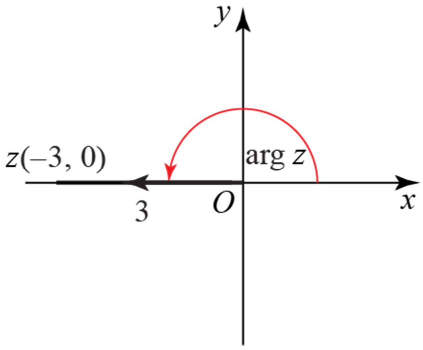
Complex numbers 1A

1 **a** -3

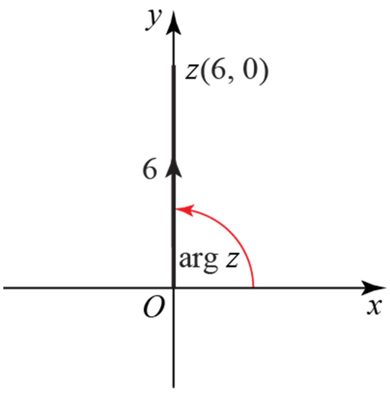


$r = 3$

$\theta = \arg z = \pi$

$\therefore -3 = 3e^{\pi i}$

b 6i

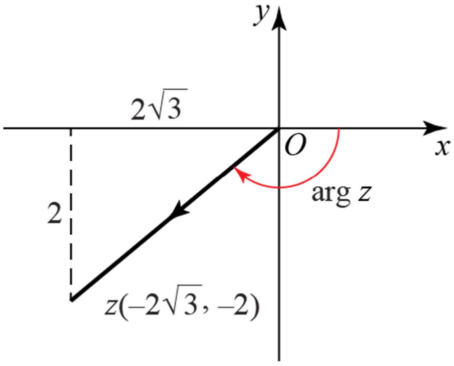


$r = 6$

$\theta = \arg z = \frac{\pi}{2}$

$\therefore \ 6i = 6e^{\frac{\pi i}{2}}$

1 c $-2\sqrt{3}-2\text{i}$

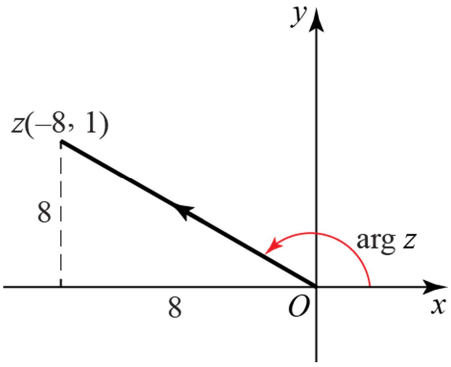


$r=\sqrt{(-2\sqrt{3})^2+(-2)^2}=\sqrt{12+4}=\sqrt{16}=4$

$\theta=\arg z=-\pi+\tan^{-1}\left(\dfrac{2}{2\sqrt{3}}\right)=-\pi+\dfrac{\pi}{6}=-\dfrac{5\pi}{6}$

$\therefore\ -2\sqrt{3}-2\text{i}=4\text{e}^{\frac{-5\pi\text{i}}{6}}$

d $-8+\text{i}$

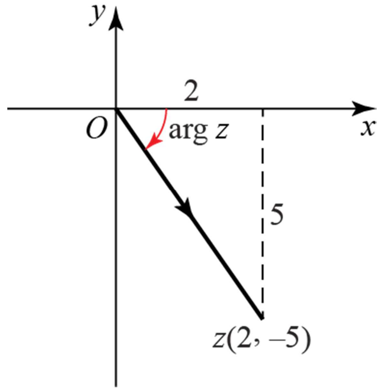


$r=\sqrt{(-8)^2+1^2}=\sqrt{65}$

$\theta=\pi-\tan^{-1}\left(\dfrac{1}{8}\right)=3.02^{\text{c}}\,(2\text{ d.p.})$

$\therefore -8+\text{i}=\sqrt{65}\,\text{e}^{3.02\text{i}}$

1 e $2-5\text{i}$

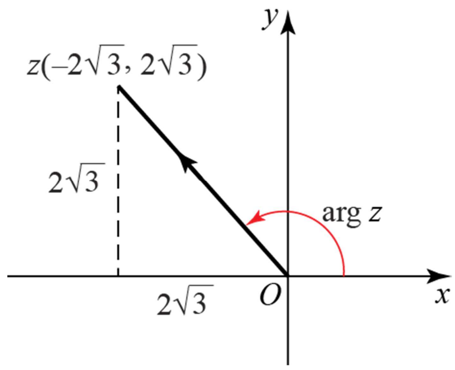


$$r=\sqrt{2^2+(-5)^2}=\sqrt{29}$$

$$\theta=-\tan^{-1}\left(\frac{5}{2}\right)=-1.19^{\text{c}}\ (2\,\text{d.p.})$$

$$\therefore\ 2-5\text{i}=\sqrt{29}\,\text{e}^{-1.19\text{i}}$$

1 f $-2\sqrt{3}+2\sqrt{3}\text{i}$



$$r=\sqrt{(-2\sqrt{3})^2+(2\sqrt{3})^2}=\sqrt{12+12}=\sqrt{24}$$

$$=\sqrt{4}\sqrt{6}=2\sqrt{6}$$

$$\theta=\pi-\tan^{-1}\left(\frac{2\sqrt{3}}{2\sqrt{3}}\right)=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$$

$$\therefore\ -2\sqrt{3}+2\sqrt{3}\,\text{i}=2\sqrt{6}\,\text{e}^{\frac{3\pi\text{i}}{4}}$$

1 g $\sqrt{8}\left(\cos\frac{\pi}{4}+\mathrm{i}\sin\frac{\pi}{4}\right)$

$=2\sqrt{2}\,\mathrm{e}^{\frac{\pi\mathrm{i}}{4}}$

$r=\sqrt{8}=\sqrt{4}\sqrt{2}=2\sqrt{2}$

$\theta=\frac{\pi}{4}$

h $8\left(\cos\frac{\pi}{6}-\mathrm{i}\sin\frac{\pi}{6}\right)$

$=8\left(\cos\left(-\frac{\pi}{6}\right)+\mathrm{i}\sin\left(-\frac{\pi}{6}\right)\right)$

$=8\mathrm{e}^{-\frac{\pi\mathrm{i}}{6}}$

$r=8,\ \theta=-\frac{\pi}{6}$

i $2\left(\cos\frac{\pi}{5}-\mathrm{i}\sin\frac{\pi}{5}\right)$

$=2\left(\cos\left(-\frac{\pi}{5}\right)+\mathrm{i}\sin\left(-\frac{\pi}{5}\right)\right)$

$=2\mathrm{e}^{\frac{-\pi\mathrm{i}}{5}}$

$r=2,\ \theta=-\frac{\pi}{5}$

2 a $\mathrm{e}^{\frac{\pi\mathrm{i}}{3}}=\cos\frac{\pi}{3}+\mathrm{i}\sin\frac{\pi}{3}$

$=\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}$

b $4\mathrm{e}^{\pi\mathrm{i}}=4(\cos\pi+\mathrm{i}\sin\pi)$

$=4(-1+\mathrm{i}(0))$

$=-4$

c $3\sqrt{2}\,\mathrm{e}^{\frac{\pi\mathrm{i}}{4}}=3\sqrt{2}\left(\cos\frac{\pi}{4}+\mathrm{i}\sin\frac{\pi}{4}\right)$

$=3\sqrt{2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\mathrm{i}\right)$

$=3+3\mathrm{i}$

2 d $8\mathrm{e}^{\frac{\pi\mathrm{i}}{6}}=8\left(\cos\frac{\pi}{6}+\mathrm{i}\sin\frac{\pi}{6}\right)$

$=8\left(\frac{\sqrt{3}}{2}+\frac{1}{2}\mathrm{i}\right)$

$=4\sqrt{3}+4\mathrm{i}$

e $3\mathrm{e}^{-\frac{\pi\mathrm{i}}{2}}=3\left(\cos\left(-\frac{\pi}{2}\right)+\mathrm{i}\sin\left(-\frac{\pi}{2}\right)\right)$

$=3(0-\mathrm{i})$

$=-3\mathrm{i}$

2 f $e^{\frac{5\pi i}{6}} = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}$

$= -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

g $e^{-\pi i} = \cos(-\pi) + i\sin(-\pi)$

$= -1 + i(0)$

$= -1$

h $3\sqrt{2}e^{-\frac{3\pi}{4}i} = 3\sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

$= 3\sqrt{2}\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i\right)$

$= -3 - 3i$

i $8e^{-\frac{4\pi i}{3}} = 8\left(\cos\left(-\frac{4\pi}{3}\right) + i\sin\left(-\frac{4\pi}{3}\right)\right)$

$= 8\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)$

$= -4 + 4\sqrt{3}i$

3 a $e^{\frac{16\pi i}{13}} = \cos\left(\frac{16\pi}{13}\right) + i\sin\left(\frac{16\pi}{13}\right)$

$= \cos\left(-\frac{10\pi}{13}\right) + i\sin\left(-\frac{10\pi}{13}\right)$

-2π from the argument.

b $4e^{\frac{17\pi i}{5}} = 4\left(\cos\left(\frac{17\pi}{5}\right) + i\sin\left(\frac{17\pi}{5}\right)\right)$

$= 4\left(\cos\left(\frac{7\pi}{5}\right) + i\sin\left(\frac{7\pi}{5}\right)\right)$

$= 4\left(\cos\left(-\frac{3\pi}{5}\right) + i\sin\left(-\frac{3\pi}{5}\right)\right)$

c $5e^{-\frac{9\pi i}{8}} = 5\left(\cos\left(-\frac{9\pi}{8}\right) + i\sin\left(-\frac{9\pi}{8}\right)\right)$

$= 5\left(\cos\left(\frac{7\pi}{8}\right) + i\sin\left(\frac{7\pi}{8}\right)\right)$

4 $e^{i\theta} = \cos\theta + i\sin\theta$ **(1)**

$e^{-i\theta} = \cos(-\theta) + i\sin(-\theta) = \cos\theta - i\sin\theta$ **(2)**

(1) − (2) $\Rightarrow e^{i\theta} - e^{-i\theta} = 2i\sin\theta$

$\frac{1}{2i}(e^{i\theta} - e^{-i\theta}) = \sin\theta$

$\therefore \sin\theta = \frac{1}{2i}(e^{i\theta} - e^{-i\theta})$ (as required)