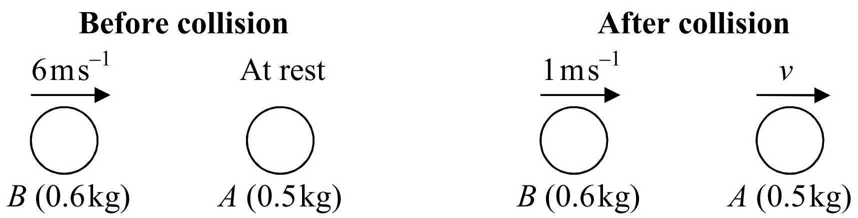
Elastic collisions in one dimension 4C

1 a



Using conservation of linear momentum for the system $(\rightarrow)$:

$0.6 \times 6 = 0.6 \times 1 + 0.5v$

$3.6 - 0.6 = 0.5v$

$\Rightarrow v = 6$

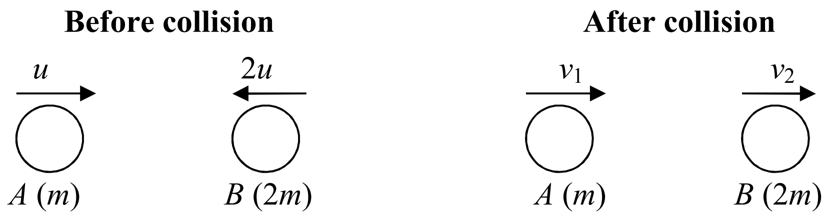
The speed of A after the collision is $6\,\text{m s}^{-1}$.

b Total kinetic energy before collision $= \frac{1}{2} \times 0.6 \times 6^2 = 10.8\,\text{J}$

Total kinetic energy after collision $= \frac{1}{2} \times 0.6 \times 1^2 + \frac{1}{2} \times 0.5 \times v^2 = 0.3 + 9 = 9.3\,\text{J}$

The loss of kinetic energy $= (10.8 - 9.3)\,\text{J} = 1.5\,\text{J}$

2



Using conservation of linear momentum for the system $(\rightarrow)$:

$mu + 2m(-2u) = mv_1 + 2mv_2$

$u - 4u = v_1 + 2v_2$

$-3u = v_1 + 2v_2$ **(1)**

Using Newton's law of restitution:

$\frac{2}{3} = \frac{v_2 - v_1}{u-(-2u)} = \frac{v_2 - v_1}{3u}$

$\Rightarrow v_2 - v_1 = 2u$ **(2)**

Adding equations **(1)** and **(2)** gives

$-u = 3v_2$

So $v_2 = -\frac{u}{3}\,\text{m s}^{-1}$

Substituting into equation **(2)** gives:

$-\frac{u}{3} - v_1 = 2u$

$v_1 = -\frac{7u}{3}\,\text{m s}^{-1}$

The direction of travel of particle A is reserved after the collision, while particle B continues to move in the same direction.

Loss of kinetic energy = initial kinetic energy − final kinetic energy

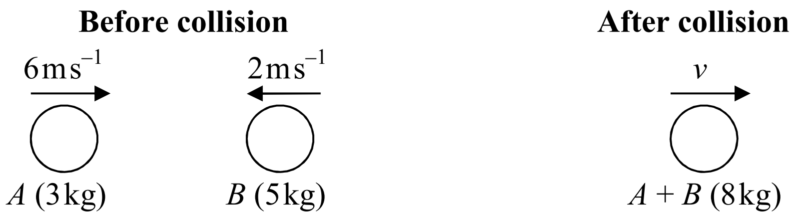
$$= \frac{1}{2}mu^2 + \frac{1}{2}2m(-2u)^2 - \frac{1}{2}m\left(-\frac{7u}{3}\right)^2 - \frac{1}{2}2m\left(-\frac{u}{3}\right)^2$$

$$= \frac{1}{2}mu^2 + 4mu^2 - \frac{49}{18}mu^2 - \frac{1}{9}mu^2$$

$$= \frac{9}{18}mu^2 + \frac{72}{18}mu^2 - \frac{49}{18}mu^2 - \frac{2}{18}mu^2$$

$$= \frac{30}{18}mu^2 = \frac{5mu^2}{3}\,\text{J}$$

3



Using conservation of linear momentum for the system $(\rightarrow)$:

$3 \times 6 + 5 \times (-2) = 8v$

$8v = 18 - 10 = 8$

$\Rightarrow v = 1\,\text{m s}^{-1}$

$$\text{Loss of kinetic energy} = \text{initial kinetic energy} - \text{final kinetic energy}$$
$$= \frac{1}{2}3(6)^2 + \frac{1}{2}5(-2)^2 - \frac{1}{2}8(1)^2$$
$$= 54 + 10 - 4 = 60\,\text{J}$$

4

After impact with the cushion the velocity of the billiard ball is $v\,\text{m s}^{-1}$, using Newton's law of restitution:

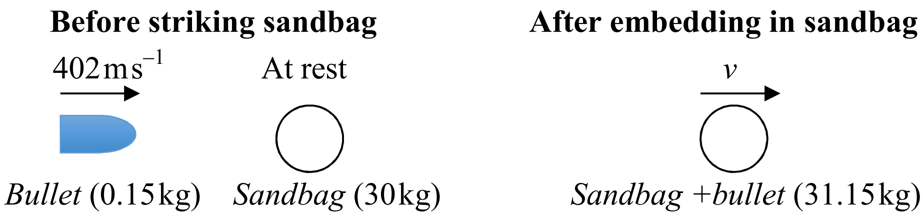
$\frac{4}{5} = \frac{v}{2.5}$

$\Rightarrow v = 2$

$\therefore$ The loss in kinetic energy is:

$$\text{Loss of kinetic energy} = \frac{1}{2} \times 0.2 \times 2.5^2 - \frac{1}{2} \times 0.2 \times 2^2$$
$$= 0.625 - 0.4 = 0.225\text{ J}$$

5 a



Using conservation of linear momentum for the system $(\rightarrow)$:

$0.15 \times 402 = 30.15v$

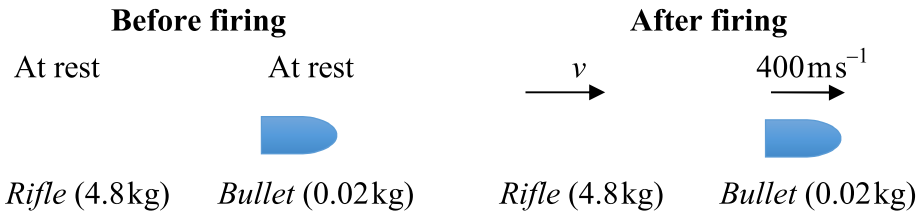
$\Rightarrow v = \frac{60.3}{30.15} = 2\,\text{m s}^{-1}$

b Total kinetic energy before impact $= \frac{1}{2} \times 0.15 \times 402^2 = 12120.3$ J

Total kinetic energy after impact $= \frac{1}{2} \times 30.15 \times 2^2 = 60.3$ J

The loss of kinetic energy $= 12\,060$ J $= 12.06$ kJ

6 a



Using conservation of linear momentum for the system $(\rightarrow)$:

$0 = 4.8v + 0.2 \times 400$

$\Rightarrow v = -\frac{8}{4.8} = -\frac{5}{3} = -1.67\,\text{m s}^{-1}$ (3 s.f.)

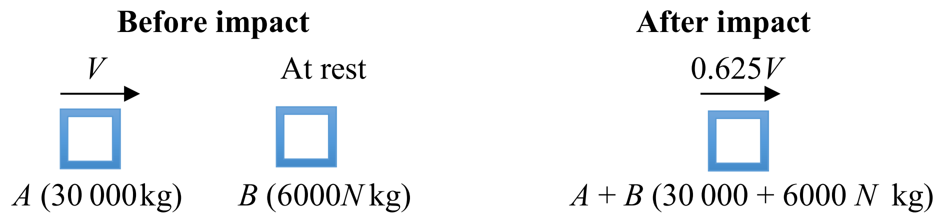
The rifle recoils with an initial speed of $\frac{5}{3}$ or $1.67\,\text{m s}^{-1}$.

b Total kinetic energy before firing $= 0$

Total kinetic energy after firing $= \frac{1}{2} \times 0.02 \times 400^2 + \frac{1}{2} \times 4.8 \times \left(\frac{5}{3}\right)^2$

$= 1600 + 6.67 = 1606.67$ J (2 d.p.)

7 a Let N be the number of stationary carriages, A be the approaching train and B be the stationary carriages.



Using conservation of linear momentum for the system $(\rightarrow)$:

$$30\,000V = (30\,000 + 6000N)0.625V$$

$48 = 30 + 6N$ (dividing both sides by $625V$)

$\Rightarrow 6N = 18$

$\Rightarrow N = 3$

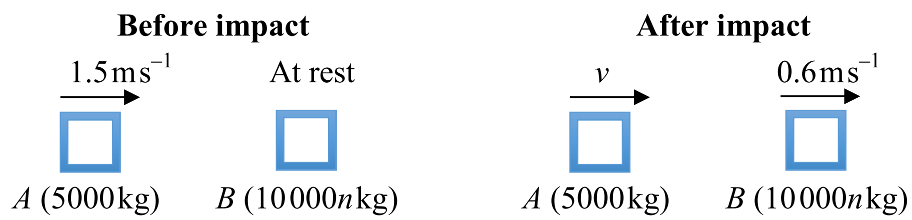
b $$\text{fraction of kinetic energy lost} = \frac{\text{initial kinetic energy} - \text{final kinetic energy}}{\text{initial kinetic energy}}$$

$$= \frac{\frac{1}{2}30\,000V^2 - \frac{1}{2}48\,000\left(\frac{5V}{8}\right)^2}{\frac{1}{2}30\,000V^2}$$

$$= \frac{15\times 64 - 24\times 25}{15\times 64} = \frac{960-600}{960} = \frac{360}{960} = \frac{6}{16} = \frac{3}{8}$$

So the fraction of kinetic energy lost is $\frac{3}{8}$

8 a



Using conservation of linear momentum for the system $(\rightarrow)$:

$$5000\times 2.5 = 5000v + 10\,000\times 0.6$$

$\Rightarrow 50v = 75 - 60 = 15$

$$\Rightarrow v = \frac{15}{50} = 0.3\,\text{m}\,\text{s}^{-1}$$

b Using Newton's law of restitution:

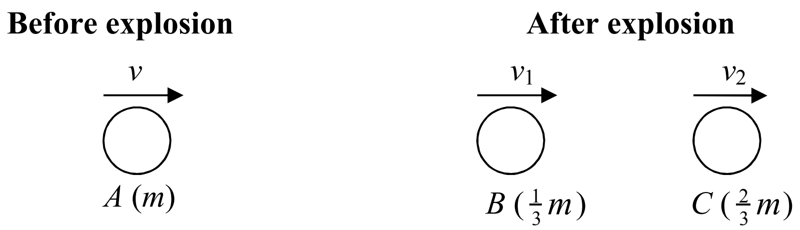
$$e = \frac{0.6-0.3}{1.5} = \frac{0.3}{1.5} = \frac{1}{5} = 0.2$$

8 c Loss of kinetic energy = initial kinetic energy – final kinetic energy

$$= \frac{1}{2} \times 5000 \times 1.5^2 - \left(\frac{1}{2} \times 5000 \times 0.3^2 + \frac{1}{2} \times 10\,000 \times 0.6^2 \right)$$

$$= 3600 \text{ J}$$

9



Using conservation of linear momentum for the system $(\rightarrow)$:

$$mv = \frac{mv_1}{3} + \frac{2mv_2}{3}$$

$$\Rightarrow v_1 = 3v - 2v_2 \qquad \textbf{(1)}$$

Increase in kinetic energy = final kinetic energy – initial kinetic energy = $\frac{1}{4}mu^2$

$$\frac{1}{2}\left(\frac{m}{3}\right)v_1^2 + \frac{1}{2}\left(\frac{2m}{3}\right)v_2^2 - \frac{1}{2}mv^2 = \frac{1}{4}mu^2$$

$$\frac{v_1^2}{3} + \frac{2v_2^2}{3} - v^2 = \frac{u^2}{2}$$

$$2v_1^2 + 4v_2^2 - 6v^2 = 3u^2 \qquad \textbf{(2)}$$

Substituting equation **(1)** into equation **(2)**:

$$2(3v - 2v_2)^2 + 4v_2^2 - 6v^2 = 3u^2$$

$$2(9v^2 - 12vv_2 + 4v_2^2) + 4v_2^2 - 6v^2 = 3u^2$$

$$12v_2^2 - 24vv_2 + 12v^2 - 3u^2 = 0$$

$$4v_2^2 - 8vv_2 + (4v^2 - u^2) = 0$$

Using the quadratic formula to solve for v_2

$$v_2 = \frac{8v \pm \sqrt{(-8v)^2 - 16(4v^2 - u^2)}}{8} = \frac{8v \pm \sqrt{64v^2 - 64v^2 + 16u^2}}{8} = \frac{8v \pm \sqrt{16u^2}}{8}$$

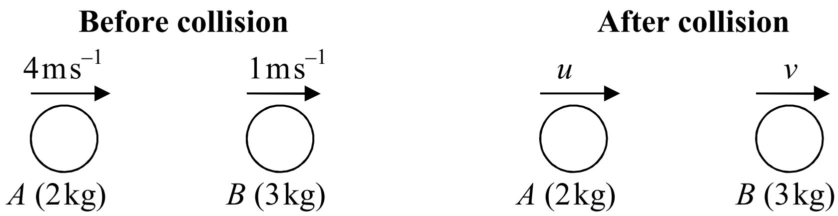
u is a positive constant so

$$v_2 = \frac{8v + 4u}{8} = v + \frac{u}{2}$$

v_1 can be found by substituting for v_2 in equation **(1)**:

$$v_2 = v + \frac{u}{2} \Rightarrow \quad v_1 = 3v - 2v - u = v - u$$

10 a



Using conservation of linear momentum for the system $(\rightarrow)$:

$2\times4+3\times1=2u+3v$

$\Rightarrow\ 11=2u+3v$ **(1)**

Loss of kinetic energy = initial kinetic energy − final kinetic energy

$$=\frac{1}{2}2(4)^2+\frac{1}{2}3(1)^2-\frac{1}{2}2u^2-\frac{1}{2}3v^2$$

$$=\frac{35}{2}-u^2-\frac{3v^2}{2}$$

So as the loss of kinetic energy due to the collision is 3 J

$$\frac{35}{2}-u^2-\frac{3v^2}{2}=3$$

$$\frac{29}{2}=\frac{3v^2}{2}+u^2$$

$58=6v^2+4u^2$ **(2)**

From equation **(1)**, $2u=11-3v$

So $(2u)^2=(11-3v)^2$

$4u^2=(11-3v)^2$

Substituting into equation **(2)**:

$58=6v^2+(11-3v)^2$

$58=6v^2+121-66v+9v^2$

$15v^2-66v+63=0$

$5v^2-22v+21=0$

b Using the quadratic formula to solve for v

$$v=\frac{22\pm\sqrt{22^2-4\times5\times21}}{10}=\frac{22\pm\sqrt{22^2-4\times5\times21}}{10}=\frac{22\pm\sqrt{64}}{10}$$

$$v=\frac{22\pm8}{10}=3\text{ or }1.4$$

Substituting into equation **(1)**:

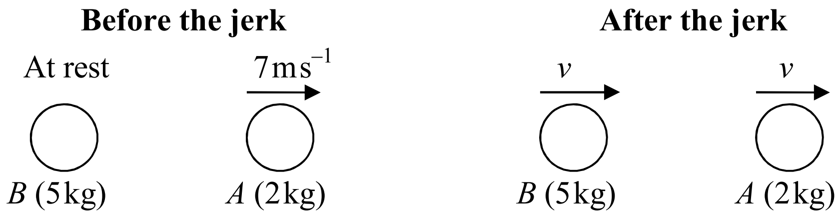
$v=1.4\Rightarrow2u=11-(3\times1.4)=6.8\Rightarrow u=3.4$

After the collision B must be moving faster than A as sphere A cannot pass through sphere B, so reject this solution since $v<u$

$v=3\Rightarrow2u=11-(3\times3)=9\Rightarrow u=1$

This is a valid solution since $v>u$, so $v=3\text{ ms}^{-1}$ and $u=1\text{ ms}^{-1}$.

11 a Let the common speed of the particles following the jerk be $v\,\text{m s}^{-1}$.



Using conservation of linear momentum for the system $(\rightarrow)$:

$2\times 7 = 2v + 5v$

$14 = 7v$

$\Rightarrow v = 2\,\text{m s}^{-1}$

b Loss of kinetic energy = initial kinetic energy – final kinetic energy

$$= \frac{1}{2}\times 2\times 7^2 - \frac{1}{2}\times 2\times 2^2 - \frac{1}{2}\times 5\times 2^2 = 49 - 4 - 10 = 35$$

So the loss of total kinetic energy is 35 J.

12

Using conservation of linear momentum for the system $(\rightarrow)$:

$mu = Mv + mv$

$\Rightarrow v = \dfrac{mu}{M+m}$ **(1)**

Kinetic energy lost = initial kinetic energy – final kinetic energy

$$= \frac{1}{2}mu^2 - \frac{1}{2}(M+m)v^2$$

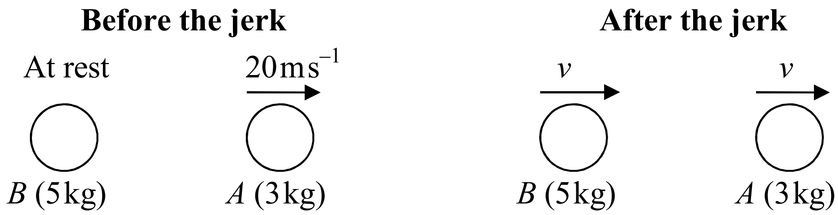
$$= \frac{1}{2}mu^2 - \frac{1}{2}(M+m)\frac{(mu)^2}{(M+m)^2}$$ Substituting for v from equation **(1)**

$$= \frac{mu^2(M+m)}{2(M+m)} - \frac{m^2u^2}{2(M+m)}$$

$$= \frac{mMu^2 + m^2u^2 - m^2u^2}{2(M+m)}$$

$$= \frac{mMu^2}{2(M+m)} = \frac{mMu^2}{2(m+M)}$$

13 a Let the common speed of the particles following the jerk be $v\ \text{m s}^{-1}$.



Using conservation of linear momentum for the system $(\rightarrow)$:

$3 \times 20 = 5v + 3v$

$60 = 8v$

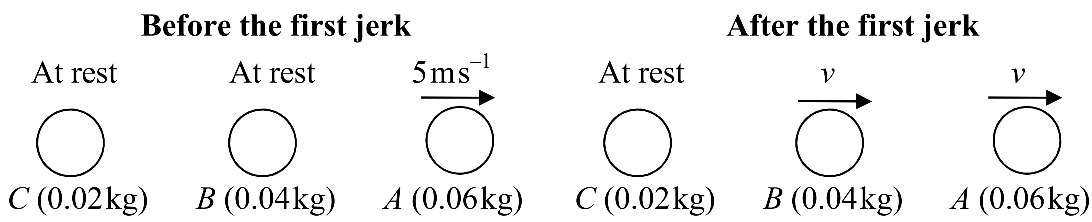
$\Rightarrow v = 7.5\ \text{m s}^{-1}$

b Initial kinetic energy $= \frac{1}{2} \times 3 \times 20^2 = 600\ \text{J}$

Final kinetic energy $= \frac{1}{2} \times 3 \times 7.5^2 + \frac{1}{2} \times 5 \times 7.5^2 = 225\ \text{J}$

So the difference between the kinetic energies is $600 - 225 = 375\ \text{J}$

14 a Let the common speed of the 40 g and 60 g masses following the first jerk be $v\,\text{m s}^{-1}$.

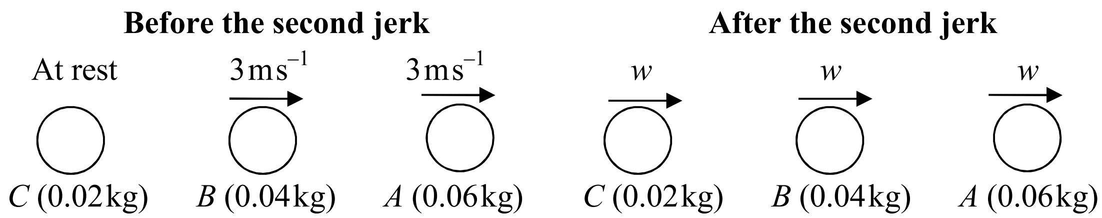


Using conservation of linear momentum for the system $(\rightarrow)$:

$0.06 \times 5 = 0.04v + 0.06v$

$\Rightarrow v = 3\,\text{m s}^{-1}$

Let the common speed of all masses following the second jerk be $w\,\text{m s}^{-1}$.



Using conservation of linear momentum for the system $(\rightarrow)$:

$(0.04 + 0.06) \times 3 = (0.02 + 0.04 + 0.06)w$

$1.2w = 3$

$\Rightarrow w = 2.5\,\text{m s}^{-1}$

Until the first jerk, the 60 g sphere moves with speed $5\,\text{m s}^{-1}$ through 0.6 m.

So the time taken is $\frac{0.6}{5} = 0.12\,\text{s}$

From the first jerk until the second jerk, the 60 g and 40 g spheres moves with speed $3\,\text{m s}^{-1}$ through 0.6 m.

So the time taken is $\frac{0.6}{3} = 0.2\,\text{s}$

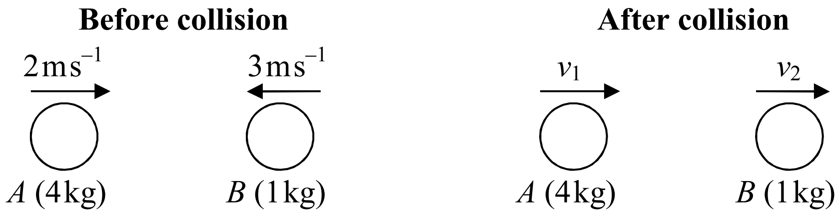
Therefore the time which elapses before the 20g sphere begins to move is $0.12 + 0.2 = 0.32\,\text{s}$

b The loss of kinetic energy = initial kinetic energy – final kinetic energy

$$= \frac{1}{2} \times 0.06 \times 5^2 - \frac{1}{2} \times (0.06 + 0.04 + 0.02) \times 2.5^2$$

$$= 0.75 - 0.375 = 0.375\ \text{J}$$

Challenge



Using conservation of linear momentum for the system $(\rightarrow)$:

$$4\times 2+1\times(-3)=4v_1+v_2$$

$$\Rightarrow 5=4v_1+v_2 \qquad \textbf{(1)}$$

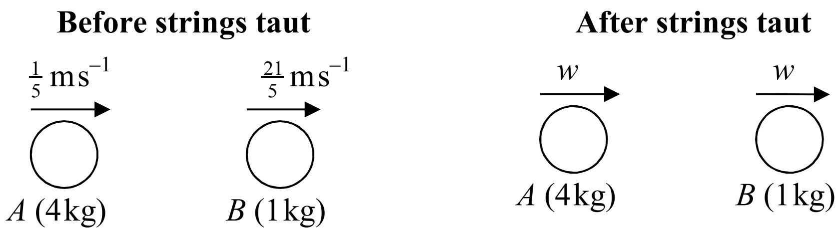
Using Newton's law of restitution gives:

$$0.8=\frac{v_2-v_1}{2+3}=\frac{v_2-v_1}{5}$$

$$\Rightarrow 4=v_2-v_1 \qquad \textbf{(2)}$$

Solving equations **(1)** and **(2)** simultaneously gives:

$v_1=\frac{1}{5}$ and $v_2=\frac{21}{5}$



Using conservation of linear momentum for the system $(\rightarrow)$:

$$4\times\frac{1}{5}+1\times\frac{21}{5}=4w+w$$

$$5w=\frac{25}{5}=5$$

$$\Rightarrow w=1\,\text{m s}^{-1}$$

Kinetic energy of the system $=\frac{1}{2}\times 4\times 1^2+\frac{1}{2}\times 1\times 1^2=2.5\text{ J}$