Modelling in mechanics 8A

1 a i $x = 0$ gives $h = 0.36 \text{ x } 0 – 0.003 \text{ x } (0)^2 = 0$

Height = 0 m

ii $x = 100$ m gives $h = 0.36 \times 100 - 0.003 \times (100)^2 = 36 – 30$

Height = 6 m

b $x = 200$ m gives $h = 0.36 \times 200 - 0.003 \times (200)^2 = 72 – 120$

Height = −48 m

c The model is not valid for this distance as it predicts the ball would be 48 m below ground level: the ball has already hit the ground at this point.

2 a 90 m (as this is the height when $t = 0$)

b i $t = 3$ gives $h = -5 \times (3)^2 + 15 \times 3 + 90 = -45 + 45 + 90$

Height above sea level = 90 m

ii $t = 5$ gives $h = -5 \times (5)^2 + 15 \times 5 + 90 = -125 + 75 + 90$

Height above sea level = 40 m

c $t = 20$ gives $h = -5 \times (20)^2 + 15 \times 20 + 90 = -2000 + 300 + 90 = -1610$

Height = 1610 m below sea level

d The prediction is incorrect because this height is below sea level, where the model is probably no longer valid, because forces acting on the ball will be different.

3 a When $h = 4$,

$4 = 2 + 1.1x - 0.1x^2$ or rearranging, $0 = -0.1x^2 + 1.1x – 2$, so using the quadratic formula,

$$x = \frac{-(1.1) \pm \sqrt{(1.1)^2 - 4 \times (-0.1) \times (-2)}}{2 \times (-0.1)}$$

$$= \frac{-1.1 \pm \sqrt{0.41}}{-0.2}$$

$$\approx \frac{-1.1 \pm 0.6403}{-0.2}$$

So $x = 2.30$ or 8.70 (to 3 s.f.)

The ball is 4 m above the ground after it has travelled both 2.30 m and 8.70 m horizontally.

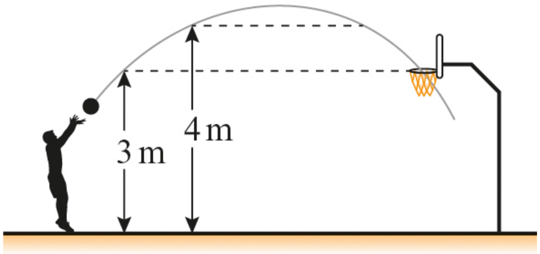
3 b When $h = 3$,

$3 = 2 + 1.1x - 0.1x^2$ or rearranging, $0 = -0.1x^2 + 1.1x - 1$, and dividing by -0.1,

$0 = x^2 - 11x + 10 = (x - 1)(x - 10)$

So $x = 1$ or 10, and the ball is at height 3 m after it has travelled both 1 m and 10 m horizontally.

At the shorter distance, the ball will be travelling upward (see diagram), so $k = 10$.



c If $x > 10$ m the equation is no longer valid as the ball will have gone through (or past) the net, and there would possibly be new forces acting on the ball.

4 a When $t = 1$, $d = 13.2$

So, substituting, $d = kt^2$ becomes $13.2 = k \times 1^2$ and therefore $k = 13.2$.

Our completed equation is $d = 13.2t^2$. When $t = 10$,

$d = 13.2 \times 10^2 = 1320$

The distance travelled is 1320 m.

b Clearly, the model is valid for positive values of t only. We are also unsure of what happens after t = 10, and we therefore can't use this model past that point. The model is valid for $0 \le t \le 10$.

5 $h \ge 0$ means that $0.36x - 0.003x^2 = x(0.36 - 0.003x) \ge 0$

We are assuming that $x \ge 0$ after the ball is struck, so we need the bracket to be non-negative:

$0.36 \ge 0.003x$

$$x \le \frac{0.36}{0.003} = 120$$

So the model is valid for $0 \le x \le 120$.

6 When the stone enters the sea, $h = 0$

$0 = -5t^2 + 15t + 90$, or, dividing by -5, $0 = t^2 - 3t - 18 = (t - 6)(t + 3)$

So, $t = -3$ or 6 and the stone hits the sea 6 seconds after it is thrown, since the model is valid only for the time **after** the stone is thrown at t = 0.

Therefore the model is valid for $0 \le t \le 6$.