Statistical distributions 6C

1 $X \sim \text{B}(9, 0.2)$

a $\text{P}(X \leq 4) = 0.9804$ (tables)

b $\text{P}(X < 3) = \text{P}(X \leq 2) = 0.7382$ (tables)

c $\text{P}(X \geq 2) = 1 - \text{P}(X \leq 1) = 1 - 0.4362 = 0.5638$ (tables)

d $\text{P}(X = 1) = \text{P}(X \leq 1) - \text{P}(X = 0) = 0.4362 - 0.1342 = 0.3020$ (tables)

2 $X \sim \text{B}(20, 0.35)$

a $\text{P}(X \leq 10) = 0.9468$ (tables)

b $\text{P}(X > 6) = 1 - \text{P}(X \leq 6) = 1 - 0.4166 = 0.5834$ (tables)

c $\text{P}(X = 5) = \text{P}(X \leq 5) - \text{P}(X \leq 4) = 0.2454 - 0.1182 = 0.1272$ (tables)

d $\text{P}(2 \leq X \leq 7) = \text{P}(X \leq 7) - (X \leq 1) = 0.6010 - 0.0021 = 0.5989$ (tables)

3 **a** Using the binomial cumulative function on a calculator where $x = 19$, $n = 40$ and $p = 0.47$,

$\text{P}(X < 20) = \text{P}(X \leq 19) = 0.5888$

b Using the binomial cumulative function on a calculator where $x = 16$, $n = 40$ and $p = 0.47$,

$\text{P}(X > 16) = 1 - \text{P}(X \leq 16) = 0.7662$

c Using the binomial cumulative function on a calculator where $x = 10$ and 15, $n = 40$ and $p = 0.47$,

$\text{P}(11 \leq X \leq 15) = \text{P}(X \leq 15) - \text{P}(X \leq 10) = 0.1478 - 0.0036 = 0.1442$

d Using the binomial cumulative function on a calculator where $x = 10$ and 16, $n = 40$ and $p = 0.47$,

$\text{P}(10 < X < 17) = \text{P}(X \leq 16) - \text{P}(X \leq 10) = 0.2338 - 0.0036 = 0.2302$

4 **a** Using the binomial cumulative function on a calculator where $x = 20$, $n = 37$ and $p = 0.65$,

$\text{P}(X > 20) = 1 - \text{P}(X \leq 20) = 0.8882$

b Using the binomial cumulative function on a calculator where $x = 26$, $n = 37$ and $p = 0.65$,

$\text{P}(X \leq 26) = 0.7992$

c Using the binomial cumulative function on a calculator where $x = 19$ and 14, $n = 37$ and $p = 0.65$,

$\text{P}(15 \leq X < 20) = \text{P}(X \leq 19) - \text{P}(X \leqslant 14) = 0.06061 - 0.00068 = 0.05993$

4 d Using the binomial cumulative function on a calculator where $x = 23$ and 22, $n = 37$ and $p = 0.65$,

$P(X = 23) = P(X \leq 23) - P(X \leq 22) = 0.4184 - 0.2926 = 0.1258$

5 $X =$ 'number of heads'

$X \sim B(8, 0.5)$ (coins are fair so $p = 0.5$)

a $P(X = 0) = (0.5)^8 = 0.0039$ (tables)

b $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.0352 = 0.9648$ (tables)

c $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.6367 = 0.3633$ (tables)

6 $X =$ 'number of plants with blue flowers on tray of 15'

a $P(X = 4) = P(X \leq 4) - P(X \leq 3) = 0.6865 - 0.4613 = 0.2252$ (tables)

b $P(X \leq 3) = 0.4613$ (tables)

c $P(3 \leq X \leq 6) = P(X \leq 6) - P(X \leq 2) = 0.9434 - 0.2361 = 0.7073$ (tables)

7 $X \sim B(50, 0.40)$

a $P(X \leq 13) = 0.0280$

$P(X \leq 14) = 0.0540$ (tables)

$\therefore k = 13$

b $P(X \leq 27) = 0.9840$

$\Rightarrow P(X > 27) = 0.0160 > 0.01$

$P(X \leq 28) = 0.9924$

$\Rightarrow P(X > 28) = 0.0076 < 0.01$

$\therefore r = 28$

8 $X \sim B(40, 0.10)$

a $P(X = 0) = 0.0148 < 0.02$

$P(X \leq 1) = 0.0805 > 0.02$ (tables)

$P(X < 1) = 0.0148 < 0.02$

$\therefore k = 1$

8 b $P(X \leq 8) = 0.9845$ (tables)

$\Rightarrow P(X > 8) = 0.0155 > 0.01$

$P(X \leq 9) = 0.9949$

$\Rightarrow P(X > 9) = 0.0051 < 0.01$

$r = 9$

c $P(k \leq X \leq r) = P(X \leq r) - P(X \leq k-1)$

$= P(X \leq 9) - P(X = 0)$

$= 0.9949 - 0.0148$

$= 0.9801$

9 a A suitable distribution is $X \sim B(10, 0.30)$. Assumptions: There are two possible outcomes of each trial listen or don't listen. There is a fixed number of trials, 10, and fixed probability of success: 0.3. Each member in the sample is assumed to listen independently.

b $P(X \geq 5) = 1 - P(X \leq 4) = 1 - 0.8497 = 0.1503$ (tables)

c $P(X \leq 6) = 0.9894$ so $P(X \geq 7) = 1 - 0.9894 = 0.0106 > 0.01$

$P(X \leq 7) = 0.9984$ so $P(X \geq 8) = 1 - 0.9984 = 0.0016 < 0.01$ (tables)

Therefore $s = 8$ is the smallest such value.

10 X = number of defects in 50 components

$X \sim B(50, 0.05)$

a $P(X < 2) = P(X \leq 1) = 0.2794$ (tables)

b $P(X > 5) = 1 - P(X \leq 5) = 1 - 0.9622 = 0.0378$ (tables)

c Seek smallest d such that $P(X > d) < 0.05$

$P(X \leq 4) = 0.8964$ so $P(X > 4) = 0.1036 > 0.05$

$P(X \leq 5) = 0.9622$ so $P(X > 5) = 0.0378 < 0.05$

$\therefore d = 5$