}{2} - \theta\right) = 1, 0 \leq \theta \leq 2\pi$

$$\begin{aligned} 3 \sin \theta &= 1 \\ \sin \theta &= \frac{1}{3} \\ \theta &= 0.340, 2.80 \end{aligned}$$

22 $\frac{\sin 2x + 0.5}{1 - \sin 2x} = 2, 0 < x < \frac{3\pi}{2}$

$$\begin{aligned} \sin 2x + 0.5 &= 2 - 2 \sin 2x \\ 3 \sin 2x &= 1.5 \\ \sin 2x &= 0.5 \\ \text{Let } X &= 2x \\ \sin X &= 0.5, 0 < X < 3\pi \\ X &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \\ x &= \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \end{aligned}$$

23 a Cosine can be negative, so do not reject

$$-\frac{1}{\sqrt{2}}$$

b Rearranged incorrectly, so incorrectly square rooted.

c $2 \cos^2 x = 1$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$$x = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$$

24 a Not all solutions have been calculated. There will be four solutions in the given interval.

b $2 \tan 2x = 5, 0 \leq x \leq 2\pi$

Let $X = 2x$

$$2 \tan X = 5, 0 \leq X \leq 4\pi$$

$$\tan X = 2.5$$

$$X = 1.19, 4.33, 7.47, 10.6$$

$$x = 0.595, 2.17, 3.74, 5.31$$

25 a $5 \sin x = 1 + \cos^2 x$

$$5 \sin x = 1 + 2(1 - \sin^2 x)$$

$$2 \sin^2 x + 5 \sin x - 3 = 0$$

b $2 \sin^2 x + 5 \sin x - 3 = 0, 0 \leq x \leq 2\pi$

$$(2 \sin x - 1)(\sin x + 3) = 0$$

$$\sin x = 0.5 \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$$

or $\sin x = -3$ (no solution)

26 a $4 \sin^2 x + 9 \cos x - 6 = 0$

$$4(1 - \cos^2 x) + 9 \cos x - 6 = 0$$

$$4 - 4 \cos^2 x + 9 \cos x - 6 = 0$$

$$4 \cos^2 x - 9 \cos x + 2 = 0$$

b $4 \cos^2 x - 9 \cos x + 2 = 0, 0 \leq \theta \leq 4\pi$

$$(4 \cos x - 1)(\cos x - 2) = 0$$

$$\cos x = 2 \text{ (no solution)}$$

$$\text{or } \cos x = 0.25 \Rightarrow x = 1.3, 5.0, 7.6, 11.2$$

27 a $\tan 2x = 5 \sin 2x$

$$\frac{\sin 2x}{\cos 2x} = 5 \sin 2x$$

$$\sin 2x = 5 \sin 2x \cos 2x$$

$$(1 - 5 \cos 2x) \sin 2x = 0$$

b $\sin 2x(1 - 5 \cos 2x) = 0, 0 \leq x \leq \pi$

Let $X = 2x$

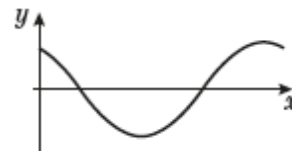
$$\sin X(1 - 5 \cos X) = 0, 0 \leq X \leq 2\pi$$

$$\sin X = 0 \Rightarrow X = 0, \pi, 2\pi$$

$$\text{or } \cos X = 0.2 \Rightarrow X = 1.37, 4.91$$

$$x = 0, 0.7, \frac{\pi}{2}, 2.5, \pi$$

28 a



b $\left(0, \frac{\sqrt{3}}{2}\right), \left(\frac{\pi}{3}, 0\right), \left(\frac{4\pi}{3}, 0\right)$

c $\cos\left(x + \frac{\pi}{6}\right) = 0.65, 0 \leq x \leq 2\pi$

Let $X = x + \frac{\pi}{6}$

$$\cos X = 0.65, \frac{\pi}{6} \leq x \leq \frac{13\pi}{6}$$

$$X = 0.863, 5.42$$

$$x = 0.34, 4.90$$

Challenge

a $9 \sin \theta \tan \theta + 25 \tan \theta = 6$

When θ is small:

$$\text{LHS} \approx 9\theta^2 + 25\theta$$

$$\text{so } 9\theta^2 + 25\theta = 6$$

$$9\theta^2 + 25\theta - 6 = 0$$

$$(9\theta - 2)(\theta + 3) = 0$$

$$\theta = \frac{2}{9} \text{ or } \theta = -3$$

 $\theta = \frac{2}{9}$ is 'small', so this value is valid. $\theta = -3$ is not 'small', so this value is not valid. 'Small' in this context is 'close to 0'.

b $2 \tan \theta + 3 = 5 \cos 4\theta$

When θ is small:

$$\text{LHS} \approx 2\theta + 3 \text{ and } \text{RHS} \approx 5 \left(1 - \frac{(4\theta)^2}{2} \right)$$

$$\text{so } 2\theta + 3 = 5 - 40\theta^2$$

$$40\theta^2 + 2\theta - 2 = 0$$

$$20\theta^2 + \theta - 1 = 0$$

$$(4\theta + 1)(5\theta - 1) = 0$$

$$\theta = -\frac{1}{4}, \theta = \frac{1}{5}$$

Both values of θ could be considered 'small' in this case so both solutions are valid.

c $\sin 4\theta = 37 - 2 \cos 2\theta$

When θ is small:

$$\text{LHS} \approx 4\theta$$

$$\text{and } \text{RHS} \approx 37 - 2 \left(1 - \frac{(2\theta)^2}{2} \right)$$

$$\text{so } 4\theta = 37 - 2 + 4\theta^2$$

$$4\theta^2 - 4\theta + 35 = 0$$

$$b^2 - 4ac < 0$$

So there are no solutions.