Algorithms on graphs 3C

1 a Arcs in order
AF (9)
FB (14)
AC (20)
AE (25)
DE (26)

$\text{weight} = 9 + 14 + 20 + 25 + 26$
$= 94$

	↓1 A	↓3 B	↓4 C	↓6 D	↓5 E	↓2 F
A	-	15	20	34	25	9
B	15	-	36	38	28	(14)
C	(20)	36	-	43	38	22
D	34	38	43	-	(26)	40
E	(25)	28	38	26	-	31
F	(9)	14	22	40	31	-

b Arcs in order
RS (28)
ST (16)
SU (19)
UV (37)

$\text{weight} = 28 + 16 + 19 + 37$
$= 100$

	↓1 R	↓2 S	↓3 T	↓4 U	↓5 V
R	-	28	30	31	41
S	(28)	-	16	19	43
T	30	(16)	-	22	41
U	31	(19)	22	-	37
V	41	43	41	(37)	-

2 Arcs in order
BS (49)
SM (44)
SN (56)
NL (37)
$\text{weight} = 186$

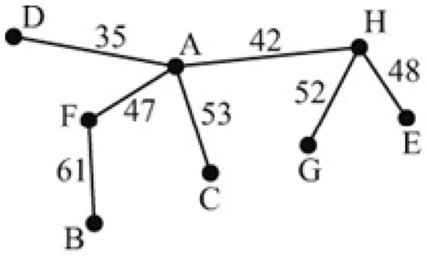
	↓1 B	↓4 N	↓5 L	↓2 S	↓3 M
B	-	164	100	49	88
N	164	-	37	(56)	74
L	100	(37)	-	90	86
S	(49)	56	90	-	44
M	88	74	86	(44)	-

3 a Arcs in order
DA (35)
AH (42)
AF (47)
HE (48)
HG (52)
AC (53)
FB (61)

$\text{weight} = 338$
$\therefore \text{cost} = 3 \times 338$
$= €1014$

	↓2 A	↓8 B	↓7 C	↓1 D	↓5 E	↓4 F	↓6 G	↓3 H
A	-	84	53	(35)	-	47	-	42
B	84	-	71	113	142	(61)	75	-
C	(53)	71	-	-	-	-	59	-
D	35	113	-	-	58	67	151	-
E	-	142	-	58	-	168	159	(48)
F	(47)	61	-	67	168	-	-	73
G	-	75	59	151	159	-	-	(52)
H	(42)	-	-	-	48	73	52	-

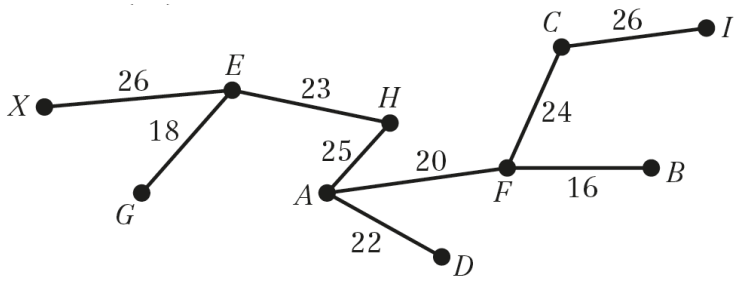
3 b



c i It is cheaper to translate from **E** to **H** then from **H** to **G** at a cost of $48+52=100$ euro rather than 159 euro per 1000 words.

ii A direct translation is likely to be more accurate than a translation via another language.

4 a Starting from X, we pick the lowest value down the X column, which is 26 at vertex E.
We now seek the lowest value along the X and E columns. We thus add EG 18 to the network.
Next, we inspect the values along X, E and G columns to find the next vertex. It turns out to be EH 23. The lowest value along the new set of columns, X, E, G and H is HA 25. Thus we now inspect columns X, A, E, G and H to find the next lowest value. It is AF 20. Searching the columns X, A, E, F, G and H we find that the next lowest value is BF 16. Next step involves looking at columns X, A, B, E, F, G and H – we discover that the lowest value now is AD 22. The only remaining vertices now are C and I. We find that the next smallest value is FC 24, which leaves the last connection to be CI 26. The total weight of this spanning tree is 200.



b 9 oil rigs and 1 depot make 10 nodes.
24 oil rigs and 1 depot make 25 nodes.

$$\text{Estimated time} = 0.7\times\left(\frac{25}{10}\right)^3 = 10.9 \text{ seconds}$$

c i Any distance less than 26 miles will change the minimum connector as *I* will link directly to *X*.

ii Any distance of 26 miles or more will not change the minimum connector as the shortest way to connect *I* to the rest of the tree will be to connect to *C*.