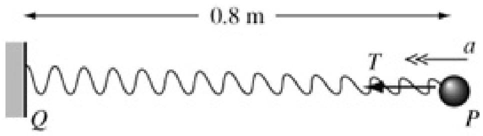
Elastic strings and springs 3B

1 a



$(\leftarrow)\ T = 4a$

$$T = \frac{40 \times 0.3}{0.5}$$

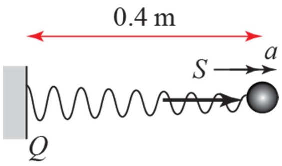
$$= 24\text{ N}$$

$$\therefore\quad 24 = 4a$$

$$6 = a$$

Initial acceleration is 6 m s^{-2}.

b



$(\rightarrow)\ S = 4a$

$$S = \frac{40 \times 0.1}{0.5}$$

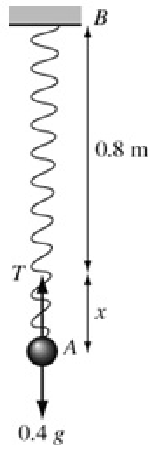
$$= 8\text{ N}$$

$$\therefore\quad 8 = 4a$$

$$2 = a$$

Initial acceleration is 2 m s^{-2}.

2



In equilibrium position

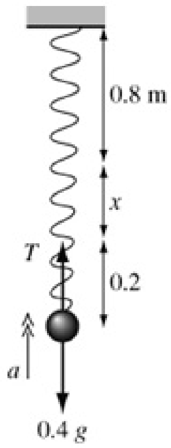
$(\uparrow)\ T = 0.4g$

$$T = \frac{20x}{0.8}$$

$$= 25x$$

$$25x = 0.4g$$

$$x = \frac{2g}{125}$$



After further extension,

$(\uparrow)\ T - 0.4g = 0.4a$

$$T = \frac{20(x+0.2)}{0.8}$$

$$= 25x + 5$$

So, $\quad 25x + 5 - 0.4g = 0.4a$

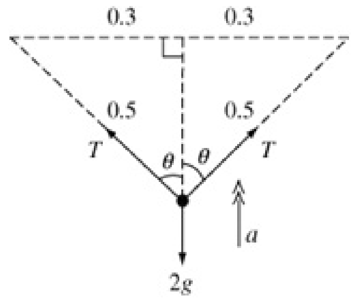
$$\left(25 \times \frac{2g}{125}\right) + 5 - 0.4g = 0.4a$$

$$a = \frac{5}{0.4}$$

$$= 12.5$$

Initial acceleration is 12.5 m s^{-2}.

3



$(\uparrow)\ 2T\cos\theta - 2g = 2a$

$$\frac{4T}{5} - g = a$$

Using Hooke's law:

$$T = \frac{20 \times (1.0 - 0.4)}{0.4} = 30$$

$$\therefore\quad \frac{4}{5} \times 30 - 9.8 = a$$

$$14.2 = a$$

Initial acceleration is 14.2 m s^{-2} upwards.

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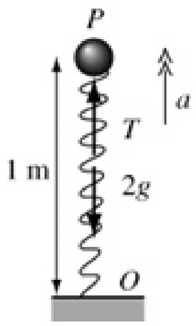
4



$$T = \frac{40 \times 0.5}{1.5}$$
$$= \frac{40}{3}$$

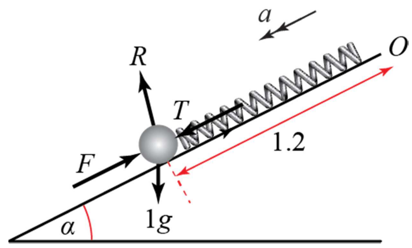
$(\uparrow)\ T - 2g = 2a$

So, $\frac{40}{3} - 19.6 = 2a$

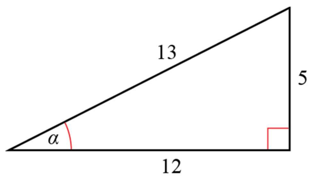
$$a = -3.13\ldots$$

Magnitude of initial acceleration is 3.13 m s^{-2} (3 s.f.) and the direction is downwards.

5 a



$\tan\alpha = \frac{5}{12}$, so from the right-angled triangle:



$\sin\alpha = \frac{5}{13}$ and $\cos\alpha = \frac{12}{13}$

By Hooke's law:

$$T = \frac{21.5 \times 0.4}{1.6} = 5.375 \text{ N}$$

$(\nwarrow)\ R = 1g\cos\alpha = 9.8 \times \frac{12}{13} = \frac{117.6}{13}$

so, $F = 0.5 \times \frac{117.6}{13} = \frac{58.8}{13}$

$(\swarrow)\quad 1g\sin\alpha + T - F = 1a$

$$\left(9.8 \times \frac{5}{13}\right) + 5.375 - \frac{58.8}{13} = a$$
$$4.621\ldots = a$$

Initial acceleration is 4.62 m s^{-2} (2 s.f.).

b Resultant force down the plane is $T + g\sin\alpha - \mu R = ma$, so if μ increases, the acceleration a would decrease.

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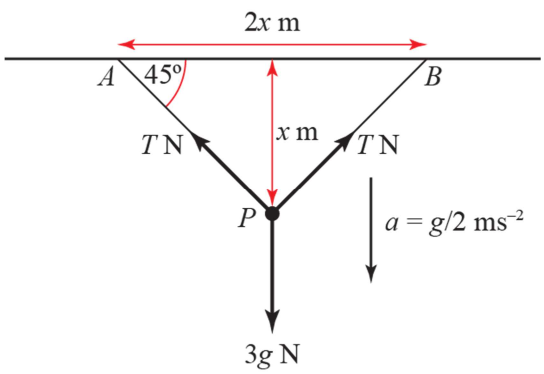
Challenge
We make use of the following diagram:



a Use Newton's Second Law for forces acting vertically at the point P.
Take upwards as the positive direction:

$$2T\cos 45^\circ - 3g = -3\times\frac{g}{2}$$

$$\Rightarrow 2T\cos 45^\circ = \frac{3}{2}g$$

$$\Rightarrow 2T\cdot\frac{\sqrt{2}}{2} = \frac{3g}{2}$$

$$\Rightarrow T = \frac{3g}{2\sqrt{2}}\text{ N} = \frac{3\sqrt{2}g}{4}\text{ N}$$

b Now use $T = kx = \frac{\lambda x}{l}$ where $x = \frac{l}{4}$:

$$\Rightarrow T = \frac{\lambda}{l}\times\frac{l}{4} = \frac{\lambda}{4}\text{ N}$$

Equating this with the expression found in **a**, we see that:

$$\frac{\lambda}{4} = \frac{3\sqrt{2}g}{4} \Rightarrow \lambda = 3\sqrt{2}g$$

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