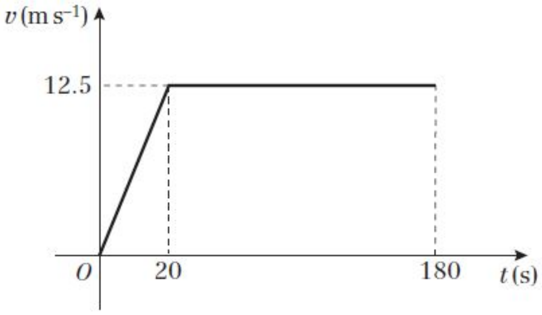
Constant acceleration, Mixed Exercise 9

1 a $45 \text{ km h}^{-1} = \frac{45 \times 1000}{3600} \text{ m s}^{-1}$

$= 12.5 \text{ m s}^{-1}$

$3 \text{ min} = 180 \text{ s}$

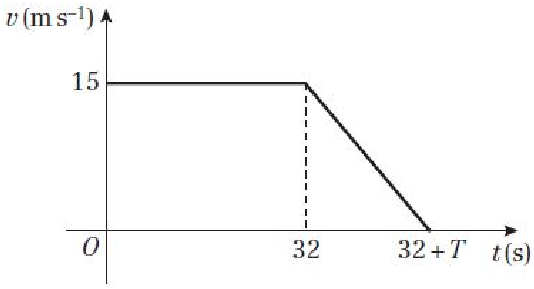


b $s = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(160 + 180) \times 12.5 = 2125$

The distance from A to B is 2125 m.

2 a



b $s = \frac{1}{2}(a + b)h$

$570 = \frac{1}{2}(32 + 32 + T) \times 15$

$\frac{15}{2}(T + 64) = 570$

$T + 64 = \frac{570 \times 2}{15} = 76$

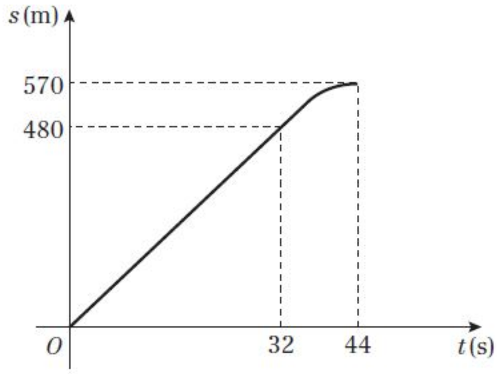
$T = 76 - 64 = 12$

c At $t = 32$, $s = 32 \times 15 = 480$

At $t = 44$, $s = 480 + \text{area of the triangle}$

$= 480 + \frac{1}{2} \times 12 \times 15 = 570$

2 c



3 a i Gradient of line = $\frac{v-u}{t}$

$a = \frac{v-u}{t}$

Rearranging: $v = u + at$

ii Shaded area is a trapezium

area = $\left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+v}{2}\right)t$

b i Rearrange $v = u + at$

$t = \frac{v-u}{a}$

Substitute into $s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$

$2as = v^2 - u^2$
$v^2 = u^2 + 2as$

ii Substitute $v = u + at$ into $s = \left(\frac{u+v}{2}\right)t$

$s = \left(\frac{u+u+at}{2}\right)t$

$s = \left(\frac{2u}{2} + \frac{at}{2}\right)t$

$s = ut + \frac{1}{2}at^2$

3 b iii Substitute $u = v - at$ into $s = ut + \frac{1}{2}at^2$

$s = (v - at)t + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

4 $s = \frac{1}{2}(a+b)h$

$152 = \frac{1}{2}(15+23)u = 19u$

$u = \frac{152}{19} = 8$

5 $40\text{ km h}^{-1} = \frac{40\times1000}{3600}\text{ m s}^{-1} = \frac{100}{9}\text{ m s}^{-1}$

$24\text{ km h}^{-1} = \frac{24\times1000}{3600}\text{ m s}^{-1} = \frac{20}{3}\text{ m s}^{-1}$

$u = \frac{100}{9},\ v = \frac{20}{3},\ s = 240,\ a = ?$

$v^2 = u^2 + 2as$

$\left(\frac{20}{3}\right)^2 = \left(\frac{100}{9}\right)^2 + 2\times a\times 240$

$a = \frac{\left(\frac{20}{3}\right)^2 - \left(\frac{100}{9}\right)^2}{2\times240} = -0.165$ (to 2 s.f.)

The deceleration of the car is 0.165 m s^{-2}.

6 a $a = -2.5,\ u = 20,\ t = 12,\ s = ?$

$s = ut + \frac{1}{2}at^2$

$= 20\times12 - \frac{1}{2}\times2.5\times12^2$

$= 240 - 180 = 60$

$OA = 60$ m

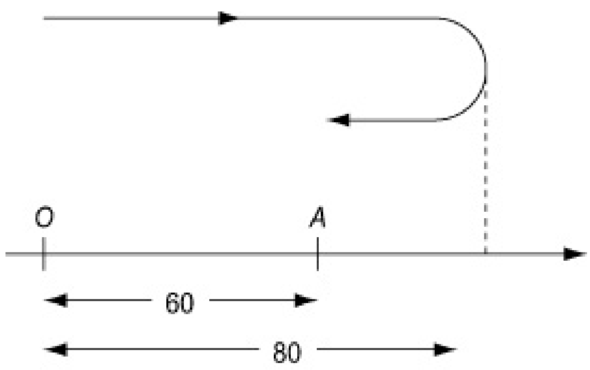
b The particle will turn round when $v = 0$

$a = -2.5,\ u = 20,\ v = 0,\ s = ?$

$v^2 = u^2 + 2as$

$0^2 = 20^2 - 5s \Rightarrow s = 80$

The total distance P travels is $(80+20)$ m $= 100$ m



7 $u = 6,\ v = 25,\ a = 9.8,\ t = ?$

$$v = u + at$$
$$25 = 6 + 9.8t$$
$$t = \frac{25-6}{9.8} = 1.9 \text{ (to 2 s.f.)}$$

The ball takes 1.9 s to move from the top of the tower to the ground.

8 Take downwards as the positive direction.

a $u = 0,\ s = 82,\ a = 9.8,\ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$82 = 0 + 4.9t^2$$
$$t = \sqrt{\frac{82}{4.9}} = 4.1 \text{ (to 2 s.f.)}$$

The time taken for the ball to reach the sea is 4.1 s.

b $u = 0,\ s = 82,\ a = 9.8,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 0 + 2 \times 9.8 \times 82 = 1607.2$$
$$v = \sqrt{1607.2} = 40 \text{ (to 2 s.f.)}$$

The speed at which the ball hits the sea is 40 m s^{-1}.

c Air resistance/wind/turbulence

9 a distance = area of triangle + area of rectangle + area of trapezium

$$451 = \tfrac{1}{2} \times 8 \times 2u + 12 \times 2u + \tfrac{1}{2} \times (u + 2u) \times 6$$
$$= 8u + 24u + 9u = 41u$$
$$u = \tfrac{451}{41} = 11$$

b The particle is moving with speed less than u m s^{-1} for the first 4 s

$$s = \tfrac{1}{2} \times 4 \times 11 = 22$$

The distance moved with speed less than u m s^{-1} is 22 m.

10 a From O to P, $u = 18,\ t = 12,\ v = 24,\ a = ?$

$u = 18,\ t = 12,\ v = 24,\ a = ?$

$$v = u + at$$
$$24 = 18 + 12a$$

10 a $a = \frac{24-18}{12} = \frac{1}{2}$

From O to Q, $u = 18$, $t = 20$, $a = \frac{1}{2}$, $v = ?$

$$v = u + at$$
$$= 18 + \tfrac{1}{2} \times 20 = 28$$

The speed of the train at Q is 28 m s^{-1}.

b From P to Q

$u = 24$, $v = 28$, $t = 8$, $s = ?$

$$s = \left(\frac{u+v}{2}\right)t = \left(\frac{24+28}{2}\right) \times 8 = 208$$

The distance from P to Q is 208 m.

11 a $s = 104$, $t = 8$, $v = 18$, $u = ?$

$$s = \left(\frac{u+v}{2}\right)t$$
$$104 = \left(\frac{u+18}{2}\right) \times 8 = (u+18) \times 4 = 4u + 72$$
$$u = \frac{104-72}{4} = 8$$

The speed of the particle at X is 8 m s^{-1}

b $s = 104$, $t = 8$, $v = 18$, $a = ?$

$$s = vt - \tfrac{1}{2}at^2$$
$$104 = 18 \times 8 - \tfrac{1}{2}a \times 8^2 = 144 - 32a$$
$$a = \frac{144-104}{32} = 1.25$$

The acceleration of the particle is 1.25 m s^{-2}.

c From X to Z, $u = 8$, $v = 24$, $a = 1.25$, $s = ?$

$$v^2 = u^2 + 2as$$
$$24^2 = 8^2 + 2 \times 1.25 \times s$$
$$s = \frac{24^2 - 8^2}{2.5} = 204.8$$

$XZ = 204.8 \text{ m}$

12 a Take upwards as the positive direction.

$u = 21,\ s = -32,\ a = -9.8,\ v = ?$

$v^2 = u^2 + 2as$

$= 21^2 + 2 \times (-9.8) \times (-32) = 441 + 627.2 = 1068.2$

$v = \sqrt{1068.2} = \pm 33$ (to 2 s.f.)

The velocity with which the pebble strikes the ground is -33 m s^{-1}.

The speed is 33 m s^{-1}.

b 40 m above the ground is 8 m above the point of projection.

$u = 21,\ s = 8,\ a = -9.8,\ t = ?$

$s = ut + \frac{1}{2}at^2$

$8 = 21t - 4.9t^2$

$0 = 4.9t^2 - 21t + 8$, so using the quadratic formula,

$$t = \frac{21 \pm \sqrt{21^2 - 4 \times 4.9 \times 8}}{9.8} = \frac{21 \pm \sqrt{284.2}}{9.8} = 3.86,\ 0.423 \text{ (to 3 s.f.)}$$

The pebble is above 40 m between these times: $3.863... - 0.423... = 3.44$ (to 3 s.f.)

The pebble is more than 40 m above the ground for 3.44 s.

c Take upwards as the positive direction.

$u = 21,\ a = -9.8$

$v = u + at = 21 - 9.8t \Rightarrow t = \dfrac{21 - v}{9.8}$

From part **a**, the pebble hits the ground when $v = -33$.

$$t = \frac{21 - v}{9.8} = \frac{21 - (-33)}{9.8} = \frac{54}{9.8} = 5.5 \text{ (to 2 s.f.)}$$

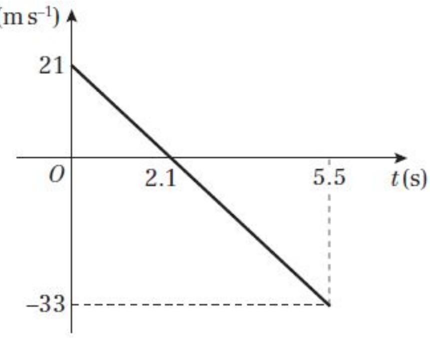
This is shown on the graph at point $(5.5, -33)$

The graph crosses the t-axis when $v = 0$.

$$t = \frac{21 - v}{9.8} = \frac{21 - 0}{9.8} = \frac{21}{9.8} = 2.1 \text{ (to 2 s.f.)}$$

So the graph passes through point $(2.1,\ 0)$

12 c



13 a $u = 12,\ v = 32,\ s = 1100,\ t = ?$

$$s = \left(\frac{u+v}{2}\right)t$$

$$1100 = \left(\frac{12+32}{2}\right)t = 22t \Rightarrow t = \frac{1100}{22} = 50$$

The time taken by the car to move from A to C is 50 s.

b Find a first.

From A to C, $u = 12,\ v = 32,\ t = 50,\ a = ?$

$$v = u + at$$
$$32 = 12 + a \times 50$$
$$a = \frac{32-12}{50} = 0.4$$

From A to B, $u = 12,\ s = 550,\ a = 0.4,\ v = ?$

$$v^2 = u^2 + 2as$$
$$= 12^2 + 2 \times 0.4 \times 550 = 584 \Rightarrow v = 24.2 \text{ (to 3 s.f.)}$$

The car passes B with speed 24.2 m s^{-1}.

14 Take upwards as the positive direction.

At the top:

$$u = 30,\ v = 0,\ a = -9.8,\ t = ?$$
$$v = u + at$$
$$0 = 30 - 9.8t \Rightarrow t = \frac{30}{9.8}$$

14 The ball spends 2.4 seconds above h, thus (by symmetry) 1.2 seconds rising between h and the top.

So it passes h 1.2 seconds earlier, at $t = \frac{30}{9.8} - 1.2 = 1.86$ (to 3 s.f.)

At h, $u = 30$, $a = -9.8$, $t \approx 1.86$, $s = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$= 30 \times 1.86 + \tfrac{1}{2}(-9.8) \times 1.86^2 = 39 \text{ (to 2 s.f.)}$$

15 a $u = 20$, $a = 4$, $s = 78$, $v = ?$

$$v^2 = u^2 + 2as$$
$$= 20^2 + 2 \times 4 \times 78 = 1024$$
$$v = \sqrt{1024} = 32$$

The speed of B when it has travelled 78 m is 32 m s^{-1}.

b Find time for B to reach the point 78 m from O.

$v = 32$, $u = 20$, $a = 4$, $t = ?$

$$v = u + at$$
$$32 = 20 + 4t \Rightarrow t = \frac{32 - 20}{4} = 3$$

For A, distance = speed × time

$$s = 30 \times 3 = 90$$

The distance from O of A when B is 78 m from O is 90 m.

c At time t seconds, for A, $s = 30t$

for B, $s = ut + \tfrac{1}{2}at^2 = 20t + 2t^2$

On overtaking the distances are the same.

$$20t + 2t^2 = 30t$$
$$t^2 - 5t = t(t - 5) = 0$$

$t = 5$ (at $t = 0$, A overtakes B)

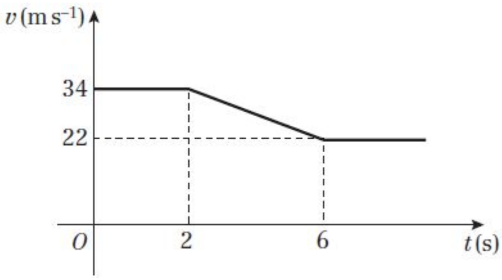
B overtakes A 5 s after passing O.

16 a To find time decelerating:

$u = 34,\ v = 22,\ a = -3,\ t = ?$

$v = u + at$

$22 = 34 - 3t \Rightarrow t = \frac{34 - 22}{3} = 4$

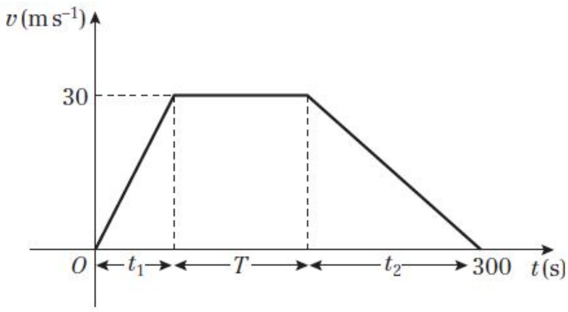


b distance = rectangle + trapezium

$s = 34 \times 2 + \frac{1}{2}(22 + 34) \times 4$
$= 68 + 112 = 180$

Distance required is 180 m.

17 a



b Acceleration is the gradient of a line.

For the first part of the journey, $3x = \frac{30}{t_1} \Rightarrow t_1 = \frac{30}{3x} = \frac{10}{x}$

For the last part of the journey, $-x = -\frac{30}{t_2} \Rightarrow t_2 = \frac{30}{x}$

$t_1 + T + t_2 = 300$

$\frac{10}{x} + T + \frac{30}{x} = 300 \Rightarrow \frac{40}{x} + T = 300$, as required

c $s = \frac{1}{2}(a + b)h$
$6000 = \frac{1}{2}(T + 300) \times 30 = 15T + 4500$

17 c $T = \dfrac{6000 - 4500}{15} = 100$

Substitute into the result in part **b**:

$\frac{40}{x} + 100 = 300 \Rightarrow \frac{40}{x} = 200$

$x = \frac{40}{200} = 0.2$

d From part **c**, $T = 100$

At constant velocity, distance = velocity $\times$ time $= 30 \times 100 = 3000$ (m)

The distance travelled at a constant speed is 3 km.

e From part **b**, $t_1 = \frac{10}{x} = \frac{10}{0.2} = 50$

Total distance travelled = 6 km (given) so halfway = 3 km = 3000 m

While accelerating, distance travelled is $\left(\frac{1}{2} \times 50 \times 30\right)$ m $= 750$ m.

At constant velocity, the train must travel a further 2250 m.

At constant velocity, time $= \dfrac{\text{distance}}{\text{velocity}} = \dfrac{2250}{30}$ s $= 75$ s

Time for train to reach halfway is $(50 + 75)$ s $= 125$ s

Challenge

Find the time taken by the first ball to reach 25 m below its point of projection (25 m above the ground). Take upwards as the positive direction.

$u = 10,\ \ s = -25,\ \ a = -9.8,\ \ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$-25 = 10t - 4.9t^2$$
$$0 = 4.9t^2 - 10t - 25$$
$$t = 10 \pm \frac{\sqrt{102 + 4 \times 4.9 \times 25}}{9.8}$$
$$= 3.5 \text{ (to 2 s.f.)}$$

As we discard the negative solution. Find the time taken by the second ball to reach 25 m below its point of projection. Take downwards as the positive direction.

$u = 0,\ \ s = 25,\ \ a = 9.8,\ \ t = ?$

$$s = ut + \tfrac{1}{2}at^2$$
$$25 = 4.9t^2$$
$$t = 2.3 \text{ (to 2 s.f.)}$$

Combining the two results:

$T = 3.4989\ldots - 2.2587\ldots = 1.2$ (to 2 s.f. using exact figures)