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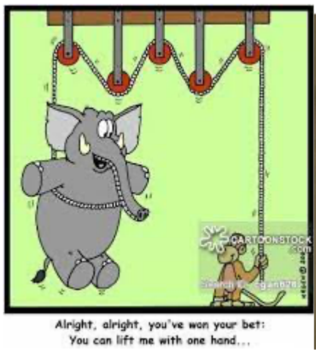
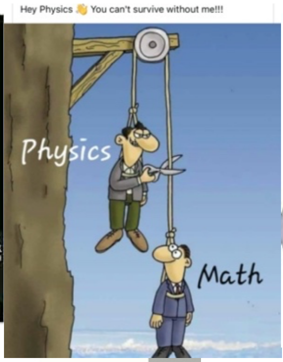
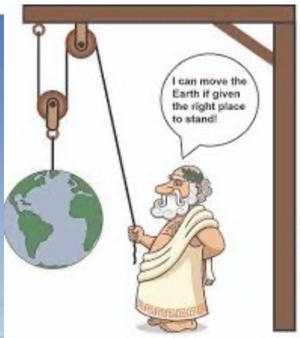
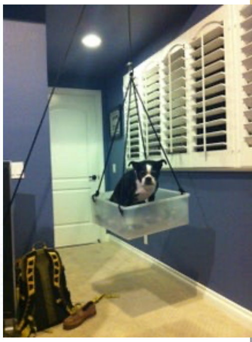
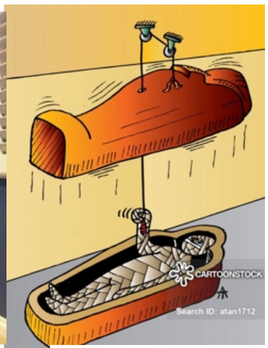
Pulleys – Vertical & Horizontal Solutions

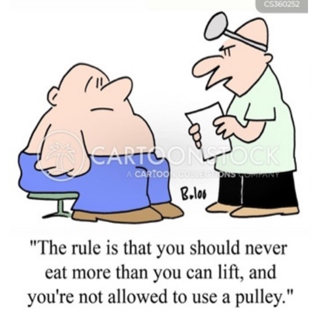


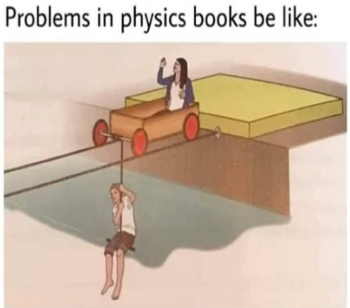


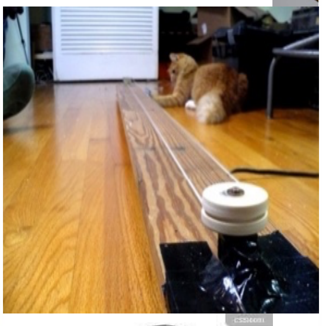


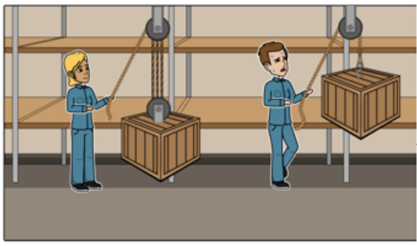












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1.1 Vertical – Known Masses

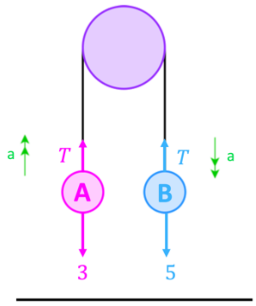
1)

Let's put all the common forces that exist for these types of questions (tension and weight) on a labelled diagram. Remember that weight is equal to mass × gravity.

For your course our assumptions are that:

- the tensions are the **same** on both sides of the pulley (since pulley is smooth) so we label both sides as T
- the accelerations are the **same** on both sides of the pulley (since string is inextensible) so we label both sides as a

We know B is heavier so B must move downwards hence we know the directions of acceleration



Remember the weight is mass × gravity hence our weight forces are 3g and 5g

Let's build our equations for each object (object A and object B) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider A:	Consider B:
Take ↑ as positive since A is moving upwards This means every force going downwards is a positive sign and every force going upwards is a negative sign Note: we could have taken ↓ as positive, but then it means we'd have to make accel a neg sign in the equation below) Follow the template $f = ma$ Remember m is the mass, not the weight! $\uparrow: T - 3g = 3a$ ①	Take ↓ as positive since going B is moving downwards This means every force going upwards is a positive sign and every force going downwards is a negative sign Note: we could have taken ↑ as positive, but then it means we'd have to make accel a neg sign in the equation below) Follow the template $f = ma$ Remember m is the mass, not the weight! $\downarrow: = -T + 5g = 5a$ ②

Notice how we have 2 equations and 2 unknowns, so we can find both T and a. Rememebr that g is not an unknown, it is gravity which we know is 9.8.

Let's solve our equations simultaneously

$$T - 3g = 3a \quad ①$$

$-T + 5g = 5a$ (2)

Way 1: Use elimination

$T - 3g = 3a$ (1)
$-T + 5g = 5a$ (2)

You can re-arrange to make the equations look more familiar if you like
(have the variables on the left and numbers on the right)

$T - 3a = 3g$ (1)
$-T - 5a = -5g$ (2)

Now we add in order to eliminate T

$$-8a = -2g$$

$$a = \frac{1}{4}g = \frac{1}{4}(9.8) = 2.45$$

Sub this into any equation

Let's choose $T - 3g = 3a$ (1)

$$T - 3g = 3(2.45)$$

$$T = 3g + 3(2.45) = 36.75\ N$$

Way 2: re-arrange both equations for T and set them equal (Best Method)

$T = 3a + 3g$
$T = 5g - 5a$

Now we can set both equations equal

$$3a + 3g = 5g - 5a$$

Group common terms

$$8a = 2g$$

$$a = \frac{1}{4}g = \frac{1}{4}(9.8) = 2.45$$

Sub this into any equation

Let's choose $T - 3g = 3a$ (1)

$$T - 3g = 3(2.45)$$

$$T = 3g + 3(2.45) = 36.75\ N$$

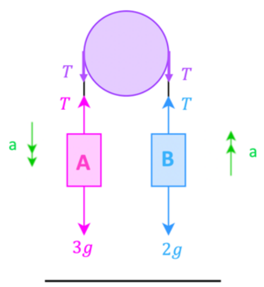
2)

Let's put all the common forces that exist for these types of questions (tension and weight) on a labelled diagram. Remember that weight is equal to mass × gravity.

For your course our assumptions are that:

- the tensions are the **same** on both sides of the pulley (since pulley is smooth) so we label both sides as T
- the accelerations are the **same** on both sides of the pulley (since string is inextensible) so we label both sides as a

We are told A moves downwards so we know the directions of the accelerations (A moves downwards which means B moves upwards). Also we know A is heavier so A must move downwards.



Remember the weight is mg hence we have 3g and 2g

Let's build our equations for each object (object A and object B) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider A:	Consider B:
Take ↓ as positive since A is moving downwards This means every force going downwards is a positive sign and every force going upwards is a negative sign	Take ↑ as positive since going B is moving upwards This means every force going upwards is a positive sign and every force going downwards is a negative sign

Note: we could have taken ↑ as positive, but then it means we'd have to make accel a neg sign in the equation below) Follow the template $f = ma$ Remember m is the mass, not the weight! $\downarrow: -T + 3g = 3a$ ①	Note: we could have taken ↓ as positive, but then it means we'd have to make accel a neg sign in the equation below) Follow the template $f = ma$ Remember m is the mass, not the weight! $\uparrow: T - 2g = 2a$ ②

Notice how we have 2 equations and 2 unknowns, so we can find both T and a. Rememebr that g is not an unknown, it is gravity which we know is 9.8.

Let's solve our equations simultaneously

$-T + 3g = 3a$ ①
$T - 2g = 2a$ ②

Way 1: Use elimination	Way 2: re-arrange both equations for T and set them equal (Best Method)
$-T + 3g = 3a$ ① $T - 2g = 2a$ ② You can re-arrange to make the equations look more familiar if you like (have the variables on the left and numbers on the right) $-T - 3a = -3g$ ① $T - 2a = 2g$ ② Now we add in order to eliminate T $-5a = -g$ $a = \frac{1}{5}g = \frac{1}{5}(9.8) = 1.96$ Sub this into any equation Let's choose $-T + 3g = 3a$ ① $-T + 3g = 3(1.96)$ $T = 3g - 3(1.96) = 23.52\ N$	$T = 3g - 3a$ $T = 2g + 2a$ Now we can set both equations equal $3g - 3a = 2g + 2a$ Group common terms $5a = g$ $a = \frac{1}{5}g = \frac{1}{5}(9.8) = 1.96$ Sub this into any equation Let's choose $-T + 3g = 3a$ ① $-T + 3g = 3(1.96)$ $T = 3g - 3(1.96) = 23.52\ N$

1.1.1 With SUVAT –Distance Travelled and Greatest Height

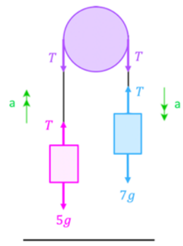
3)

Let's put all the common forces that exist for these types of questions (tension and weight) on a labelled diagram. Remember that weight is equal to mass × gravity.

For your course our assumptions are that:

- the tensions are the same on both sides of the pulley (since pulley is smooth)
- the accelerations are the same on both sides of the pulley (since string is inextensible)

We know that $7 > 5$ so the 7 kg mass must move moves downwards. This means we know the directions of the accelerations (The 7 kg mass moves downwards which means the 5 kg mass moves upwards)



Let's build our equations for each object (object with 5 kg mass and object with 7 kg mass) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider the 5 kg mass:
Take ↑ as positive since moving upwards
This means every force going upwards is a positive sign and every force going downwards is a negative sign

Follow the template $f = ma$

$\downarrow : T - 5g = 5a$ ①

Consider the 7 kg mass:
Take ↓ as positive since moving downwards
This means every force going downwards is a positive sign and every force going upwards is a negative sign

Follow the template $f = ma$

$\uparrow : -T + 7g = 7a$ ②

Notice how we have 2 equations and 2 unknowns, so we can find both T and a. Rememebr that g is not an unknown, it is gravity which we know is 9.8.

Let's solve our equations simultaneously

$$T - 5g = 5a \quad ①$$
$$-T + 7g = 7a \quad ②$$

Way 1: Use elimination

$$T - 5g = 5a \quad ①$$
$$-T + 7g = 7a \quad ②$$

You can re-arrange to make the equations look more familiar if you like
(have the variables on the left and numbers on the right)

$$T - 5a = 5g \quad ①$$
$$-T - 7a = -7g \quad ②$$

Now we add in order to eliminate T

$$-12a = -2g$$

$$a = \frac{2}{12}g = \frac{1}{6}(9.8) = 1.633$$

Sub this into any equation

Let's choose $T - 5g = 5a$ ①

$$T - 5g = 5(1.633)$$

$$T = 5g + 5(1.633) = 57.165\ N$$

Way 2: re-arrange both equations for T and set them equal

$$T = 5a + 5g$$
$$T = 7g - 7a$$

Now we can set both equations equal

$$5a + 5g = 7g - 7a$$

Group common terms

$$12a = 2g$$

$$a = \frac{2}{12}g = \frac{1}{6}(9.8) = 1.633$$

Sub this into any equation

Let's choose $T - 5g = 5a$ ①

$$T - 5g = 5(1.633)$$

$$T = 5g + 5(1.633) = 57.165\ N$$

i.

$$a = 1.63$$

ii.

$$T = 57.2\ N$$

ii.

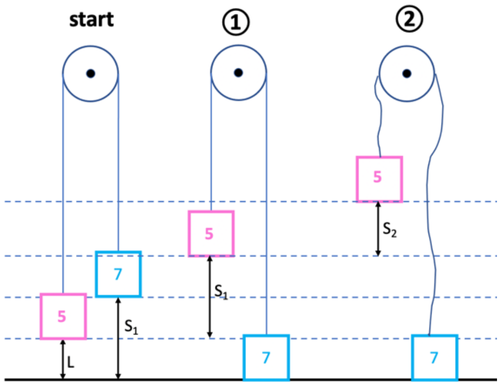
Let's look at what is happening in words and then a picture

- Firstly, the 7 kg moves down to hit the ground
- Secondly, once the 7 kg hits the ground the string goes slack and therefore the string has some give in it and the 5 kg object can move up a little bit more before it comes to rest

The important part here is to realise that:

- The speed that the 7 kg object hits the ground in the middle diagram below will be the starting speed for the next motion for the 5 kg object when it moves up slightly in the right most diagram
- Once the 7 kg object hits the ground, the string is slack and therefore the acceleration is no longer the acceleration in the system (it is due to gravity instead and always equal to -9.8)

Now a picture:



①

Consider 7 kg object

$S = s_1$

$U = 0$

$V = v$

$A = 1.633$

$T = 3$

$$v = u + at$$
$$v = 0 + 1.633(3)$$
$$v = 4.899$$

$$s = ut + \frac{1}{2}at^2$$
$$s_1 = 0(3) + \frac{1}{2}(1.633)(3)^2$$
$$s_1 = 7.3485$$

②

Consider 5 kg object

$S = S_2$

$U = 4.899$

$V = 0$ (at rest)

$A = -9.8$ (string slack)

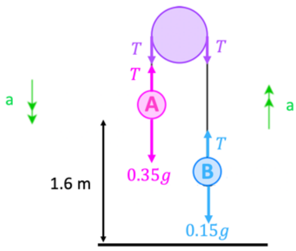
$T = t$

$$v^2 = u^2 + 2as$$
$$0^2 = 4.899^2 + 2(-9.8)s_2$$
$$s_2 = 1.2245$$

Total distance$= 7.3485 + 1.2245 = 8.57\ m$

4)

A is hitting the ground (since heavier) so we know which way the system is moving



Consider A:

Take ↓ as positive since A is moving downwards

Consider B:

Take ↑ as positive since going B is moving upwards

This means every force going downwards is a positive and every force going upwards is a positive

This means every force not going upwards is a positive and every force going downwards is a negative

Follow the template $f = ma$

$\downarrow: -T + 0.35g = 0.35a$ ①

Follow the template $f = ma$

$\uparrow: T - 0.15g = 0.15a$ ②

Solve ① and ② simultaneously by re-arranging both for T

$T = 0.35g - 0.35a$ ①

$T = 0.15a + 0.15g$ ②

Setting them equal

$$0.35g - 0.35a = 0.15a + 0.15g$$

$$0.5a = 0.2g$$

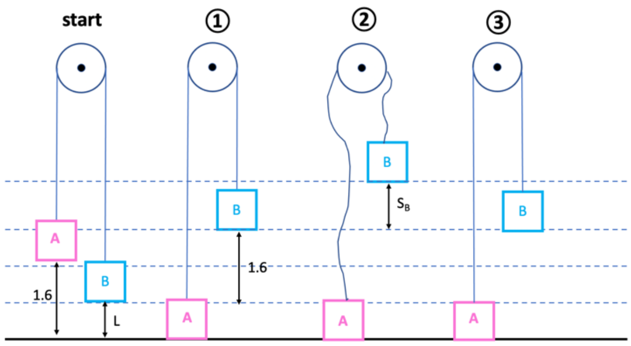
$$a = \frac{0.2g}{0.5} = 3.92\ ms^{-2}$$

Sub a into $T = 0.35g - 0.35a$ ①

$$T = 0.35g - 0.35(3.92) = 2.058$$

$$2.06\ N$$

ii.



① Consider A

S=1.6
U=0
V=
A=3.92
T=

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2(3.92)(1.6)$$

$$v = 3.54$$

② Consider B

S=S
U=3.54
V=0
A=−9.8 (string slack)

$$v^2 = u^2 + 2as$$

$$0^2 = 3.54^2 + 2(-9.8)s$$

$$s = 0.639$$

We don't care about this motion since this is when the string becomes taut again and we aren't asked for this

$$1.6 + 0.639 = 2.24\ m$$

Note: B starts off of the ground so we had to add the 1.6 which it was already off from the ground originally

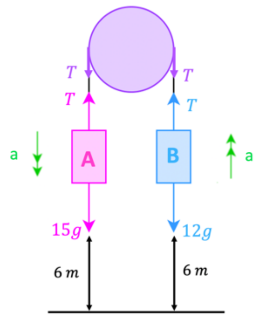
5)

Let's put all the common forces that exist for these types of questions (tension and weight) on a labelled diagram. Remember that weight is equal to mass × gravity.

For your course our assumptions are that:

- the tensions are the same on both sides of the pulley (since pulley is smooth)
- the accelerations are the same on both sides of the pulley (since string is inextensible)

We are told A is heavier so A must move moves downwards, so we know the directions of the accelerations (A moves downwards which means B moves upwards)



Let's build our equations for each object (object A and object B) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider A: Take ↓ as positive since A is moving downwards This means every force going downwards is a positive sign and every force going upwards is a negative sign Follow the template $f = ma$ $\downarrow: -T + 15g = 15a$ (1)	Consider B: Take ↑ as positive since going B is moving upwards This means every force going upwards is a positive sign and every force going downwards is a negative sign Follow the template $f = ma$ $\uparrow: T - 12g = 12a$ (2)

Notice how we have 2 equations and 2 unknowns, so we can find both T and a. Rememebr that g is not an unknown, it is gravity which we know is 9.8.

Let's solve our equations simultaneously

$$-T + 15g = 15a \quad (1)$$
$$T - 12g = 12a \quad (2)$$

Way 1: Use elimination	Way 2: re-arrange both equations for T and set them equal
$-T + 15g = 15a$ (1) $T - 12g = 12a$ (2) You can re-arrange to make the equations look more familiar if you like (have the variables on the left and numbers on the right) $-T - 15a = -15g$ (1) $T - 12a = 12g$ (2) Now we add in order to eliminate T $-27a = -3g$ $a = \frac{3}{27}g = \frac{1}{9}(9.8) = 1.089$	$T = 15g - 15a$ $T = 12g + 12a$ Now we can set both equations equal $15g - 15a = 12g + 12a$ Group common terms $-27a = -3g$ $a = \frac{3}{27}g = \frac{1}{9}(9.8) = 1.09$ Sub this into any equation

Sub this into any equation Let's choose $-T + 15g = 15a$ ① $-T + 15g = 15(1.09)$ $T = 15g - 15(1.09) = 130.7\ N$	Let's choose $-T + 15g = 15a$ ① $-T + 15g = 15(1.09)$ $T = 15g - 15(1.09) = 130.7\ N$

i.

$$a = 1.089, T = 130.7\ N$$

ii.

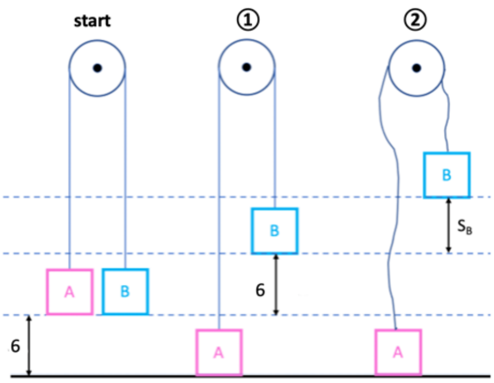
Let's look at what is happening in words and then a picture

- Firstly A moves down to hit the ground
- Secondly once A hits the ground the string goes slack and therefore the string has some give in it and B can move up a little bit more before it comes to rest

The important part here is to realise that:

- the speed that A hits the ground in the middle diagram below will be the starting speed for the next motion for B when it moves up slightly in the right most diagram
- Once A hits the ground, the string is slack and therefore the acceleration is no longer the acceleration in the system (it is due to gravity instead and always equal to -9.8)

Now a picture:



①

Consider A
S = 6
U = 0
V = v
A = 1.089
T = 1.2

$v^2 = u^2 + 2as$
$v^2 = 0^2 + 2(1.089)(6)$
$v = 3.615$

②

Consider B
S = S_B
U = 3.615
V = 0 (at rest)
A = -9.8 (string slack)
T = t

$v = u + at$
$0 = 3.615 - 9.8t$
$t = 0.369$

$v^2 = u^2 + 2as$
$0^2 = 3.615^2 + 2(-9.8)S_B$
$S_B = 0.667$

iv.

It means negligible mass of string and for vertical systems this means the acceleration is the same on both sides of the pulley (and the tensions are the same since smooth also).

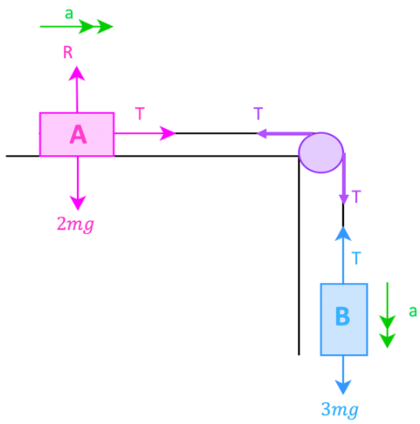
1.2 Horizontal - Known Masses

6)

Let's put all the common forces that exist for these types of questions (tension, weight and now friction) on a labelled diagram. Remember that weight is equal to $\text{mass} \times$ gravity and friction only exists if the surface is rough. Here we have a smooth table and hence no friction.

For your course our assumptions are that:

- the tensions are the same on both sides of the pulley (since pulley is smooth)
- the accelerations are the same on both sides of the pulley (since string is inextensible)



Let's build our equations for each object (object A and object B) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider A (we have to look at **2 directions now** since we have forces in the horizontal AND vertical direction)		Consider B: (we only look at the vertical direction since we only have forces in this direction)
Vertical:	Horizontal	Vertical:
Take ↑ as positive	Take → as positive since moving right. This means every force going to the right is a positive sign and every force going to the left is a negative sign	Take ↓ as positive since moving downwards. This means every force going downwards is a positive sign and every force going upwards is a negative sign
There is no acceleration (hence $a = 0$) in this direction since the motion is horizontal		$\downarrow: -T + 3mg = 3ma$
$\uparrow: R - 2mg = 2(0)$	$\rightarrow: T = 2ma$ ②	$T = 3mg - 3ma$ ③
$R = 2mg$ ①		

ii.

We had the following equations

$R = 2mg$ ①

$T = 2ma$ ②

$T = 3mg - 3ma$ ③

We can set ② and ③ equal to find the tension

$$2ma = 3mg - 3ma$$

Cancel an m from all terms

$$2a = 3g - 3a$$

$$5a = 3g$$

$$a = \frac{3}{5}g = 5.88\ ms^{-2}$$

iii.

We can sub a into (3) to find T

$$T = 3mg - 3ma \quad (3)$$

$$T = 3mg - 3m\left(\frac{3}{5}g\right)$$

$$T = 3mg - \frac{9}{5}mg$$

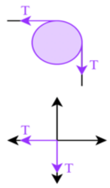
$$T = \frac{6}{5}mg$$

Can't simplify further since don't know the know the mass m

iii.

We now have to consider the purple tensions since they are acting on the pulley and the question wants the forces exerted **on the pulley.**

Let's resolve as usual



$$\uparrow = -T$$
$$\rightarrow = -T$$

Resultant$= \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \rightarrow \\ \uparrow \end{pmatrix} = \begin{pmatrix} -T \\ -T \end{pmatrix} = \begin{pmatrix} -\frac{6}{5}mg \\ -\frac{6}{5}mg \end{pmatrix}$

Mag$=\sqrt{(-\frac{6}{5}mg)^2 + (-\frac{6}{5}mg)^2} = \sqrt{\frac{36}{25}m^2g^2 + \frac{36}{25}m^2g^2} = \sqrt{\frac{72}{25}m^2g^2} = \frac{\sqrt{72}}{5}m^2g^2 = \frac{6\sqrt{2}}{5}mg$

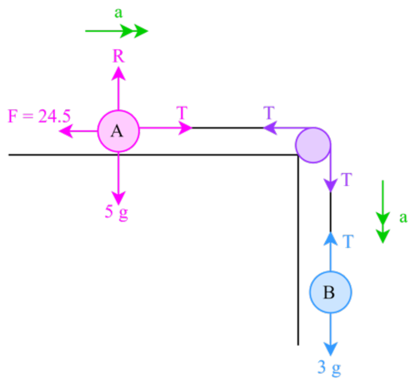
$\frac{6\sqrt{2}}{5}mg$ acting 45 degrees below the horizontal (bearing of 225°)

7)

Let's put all the common forces that exist for these types of questions (tension, weight and now friction) on a labelled diagram. Remember that weight is equal to mass × gravity and friction only exists if the surface is rough.

For your course our assumptions are that:

- the tensions are the same on both sides of the pulley (since pulley is smooth)
- the accelerations are the same on both sides of the pulley (since string is inextensible)



Let's build our equations for each object (object A and object B) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider A (we have to look at **2 directions now** since we have forces in the horizontal AND vertical direction)		Consider B: (we only look at the vertical direction since we only have forces in this direction)
Vertical:	Horizontal	Vertical:
Take ↑ as positive There is no acceleration (hence $a = 0$) in this direction since the motion is horizontal	Take → as positive since moving right. This means every force going to the right is a positive sign and every force going to the left is a negative sign	Take ↓ as positive since moving downwards. This means every force going downwards is a positive sign and every force going upwards is a negative sign
$\uparrow: R - 5g = 5(0)$ $R = 5g$ (1)	$\rightarrow: T - 24.5 = 5a$ $T = 5a + 24.5$ (2)	$\downarrow: -T + 3g = 3a$ $T = 3g - 3a$ (3)

Solve (2) and (3) simultaneously

$$5a + 24.5 = 3g - 3a$$
$$8a = 4.9$$
$$a = 0.6125$$

Sub a into (2)

$$T = 5(0.6125) + 24.5 = 27.6\ N$$

Note: Notice how we didn't need (1) to solve this

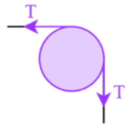
i.

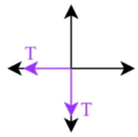
$$a = 0.6125$$

ii.

$$T = 27.6\ N$$

iii.
We now have to consider the purple tensions since they are acting on the pulley and the question wants the forces exerted **on the pulley.**





Let's resolve as usual

$\uparrow = -T$
$\rightarrow = -T$

Resultant= $\binom{x}{y} = \binom{\rightarrow}{\uparrow} = \binom{-T}{-T} = \binom{-27.6}{-27.6}$

Mag= $\sqrt{(-27.6)^2 + (-27.6)^2} = 38.979$

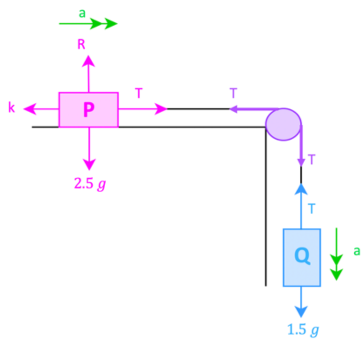
1.2.1 With SUVAT – Finding the Acceleration / Time Taken To Hit The Pulley

8)

Let's put all the common forces that exist for these types of questions (tension, weight and now friction) on a labelled diagram. Remember that weight is equal to mass × gravity and friction only exists if the surface is rough. Here we have a rough surface so the there is friction which we told is k.

For your course our assumptions are that:

- the tensions are the same on both sides of the pulley (since pulley is smooth)
- the accelerations are the same on both sides of the pulley (since string is inextensible)



Let's build our equations for each object (object P and object Q) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider P (we have to look at **2 directions now** since we have forces in the horizontal AND vertical direction)		Consider Q: (we only look at the vertical direction since we only have forces in this direction)
Vertical:	Horizontal	Vertical:
Take ↑ as positive There is no acceleration (hence $a = 0$) in this direction since the motion is horizontal $\uparrow: R - 2.5g = 2.5(0)$ $R = 2.5g$ ①	Take → as positive since moving right. This means every force going to the right is a positive sign and every force going to the left is a negative sign $\rightarrow: T - k = 2.5a$	Take ↓ as positive since moving downwards. This means every force going downwards is a positive sign and every force going upwards is a negative sign $\downarrow: -T + 1.5g = 1.5a$ $T = 1.5g - 1.5a$ ③

$T = 2.5a + k$ (2)

We have too many unknowns to solve these. We have enough info though to use SUVAT in order to find a first

S=0.8
U=0
V=
A=
T=0.75

$$s = ut + \frac{1}{2}at^2$$

$$0.8 = (0)(0.75) + \frac{1}{2}a(0.75)^2$$

$$a = 2.84\,ms^{-2}$$

ii.

We had the following equations

$R = 2.5g$ (1)
$T = 2.5a + k$ (2)
$T = 1.5g - 1.5a$ (3)

We can sub a in now to (3) to find the tension

$T = 1.5g - 1.5a$ (3)

$$T = 1.5g - 1.5(2.844) = 10.434$$

$$T = 10.4\,N$$

iii.

We can sub a and T into (2) to find k

$T = 2.5a + k$ (2)

$$10.4 = 2.5(2.844) + k$$

$$10.4 = 7.11 + k$$

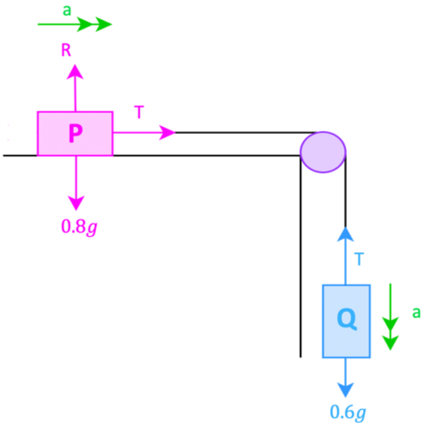
$$k = 3.29\,N$$

Note: Notice how we didn't need (1) to solve ii or iii

iv.
The acceleration the same on both sides of pulley

9)

Smooth table hence no friction



Consider object P:
(we have to look at 2 directions since we have forces in the horizontal and vertical direction)

Consider object Q:
(we only look at the vertical direction since only have forces in this direction)

Vertical:
Take ↑ as positive
There is no acceleration in this direction since the motion is horizontal

$\uparrow: R - 0.8g = 0.8(0)$

$R = 0.8g$ ①

Horizontal
Take → as positive

$\rightarrow: T = 0.8a$

$T = 0.8a$ ②

Vertical:
Take ↓ as positive

$\downarrow: -T + 0.6g = 0.6a$

$T = 0.6g - 0.6a$ ③

i.

We had the following equations

$R = 0.8g$ ①

$T = 0.8a$ ②

$T = 0.6g - 0.6a$ ③

Let's set ② and ③ equal

$$0.8a = 0.6g - 0.6a$$

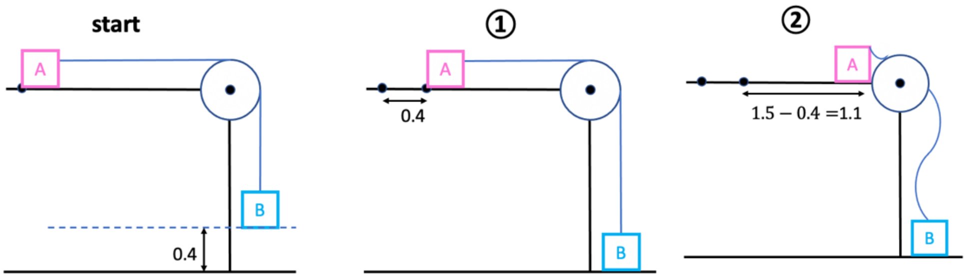
$$1.4a = 0.6g$$

$$a = \frac{0.6g}{1.4}$$

$$a = 4.2\ ms^{-2}$$

Note: Notice how we didn't need ① to solve i.

ii.



①

Consider B
S = 0.4
U = 0
V = v
A = 4.2
T =

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2(4.2)(0.4)$

$v^2 = 3.36$

$v = 1.833$

Now use $v = u + at$

$1.833 = 0 + 4.2t$

$t = 0.46\ s$

②

Consider A
S = $1.5 - 0.4 = 1.1$
U = 1.833
V =
A = 0 (see * below)
T =

$s = ut + \frac{1}{2}at^2$

$1.1 = 1.833t + \frac{1}{2}(0)t^2$

$t = 0.6\ s$

Total time =0.6+0.436=1.04s

*once the string went slack (i.e. once B hit the ground) we needed to find the new acceleration. This will not be due to gravity like for the vertical pulleys, since A is moving horizontally and gravity only acts vertically!

We re-resolve to find the new acceleration. We do what we did when we considered A horizontally last time, except we delete T since no tension in the strong

$$\rightarrow: \not{T} = 0.8a$$

Deleting T gives

$$0 = 0.8a$$

$$a = 0$$

iii.

rope is light and inextensible and pulley is smooth

2 Silver

2.1 Vertical – Unknown Masses

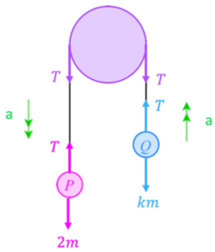
10)

Let's put all the common forces that exist for these types of questions (tension and weight) on a labelled diagram. Remember that weight is equal to mass × gravity.

For your course our assumptions are that:

- the tensions are the same on both sides of the pulley (since pulley is smooth)
- the accelerations are the same on both sides of the pulley (since string is inextensible)

We are told P moves downwards so we know the directions of the accelerations (P moves downwards which means Q moves upwards)



Let's build our equations for each object (object P and Object Q) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider P:	Consider Q:
Take ↓ as positive since P is moving downwards This means every force going downwards is a positive sign and every force going upwards is a negative sign Follow the template $f = ma$ $\downarrow: -T + 2mg = 2m\left(\frac{5g}{7}\right)$ ①	Take ↑ as positive since going Q is moving upwards This means every force going upwards is a positive sign and every force going downwards is a negative sign Follow the template $f = ma$ $\uparrow: T - kmg = km\left(\frac{5g}{7}\right)$ ②

Notice how we have 2 equations and 3 unknowns, so we will never be able to find all unknowns in terms of a number (this is why the best we can do is get T in term of m)

Let's solve simultaneously

Way 1: work on one equation at a time	Way 2: re-arrange both equations for T and set them equal (BETTER METHOD)
① tells us that $-T + 2mg = 2m\left(\frac{5g}{7}\right)$	

Re-arranging for T gives $T = 2mg - \frac{10}{7}mg$ $T = \frac{4}{7}mg$ Plug T into (2) $\frac{4}{7}mg - kmg = km\left(\frac{5g}{7}\right)$ Cancel an m and g from each term $\frac{4}{7} - k = \frac{5}{7}k$ Solve for k $\frac{12}{7}k = \frac{4}{7}$ $k = \frac{\frac{4}{7}}{\frac{12}{7}} = \frac{4}{12} = \frac{1}{3}$	$T = \frac{10}{7}mg - 2mg$ $T = \frac{5}{7}mg + kmg$ Now we can set both equations equal $\frac{10}{7}mg - 2mg = \frac{5}{7}mg + kmg$ We can cancel an m and g from each term $\frac{10}{7} - 2 = \frac{5}{7} + k$ $k = \frac{10}{7} - 2 - \frac{5}{7}$ $k = -\frac{9}{7}$

So, now we can answer the question

i.
$T = \frac{4}{7}mg$

ii. The string is modelled as inextensible

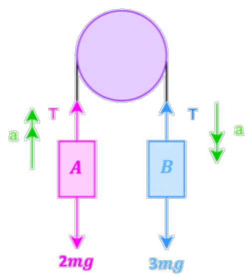
iii.
$k = \frac{1}{3}$

iv.
The pulley may not be smooth

2.1.1 With SUVAT – Greatest Height and Taut Again

11)

$3m > 2m$ so we know which way the system is moving (the heavier object moves down)



Consider A:
Take ↑ as positive since A is moving upwards

Consider B:
Take ↓ as positive since B is moving downwards

This means every force going upwards is a positive and every force going upwards is a negative

This means every force going upwards is a negative and every force going downwards is a positive

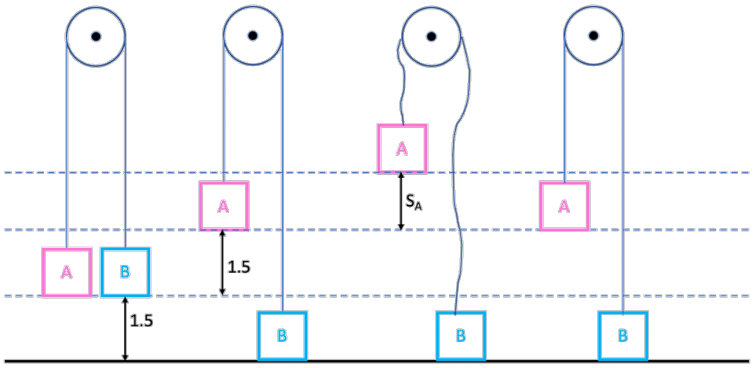
Using template $F = ma$ we get
$\uparrow: T - 2mg = 2ma$ (1)

Using template $F = ma$ we get
$\downarrow: -T + 3mg = 3ma$ (2)
$T = 3mg - 3ma$ (2)

Solve (1) and (2) simultaneously by re-arranging both for T

$T = 2mg + 2ma$
$T = 3mg - 3ma$

$2mg + 2ma = 3mg - 3ma$
$2g + 2a = 3g - 3a$
$a = \frac{g}{5}$
Sub into $T = 2mg + 2ma$
$\mathrm{T} = 2\mathrm{mg} + 2\mathrm{m}\left(\frac{g}{5}\right) = \frac{12}{5}mg$



Consider B
$S = 1.5$
$U = 0$
$V = V_B$
$A = \frac{g}{5}$
$T = T_B$

$v^2 = u^2 + 2as$
$v = \sqrt{0.6g}$

Consider A
$S = S_A$
$U = \sqrt{0.6g}$
$V = 0$
$A = -9.8$
$T = T_A$

$v^2 = u^2 + 2as$
$0 = 0.6g + 2(-9.8)S_A$
$S_A = 0.3$

Greatest height:
$1.5 + 1.5 + 0.3 = 3.3$

Consider A
$S = S_A$
$U = \sqrt{0.6g}$
$V = 0$
$A = -9.8$
$T = T_A$

$v = u + at$
$0 = \sqrt{0.6g} - 9.8t$
$t = 0.247$

Time: 2(0.247)=0.495
Distance: 0.3+0.3=0.6

2.2 Horizontal

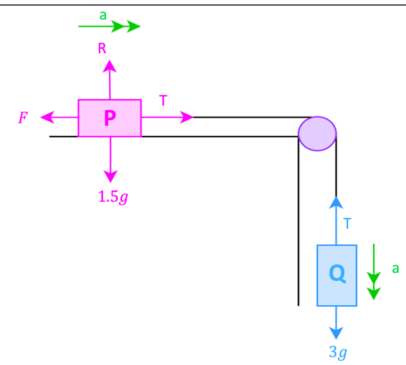
2.2.1 With Friction (year 2 only)

12)

Let's put all the common forces that exist for these types of questions (tension, weight and now friction) on a labelled diagram. Remember that weight is equal to mass × gravity and friction only exists if the surface is rough. Here we have a rough table and hence friction.

For your course our assumptions are that:

- the tensions are the same on both sides of the pulley (since pulley is smooth)
- the accelerations are the same on both sides of the pulley (since string is inextensible)



Let's build our equations for each object (object P and object Q) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider P (we have to look at **2 directions now** since we have forces in the horizontal AND vertical direction)		Consider Q: (we only look at the vertical direction since we only have forces in this direction)
Vertical: Take ↑ as positive	Horizontal: Take → as positive since moving right.	Vertical: Take ↓ as positive since moving downwards. This means every force going downwards is a positive sign and every force going upwards is a negative sign
There is no acceleration (hence $a = 0$) in this direction since the motion is horizontal	This means every force going to the right is a positive sign and every force going to the left is a negative sign	$\downarrow: -T + 3g = 3a$
$\uparrow: R - 1.5g = 1.5(0)$		$T = 3g - 3a$ ③
$R = 1.5g$ ①	$\rightarrow: T - F = 1.5a$	
	$T = 1.5a + F$ ②	

ii.

We had the following equations

$R = 1.5g$ ①

$T = 1.5a + F$ ②

$T = 3g - 3a$ ③

We also have a fourth equation: $F = \mu R = \frac{1}{5}R$ ④

Sub ④ and ③ into ②

$T = 1.5a + F$ ②

$$3g - 3a = 1.5a + \frac{1}{5}R$$

Now sub in ①

$$3g - 3a = 1.5a + \frac{1}{5}(1.5g)$$

$$3g - 3a = 1.5a + 0.3g$$

$$4.5a = 2.7g$$

$$a = 5.88$$

Sub this into $T = 3g - 3a$ ③ to find T

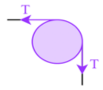
$$T = 3g - 3(5.88) = 11.76$$

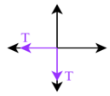
$$11.8\ N$$

ii.

We now have to consider the purple tensions since they are acting on the pulley and the question wants the forces exerted **on the pulley.**

Let's resolve as usual





$\uparrow = -T$

$\rightarrow = -T$

Resultant= $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \rightarrow \\ \uparrow \end{pmatrix} = \begin{pmatrix} -T \\ -T \end{pmatrix} = \begin{pmatrix} -11.8 \\ -11.8 \end{pmatrix}$

Mag= $\sqrt{11.8^2 + 11.8^2} = \sqrt{278.48} = 16.7\ N$

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3 Gold

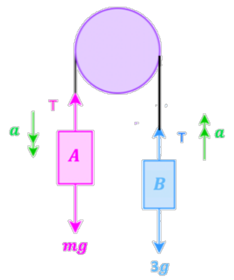
3.1 Vertical

3.1.1 Using SUVAT Multiple Times

13)

We set the total/resultant/net force which is F equal to ma for each object (pink and blue sections below)

$m > 3$ so we know which way the system is moving (the heavier object moves down)



Consider A:
Take ↓ as positive since A is moving downwards
This means every force going downwards is a positive and every force going upwards is a positive
$\downarrow: -T + mg = ma$ ①

Consider B:
Take ↑ as positive since B is moving upwards
This means every force not going upwards is a positive and every force going downwards is a negative
$\uparrow: T - 3g = 3a$ ②

i.

Solve ① and ② simultaneously by re-arranging both for T

$$T = mg - ma$$
$$T = 3g + 3a$$

We have 2 equations and 3 unknowns. We need an extra equation first. Let's use SUVAT

S=2.5
U=0
V=
A= a
T=1.25

$s = ut + \frac{1}{2}at^2$

$2.5 = 0 + \frac{1}{2}a(1.25)^2$

$a = 3.2$

ii and iii.

Our 2 equations become

$$T = mg - 3.2m$$
$$T = 3g + 9.6$$

$mg - 3.2m = 3g + 9.6$

$6.6m = 39$

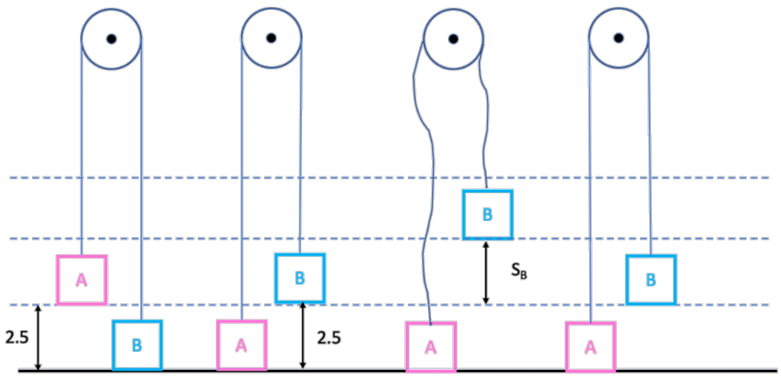
$m = \frac{65}{11}$

$T = 3g + 9.6$ gives $T = 39\ N$

iv.

Inextensible ⇒ magnitude of **acceleration is the same** on both sides of pulley

v and vi.



Consider A

$S = 2.5$

$U = 0$

$V = v$

$A = 3.2$

$T = 1.25$

$v = u + at$

$v = 0 + 3.2(1.25)$

$v = 4$

Consider B

$S =$

$U = 4$

$V = 0$

$A = -9.8$

$T = T_B$

$v^2 = u^2 + 2as$

$0 = 4^2 + 2(-9.8)S_B$

$S_B = 0.816$

Greatest height:

$2.5 + 0.816 = 3.316$

Use SUVAT again

$v = u + at$

$0 = 4 - 9.8t$

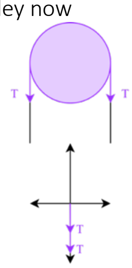
$t = \frac{20}{49}$

Time until taut:

$2\left(\frac{20}{49}\right) = \frac{40}{49}$

vii.

we need to consider the forces acting on the pulley now



$\downarrow = T + T$

$\rightarrow = 0$

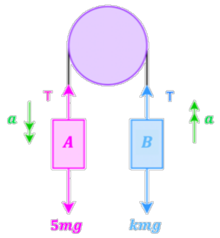
Resultant$= \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \rightarrow \\ \uparrow \end{pmatrix} = \begin{pmatrix} 0 \\ 2T \end{pmatrix} = \begin{pmatrix} 0 \\ 2(39) \end{pmatrix} = \begin{pmatrix} 0 \\ 78 \end{pmatrix}$

Mag$=\sqrt{0^2 + 78^2} = 78$ N

14)

We set the total/resultant/net force which is F equal to ma for each object (pink and blue sections below)

$k < 5$ so we know which way the system is moving (the heavier object moves down)



Consider A:
Take ↓ as positive since A is moving downwards
This means every force going downwards is a positive and every force going upwards is a positive

$\downarrow: -T + 5mg = 5m\left(\frac{1}{4}g\right)$ ①

Consider B:
Take ↑ as positive since B is moving upwards
This means every force not going upwards is a positive and every force going downwards is a negative

$\uparrow: T - kmg = km\left(\frac{1}{4}g\right)$ ②

i. and ii.

Solve ① and ② simultaneously by re-arranging both for T

$$T = 5mg - \frac{5}{4}mg$$

$$T = kmg + \frac{1}{4}kmg$$

$5mg - \frac{5}{4}mg = kmg + \frac{1}{4}kmg$

We can cancel the $m's$ and g's

$5 - \frac{5}{4} = k + \frac{1}{4}k$

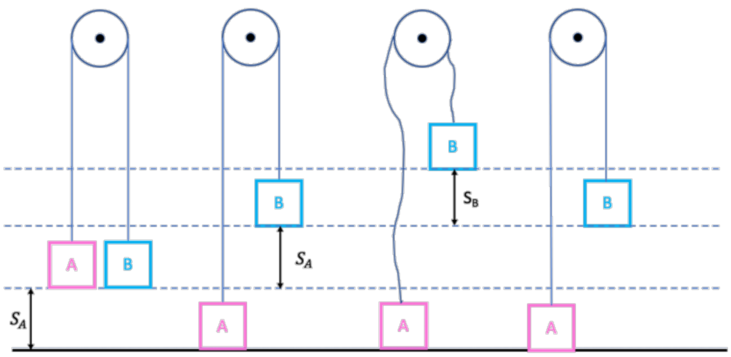
$\frac{15}{4} = \frac{5}{4}k$

$k = 3$

$T = 5mg - \frac{5}{4}mg = \left(5 - \frac{5}{4}\right)mg = \frac{15}{4}mg$

iii. The tensions are the same on both sides of the pulley

iv.



Consider A
$S = s_A$
$U = 0$
$V = v$
$A = \frac{1}{4}g$
$T = 1.2$

We need to do SUVAT twice to find s and v:

$s = ut + \frac{1}{2}at^2$

$s = 0 + \frac{1}{2}\left(\frac{1}{4}g\right)(1.2)^2$

$s = 1.764$

Consider B
$S = S_B$
$U = 2.94$
$V = 0$
$A = -9.8$
$T = t$

$v^2 = u^2 + 2as$

$0 = 2.94^2 + 2(-9.8)S_B$

$S_B = 0.441$

Greatest height:
$1.764 + 1.764 + 0.441 = 3.969$

Taut again:

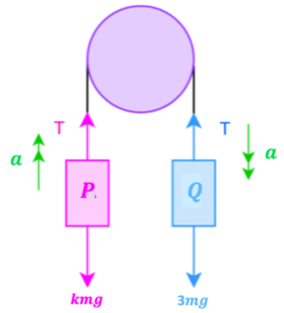
$2(0.3) = 0.6\ s$

$v = u + at$
$v = 0 + \frac{1}{4}g(1.2)$
$v = 2.94$

Use SUVAT again
$v = u + at$
$0 = 2.94 - 9.8t$
$t = 0.3$

15)

We set the total/resultant/net force which is F equal to ma for each object (pink and blue sections below)
$k < 3$ so we know which way the system is moving (the heavier object Q moves down)



Consider P:
Take ↑ as positive since A is moving upwards
This means every force going upwards is a positive
and every force going upwards is a negative
Using template $F = ma$ we get
$\uparrow: T - kmg = km\left(\frac{1}{3}g\right)$ ①

Consider Q:
Take ↓ as positive since B is moving downwards
This means every force going upwards is a negative
and every force going downwards is a positive
Using template $F = ma$ we get
$\downarrow: -T + 3mg = 3m\left(\frac{1}{3}g\right)$ ②

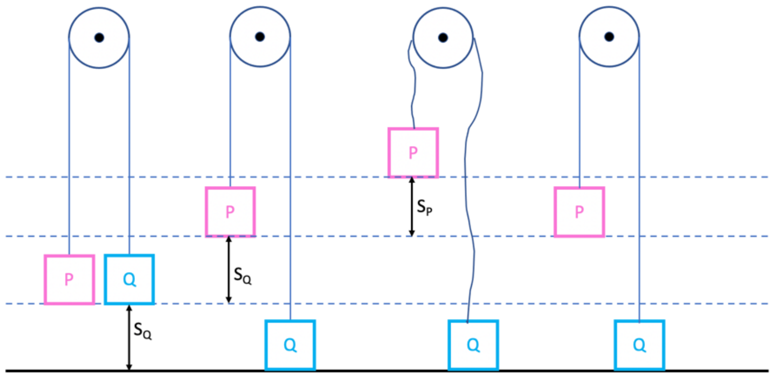
i. Solve ① and ② simultaneously by re-arranging both for T

$T = \frac{4}{3}kmg$
$T = 2mg$

$2mg = \frac{4}{3}kmg$
$k = \frac{3}{2} = 1.5$
$T = 2mg$ N

ii. Tension the same on both sides of the string

iii.



Consider Q
S = s_Q
U = 0
V =
A = $\frac{1}{3}g$
T = 1.8

We need to do SUVAT twice to find s and v:
$s = ut + \frac{1}{2}at^2$
$s = 0 + \frac{1}{2}\left(\frac{1}{3}g\right)(1.8)^2$
$s = 0.54g$
(leave in terms of g since answer is in terms of g)

$v = u + at$
$v = 0 + \frac{1}{3}g(1.8)$
$v = \frac{3}{5}g$

Consider P
S = S_P
U = $\frac{3}{5}g$
V = 0
A = $-g$
T = t

$v^2 = u^2 + 2as$
$0 = (\frac{3}{5}g)^2 + 2(-g)S_P$
$S_P = \frac{\frac{9}{25}g^2}{2g} = 0.18g$

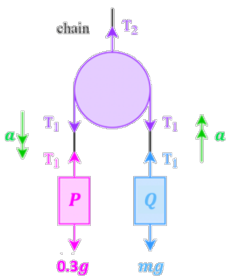
Greatest height:
$0.54g + 0.54g + 0.18g$
$= 1.2699g\ m$

Don't care about this part of motion since not looking for taut again

3.1.2 More Than 1 Tension (suspended Pulleys)

16)

P is hitting the ground so we know which way the system is moving



Consider P:
Take ↓ as positive since P is moving downwards
This means every force going downwards is a positive and every force going upwards is a positive

$\downarrow: -T_1 + 0.3g = 0.3a$ (1)

Consider Q:
Take ↑ as positive since going Q is moving upwards
This means every force not going upwards is a positive and every force going downwards is a negative

$\uparrow: T_1 - mg = ma$ (2)

i and ii.

Solve (1) and (2) simultaneously by re-arranging both for T

$T_1 = 0.3g - 0.3a$ (1)

$T_1 = mg + ma$ (2)

We have 2 equations and 3 unknowns. We need an extra equation first. Let's use SUVAT

S=0.2
U=0
V=1.4
A
T

$v^2 = u^2 + 2as$
$1.4^2 = 0^2 + 2a(0.2)$
$a = 4.9\ ms^{-2}$

i.

Sub a into $T_1 = 0.3g - 0.3a$

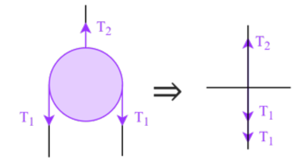
$T_1 = 0.3(9.8) - 0.3(4.9) = 1.47\ N$

Sub a and T_1 into $T_1 = mg + ma$ (2)

$1.47 = m(9.8) + m(4.9)$

$m = 0.1\ kg$

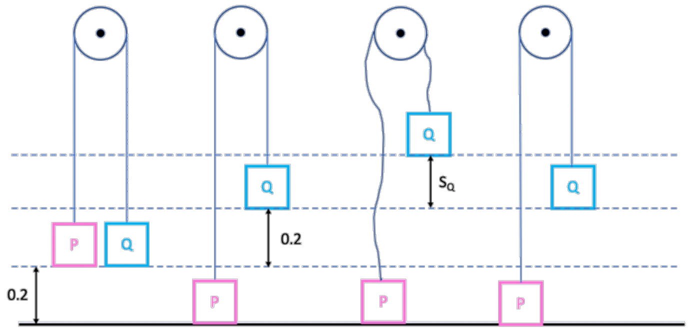
iii.



a) Hint: You have tension up T_2 for chain and 2 tensions T_1 for pulley and the weight acting down, find T_2
consider the pulley as this wants to forces on the Pulley
R(↓): $-T_2 + T_1 + T_1 + 0.5g = 0.5(0)$
$-T_2 + 2T_1 + 0.5g = 0.5(0)$
$-T_2 + 2(1.47) + 0.5g = 0.5(0)$
$T_2 = 7.84\ N$

b) Hint only have T_2 and 0.5g here, there is no tension T_1 in string when on the ground
Pulley is now on the ground so no tension T_1 now (string is slack)
R(↓): $-T_2 + 0.5g = 0.5(0)$
$-T_2 + +0.5g = 0.5(0)$
$T_2 = 4.9\ N$

iv. This is an easy SUVAT since we know the height that we started off the ground and we know the speed that p hit the ground so we don't need to find these first



Consider P
We don't need to do this since we know s and v already for P

Consider Q
S = S_Q
U = 1.4 (given)
V = 0
A = -9.8
T = T_B

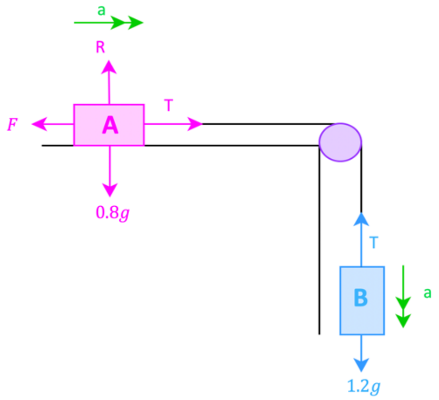
$v^2 = u^2 + 2as$
$0 = 1.4^2 + 2(-9.8)s$
$s = 0.1$

Greatest height:
$0.2 + 0.2 + 0.1 = 0.5$

We don't care about this motion since this is when the string becomes taut again

3.1.3 With SUVAT - Speed hits the pulley and total distance travelled

17)



Let's build our equations for each object (object A and object B) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider A: (we have to look at **2 directions now** since we have forces in the horizontal AND vertical direction)		Consider B: (we only look at the vertical direction since we only have forces in this direction)
Vertical:	Horizontal:	Vertical:
Take ↑ as positive	Take → as positive since moving right. This means every force going to the right is a positive sign and every force going to the left is a negative sign	Take ↓ as positive since moving downwards. This means every force going downwards is a positive sign and every force going upwards is a negative sign
There is no acceleration (hence $a = 0$) in this direction since the motion is horizontal		
$\uparrow: R - 0.8g = 0.8(0)$	$\rightarrow: T - F = 0.8a$	$\downarrow: -T + 1.2g = 1.2a$
$R = 0.8g$ (1)	$T = 0.8a + F$ (2)	$T = 1.2g - 1.2a$ (3)

i.

We had the following equations

$$R = 0.8g \quad (1)$$
$$T = 0.8a + F \quad (2)$$
$$T = 1.2g - 1.2a \quad (3)$$

We have too many unknowns, but we have been given enough info to use SUVAT first since told after release, B descends a distance of 0.9m in 0.8s.

S= 0.9
U= 0
V=
A=
T= 0.8

$$s = ut + \frac{1}{2}at^2$$

$$0.9 = 0 + \frac{1}{2}a(0.8)^2$$

$$0.9 = 0.32a$$

$$a = 2.8125$$

$$2.81ms^{-2}$$

ii.

sub a into ③

$$T = 1.2g - 1.2(2.8125) = 8.385$$

iii.

sub T and a into ②

$$8.385 = 0.8(2.8125) + F$$

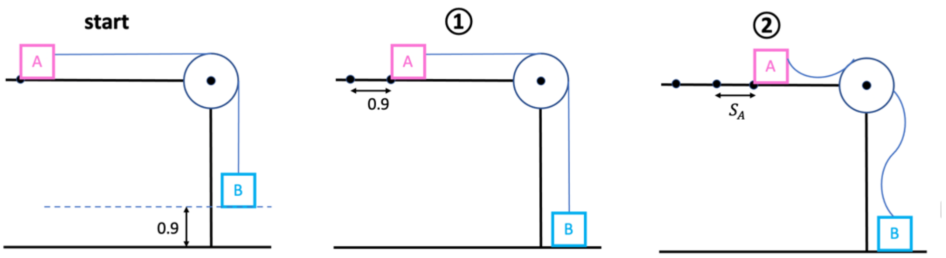
$$8.385 = 2.25 + F$$

$$F = 6.135$$

iv.

Sphere B is 0.9 m above the ground when the system is released. Given that A does not reach the pulley and the frictional force remains constant throughout,

i. find the **total distance** travelled by A (ans=0.33+0.9=1.23m)



①

②

Total distance= $0.9 + 0.33 = 1.23\ m$

Consider B

S = 0.9

U = 0

V = v

A = 2.8125

T = 0.8

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2(2.8125)(0.9)$$

$$v^2 = 5.0625$$

$$v = 2.25$$

Consider A

S = S_A

U = 2.25

V = 0 (comes to rest)

A = -7.66875 (see * below)

T =

$$v^2 = u^2 + 2as$$

$$0^2 = 2.25^2 + 2(-7.66875)S_A$$

$$-5.0625 = -15.3375S_A$$

$$S_A = 0.33$$

*once the string went slack (i.e. once B hit the ground) we needed to find the new acceleration. This will not be due to gravity like for the vertical pulleys, since A is moving horizontally and gravity only acts vertically! We re-resolve to find the new acceleration.

We do what we did when we considered A horizontally last time, except we delete T since no tension in the strong

$$\rightarrow : \not{T} - F = 0.8a$$

Deleting T gives

$$-F = 0.8a$$

$$a = \frac{-F}{0.8}$$

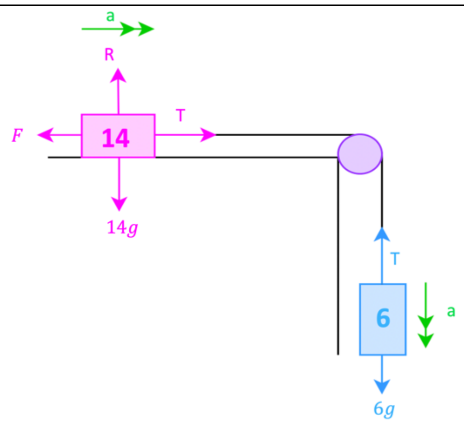
We know F=6.135 from part iii.

$$a = \frac{-6.135}{0.8}$$

$$a = -7.66875$$

3.1.4 With Friction (year 2 only)

18)



Consider the 14 kg mass
(we have to look at 2 directions since we have forces in the horizontal and vertical direction)

Vertical:
Take ↑ as positive
There is no acceleration in this direction since the motion is horizontal

$\uparrow: R - 14g = 5(0)$
$R = 14g$ ①

Horizontal
Take → as positive

$\rightarrow: T - F = 14a$
$T = 14a + F$ ②

Consider the 6 kg mass:
(we only look at the vertical direction since only have forces in this direction)

Vertical:
Take ↓ as positive

$\downarrow: -T + 6g = 6a$
$T = 6g - 6a$ ③

i.

We had the following equations

$R = 14g$ ①
$T = 14a + F$ ②
$T = 6g - 6a$ ③

We also have the equation: $F = \mu R = 0.25R = 0.25(14g) = 34.3$ ④

ii.

Let's sub ④ into ②

So ② becomes $T = 14a + 34.3$

Solve ② and ③ simultaneously:
$14a + 34.3 = 6g - 6a$
$20a = 24.5$
$a = 1.225\ ms^{-2}$

Sub a into ②
$T = 14(1.225) + 34.3 = 51.45\ N$

$T = 51.5\ N$

iii.

Consider the 14 kg mass
S=0.8
U= 0
V= v
A= 1.225
T
$v^2 = u^2 + 2as$
$v^2 = +2(1.225)(0.8) = 1.4$

iv.

Consider the 6 kg mass

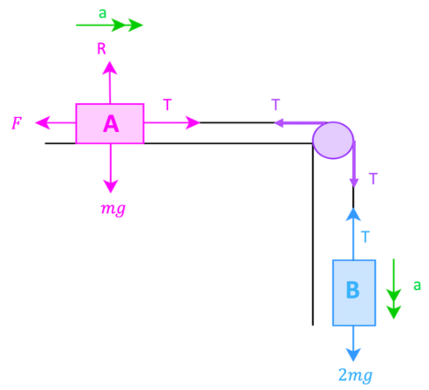
Way 1: Take down to be positive sense	Way 2: Take up to be positive sense
S=0.5	S= −0.5
U= 1.4	U= 1.4
V= v	V= v
A= 9.8 (since due to gravity)	A= −9.8
T=	T=
$v^2 = u^2 + 2as$	$v^2 = u^2 + 2as$
$v^2 = 1.4^2 + 2(9.8)(0.5)$	$v^2 = 1.4^2 + 2(-9.8)(-0.5)$
$v = 3.43\ ms^{-1}$	$v = 3.43\ ms^{-1}$

19)

Let's put all the common forces that exist for these types of questions (tension, weight and now friction) on a labelled diagram. Remember that weight is equal to **mass** × gravity and friction only exists if the surface is rough. Here we have a rough table and hence friction.

For your course our assumptions are that:

- the tensions are the same on both sides of the pulley (since pulley is smooth)
- the accelerations are the same on both sides of the pulley (since string is inextensible)



Let's build our equations for each object (object A and object B) before worrying about what the question is asking for. What they are asking for will come out if we build the equations correctly. We can ignore the purple tensions since they are acting on the pulley and we aren't needing to consider the pulley for this question (this is only when the questions talks about the forces exerted on the pulley).

Consider A
(we have to look at **2 directions now** since we have forces in the horizontal AND vertical direction)

Vertical:

Take ↑ as positive

There is no acceleration (hence $a = 0$) in this direction since the motion is horizontal

$\uparrow: R - mg = 2(0)$

$R = mg$ ①

Horizontal

Take → as positive since moving right. This means every force going to the right is a positive sign and every force going to the left is a negative sign

$\rightarrow: T - F = m\left(\frac{4}{9}g\right)$

$T = \frac{4}{9}mg + F$ ②

Consider B:
(we only look at the vertical direction since we only have forces in this direction)

Vertical:

Take ↓ as positive since moving downwards. This means every force going downwards is a positive sign and every force going upwards is a negative sign

$\downarrow: -T + 2mg = 2m\left(\frac{4}{9}g\right)$

$T = 2mg - \frac{8}{9}mg$

$T = \frac{10}{9}mg$ ③

ii.

We had the following equations

$R = mg$ ①

$T = \frac{4}{9}mg + F$ ②

$T = \frac{10}{9}mg$ ③

We also have a fourth equation: $F = \mu R$ ④

Sub ④ and ③ into ②

$T = \frac{4}{9}mg + F$ ②

$\frac{10}{9}mg = \frac{4}{9}mg + \mu R$

Now sub in ①

$\frac{10}{9}mg = \frac{4}{9}mg + \mu(mg)$

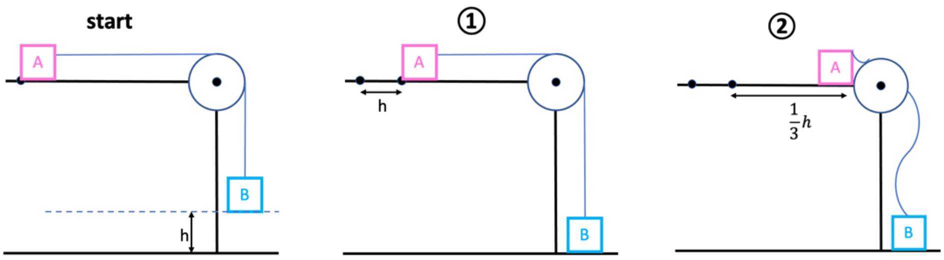
We now need to solve for μ

Cancel an m and g from all terms

$$\frac{10}{9} = \frac{4}{9} + \mu$$

$$\mu = \frac{10}{9} - \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$$

iii.



①

Consider B

S = h

U = 0

V = v

A = $\frac{4}{9}g$

T =

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2\left(\frac{4}{9}g\right)h$$

$$v^2 = \frac{8}{9}gh$$

$$v = \sqrt{\frac{8}{9}gh}$$

②

Consider A

S = $\frac{1}{3}h$

U = $\sqrt{\frac{8}{9}gh}$

V =

A = $-\frac{2}{3}g$ (see * below)

T =

$$v^2 = u^2 + 2as$$

$$v^2 = \frac{8}{9}gh + 2\left(-\frac{2}{3}g\right)\left(\frac{1}{3}h\right)$$

$$v^2 = \frac{8}{9}gh - \frac{4}{9}gh$$

$$v^2 = \frac{4}{9}gh$$

$$v = \sqrt{\frac{4}{9}gh}$$

$$v = \frac{2}{3}\sqrt{gh}$$

$$v = \frac{2}{3}\sqrt{gh}$$

*once the string went slack (i.e. once B hit the ground) we needed to find the new acceleration. This will not be due to gravity like for the vertical pulleys, since A is moving horizontally and gravity only acts vertically!
We re-resolve to find the new acceleration. We do what we did when we considered A horizontally last time, except we delete T since no tension in the string.

$$\rightarrow: \not{T} - F = m(a)$$

Deleting T gives

$$-F = ma$$

$$a = \frac{-F}{m}$$

Let's also use $F = \mu R = \frac{2}{3}R$

$$a = \frac{-\frac{2}{3}R}{m}$$

Let's also use $R = mg$ ①

$$a = \frac{-\frac{2}{3}(mg)}{m}$$

$$a = -\frac{2}{3}mg$$

iv.
same tension on both sides of pulley

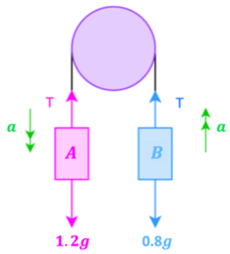
4 Diamond

4.1 Vertical

4.1.1 SUVAT Many Times (Hardest SUVAT Type)

20)

$1.2g > 0.8g$ so we know which way the system is moving (the heavier object moves down)



Consider A:
Take ↓ as positive since A is moving downwards
This means every force going downwards is a positive and every force going upwards is a positive
$\downarrow: -T + 1.2g = 1.2a$ ①

Consider B:
Take ↑ as positive since B is moving upwards
This means every force not going upwards is a positive and every force going downwards is a negative
$\uparrow: T - 0.8g = 0.8a$ ②

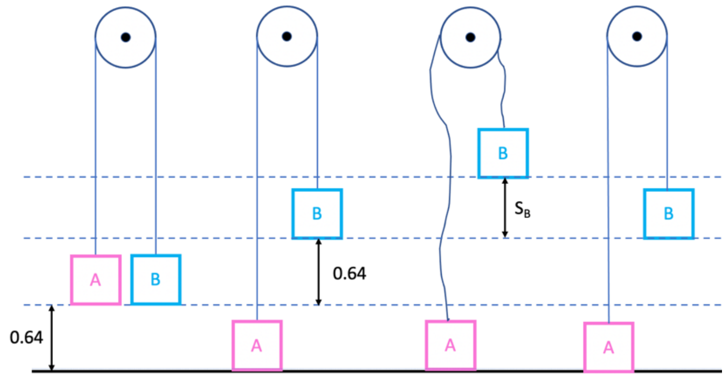
i. and ii.

Solve ① and ② simultaneously by re-arranging both for T

$T = 1.2g - 1.2a$
$T = 0.8a + 0.8g$

$1.2g - 1.2a = 0.8a + 0.8g$
$2a = 0.4g$
$a = 1.96\ ms^{-2}$

Sub back into $T = 1.2g - 1.2a$ $= 1.2g - 1.2(1.96) = 9.408\ N$



We need to know how long it takes to hit ground so that we know which motion parts to split up

Consider A
S = 0.64
U = 0
V = v
A = 1.96
T =

$v^2 = u^2 + 2as$
$v = 1.58$

Now we do SUVAT again
$v = u + at$
$t = 0.806$

Travelled 0.64 m for 0.806 s and B did this too

Consider B
S =
U = 1.58
V = 0
A = -9.8
T = t

$v = u + at$
$t = 0.16$

Now we do SUVAT again
$v^2 = u^2 + 2as$
$s = 0.128$

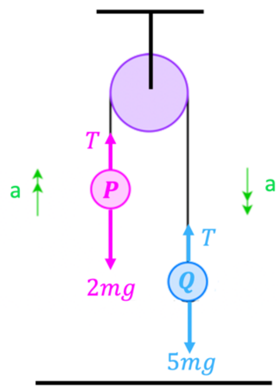
Travelled 0.128 m for 0.16 s

Consider B
S =
U = 1.58
V = 0
A = $+9.8$ (plus since looking at down motion only and take down as positive)
T = $1 - 0.806 - 0.16 = 0.034$ (this will find us the remaining time until first second)

$s = ut + \frac{1}{2}at^2$
$s = 0.008$

$0.64 + 0.128 + 0.008 = 0.776$

21)



i. and ii.

Consider P:

Take ↑ as positive since A is moving upwards
This means every force going upwards is a positive and every force going upwards is a negative

Using template $F = ma$ we get

$\uparrow : T - 2mg = 2ma$ ①

Consider Q:

Take ↓ as positive since B is moving downwards
This means every force going upwards is a negative and every force going downwards is a positive

Using template $F = ma$ we get
$\downarrow : -T + 5mg = 5ma$ ②

iii.

$T - 2mg = 2ma$ ①
$-T + 5mg = 5ma$ ②

Let's re-arrange both for T and set them equal

$T = 2ma + 2mg$ ①
$T = 5mg - 5ma$ ②

$$2ma + 2mg = 5mg - 5ma$$

Cancel the $m's$ from each term

$$2a + 2g = 5g - 5a$$

$$7a = 3g$$

$$a = \frac{3}{7}g = 4.2$$

First we consider Q to find v, since the speed Q hits the ground is the starting speed for P

Now use SUVAT to get h

S $= h$
U $= 0$
V $=$ v
A $= 4.2$ (looking at downwards motion only so accel is positive)
T $= t$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2(4.2)h$$

$$v^2 = 8.4\,h$$

$$v = \sqrt{8.4h}$$

Once Q hits the ground, P moves up a bit more since the string is slack and allows P to move up a bit. P then reaches its greatest speed and comes to rest.

Next we consider P

S $= s$
U $= \sqrt{8.4\,h}$
V $= 0$ (comes to rest)
$a = -9.8$ (string slack so accel is due to gravity)
T $= t$

$$v^2 = u^2 + 2as$$

$$0^2 = \left(\sqrt{8.4\,h}\right)^2 + 2(-9.8)s$$

$$s = \frac{8.4\,h}{2(9.8)} = \frac{3}{7}h$$

Total height = height originally off the ground + distance p moves (since Q moves the same distance) + extra distance Q moves

$$2h + h + \frac{3}{7}h$$

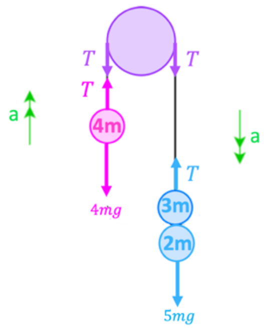
$$= \frac{24}{7}h$$

i. The distance that Q falls to the ground is not exactly h

ii. Inextensible ⟹ acceleration is the same on both sides of the pulley, but in reality the accelerations of P and Q would not have the same magnitude

4.1.2 More Than 1 Object On Either Side

22)



a) As usual consider the objects separately on each side

Consider 4 m force (left side)	Consider 3m and 2m force (right side)
$\uparrow: T - 4mg = 4ma$ $T = 4ma + 4mg$	$\downarrow: -T + 3mg + 2mg = (2+3)ma$ $T = 5mg - 5ma$ This should not be confusing. I had a question like this on my sheet and also it is just like cars/trailers.

$$4ma + 4mg = 5mg - 5ma$$

Cancel m (appears in entire equation)

$$4a + 4g = 5g - 5a$$

Group like terms to get 'a' on its own

$$g = 9a$$

$$a = \frac{g}{9}$$

Sub back in into either original equation

$$T = 4m\left(\frac{g}{9}\right) + 4mg$$

$$T = \frac{4mg}{9} + \frac{36mg}{9}$$

$$T = \frac{40mg}{9}$$

b) acceleration is the same on both sides of the pulley

c)
Don't get lost in the words! These questions are ALWAYS the same thing. Doing SUVAT on object hitting the ground to get ending speed and then doing SUVAT on object going towards pulley using the ending speed from object hitting the ground as the starting speed.

Do SUVAT first on object hitting the ground (3m and 2m)
S=d
U=0
V=0
A=−9.8
T=

$$v^2 = u^2 + 2as$$
$$v^2 = u^2 + 2\left(\frac{g}{9}\right)(d)$$
$$v^2 = \frac{2gd}{9}$$
$$v = \sqrt{\frac{2gd}{9}}$$

Do SUVAT for object going towards pulley (4m).
Remember ending speed for last part of motion is starting speed for next part, like for all questions hence we know $'u'$ now.
We usually have acceleration due to gravity once the string goes slack, but since the objects have not hit the ground the string is not slack and hence we STILL have an accel in the system (but a different one since one object is gone hence a force is gone). This is just like horizontal pulleys where we re-resolve again to get the new accel if it is not due to gravity. This is always the case as I mentioned. Re-resolve to get new 'a' if acceleration due to gravity. So, we do the exact same thing as in A, but without the 2m force since it is gone. Usually we re-resolve without the tension, but here the string is not slack since B has not hit the ground, only one object has been removed so we just discount the 2mg weight. Hence accel not due to gravity and we re-resolve to get 'a' just like we do with horizontal Pulleys. We did mention it with horizontal pulleys that we might have to re-arrange for a new accel in system and anytime accel is not due to gravity then re-resolve, so you should really have done this.

Consider 4 m force	Consider 3m and 2m force
$\uparrow: T - 4mg = 4ma$ $T = 4ma + 4mg$	Consider 3m and 2m force: $\downarrow: -T + 3mg \cancel{+ 2mg} = (\cancel{2} + 3)ma$ $-T + 3mg = 3ma$ $T = 3mg - 3ma$

$$4ma + 4mg = 3mg - 3ma$$
$$mg = -7ma$$
$$g = -7a$$
$$a = -\frac{g}{7}$$

Now we are ready to do SUVAT
S=s
U=$\sqrt{\frac{2gd}{9}}$
V=0 (comes to rest)
A=
T=

$$v^2 = u^2 + 2as$$
$$0^2 = \left(\sqrt{\frac{2gd}{9}}\right)^2 + 2\left(-\frac{g}{7}\right)s$$

$$0 = \frac{2gd^2}{9} - \frac{2gs}{7}$$

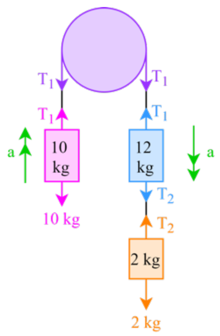
$$\frac{2gs}{7} = \frac{2gd}{9}$$

$$\frac{2s}{7} = \frac{2d}{9}$$

$$14d = 18s$$

$$s = \frac{7d}{9}$$

23)



Consider 10 kg weight:
Take ↑ as positive since moving upwards
↑: $T_1 - 10g = 10a$ (1)

Consider 12 kg weight:
Take ↓ as positive since moving downwards
↓: $-T_1 + T_2 + 12g = 12a$ (2)

(we won't use this equation since has 2 unknowns in it)

Consider 2 kg weight:
Take ↓ as positive since moving downwards
↓: $-T_2 + 2g = 2a$ (3)

Consider 12 kg and 2 kg weight:
Take ↓ as positive since moving downwards
↓: $-T_1 - T_2 + T_2 + 2g + 12g = 14a$
$-T_1 + 14g = 14a$ (4)

Solve (1) and (4) simultaneously
$T_1 - 10g = 10a \Longrightarrow T_1 = 10a + 10g$
$-T_1 + 14g = 14a \Longrightarrow T_1 = -14a + 14g$

$10a + 10g = -14a + 14g$
$24a = 4g$
$a = 1.63\ ms^{-2}$

Sub into $T_1 = 10a + 10g$
$T_1 = 10(1.63) + 10g = 114.3\ N$

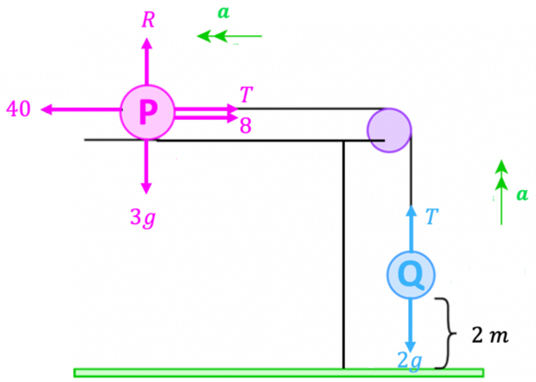
$-T_2 + 2g = 2a$
$-T_2 + 2g = 2(1.63)$
$T_2 = 16.34\ N$

4.2 Horizontal

4.2.1 SUVAT Many Times (Hardest SUVAT Type)

24)

This is different to most questions as we move AWAY from the pulley, not towards



Consider object P:

(we have to look at 2 directions since we have forces in the horizontal and vertical direction)

Consider object Q:

(we only look at the vertical direction since only have forces in this direction)

P Vertical:	P Horizontal	Q Vertical:
Take ↑ as positive There is no acceleration in this direction since the motion is horizontal $\uparrow: R - 3g = 3(0)$ $R = 3g$ ①	Take ← as positive since moving to the left now $\leftarrow: -T - 8 + 40 = 3a$ $T = -3a + 32$ ②	Take ↑ as positive since moving upwards $\downarrow: T - 2g = 2a$ $T = 2a + 2g$ ③

i.

We had the following equations

$R = 3g$ ①

$T = -3a + 32$ ②

$T = 2a + 2g$ ③

Let's set ② and ③ equal

$$-3a + 32 = 2a + 2g$$

$$5a = 32 - 2g$$

$$a = 2.48\ ms^{-2}$$

ii.

sub a into $T = 2a + 2g$ ③

$$T = 2(2.48) + 2g = 24.56\ N$$

iii.

<table>
<tr><th>Way 1:</th><th>Way 2: Longer</th></tr>
<tr><td>

Consider P
S=
U= 0
V= v
A= 2.48
T= 0.5

$v = u + at$
$v = 0 + 2.48(0.5) = 1.24$

$s = ut + \frac{1}{2}at^2$
$s = (0)(0.5) + \frac{1}{2}(2.48)(0.5)^2 = 0.31$

Now string breaks

Consider Q

Q has moved up by what P moved to the left and now needs to move down again. Don't forget that it was already $2\ m$ off the ground before P even more so we need to add this on

<table>
<tr><th>Way 1: Take down to be positive sense</th><th>Way 2: Take up to be positive sense</th></tr>
<tr><td>S= $2 + 0.31 = 2.31$
U= -1.24
V= v
A= 9.8 (due to gravity)
T=

$s = ut + \frac{1}{2}at^2$

$2.31 = (-1.24)t + \frac{1}{2}(9.8)t^2$

$t = 0.825, -0.572$

t cant be negative

$t = 0.825$</td><td>S= $-(2 + 0.31) = -2.31$
U= 1.24
V= v
A= -9.8 (due to gravity)
T=

$s = ut + \frac{1}{2}at^2$

$-2.31 = 1.24t + \frac{1}{2}(-9.8)t^2$

$t = 0.825, -0.572$

t cant be negative

$t = 0.825$</td></tr>
</table>

</td><td>

Consider P
S=
U= 0
V= v
A= 2.48
T= 0.5

$v = u + at$
$v = 0 + 2.48(0.5) = 1.24$

$s = ut + \frac{1}{2}at^2$
$s = (0)(0.5) + \frac{1}{2}(2.48)(0.5)^2 = 0.31$

Consider Q
Let's find how much more Q moves up

S= s
U= 1.24
V= 0
A= -9.8 (due to gravity)
T=

$v^2 = u^2 + 2as$

$0^2 = 1.24^2 + 2(-9.8)s$

$s = 0.0784$

Let's find how long it takes Q to come to rest (at the top and again when it has hit the ground)

Take downwards to be positive

S= $2 + 0.31 + 0.0784 = 2.3884$
U= 0
V=
A= 9.8 (due to gravity)
T=

$s = ut + \frac{1}{2}at^2$

$2.3884 = 0 + \frac{1}{2}(9.8)t^2$

$t = \pm 0.698$
$t \geq 0$, so
$t = 0.698$

But we also need to add on the time, t', it takes for Q to decelerate to 0 at its apex.

</td></tr>
</table>

	U= 1.24 V= 0 A= -9.8 (due to gravity) T= $v = u + at'$ $0 = 1.24 - 9.8t'$ $t' \approx 0.1265$ The total time is $0.1265 + 0.698 \approx 0.825$

iv.

Consider Q

S= $-(2 + 0.31) = -2.31$
U= 1.24
V=
A= -9.8 (due to gravity)
T=

$$v^2 = u^2 + 2as$$

$$v^2 = 1.24^2 + 2(-9.8)(-2.31)$$

$$v = 6.84\ ms^{-1}$$

v.

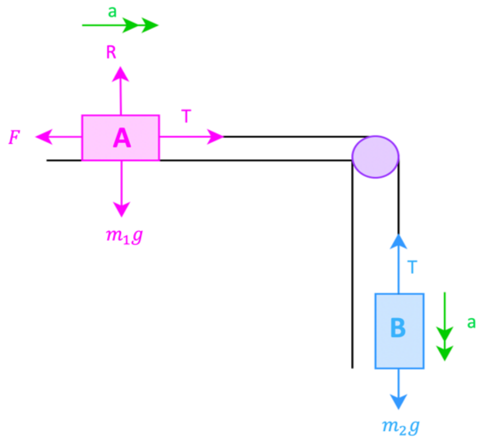
$R - 2g = 2(0)$
$R = 19.6$

vi.

- Include a more accurate value for g
- Include a variable resistance in the model rather than a constant
- Include the dimension of the pulley in the model so that the string is not parallel to the table
- Include a frictional force at the pulley

4.2.2 Harder Algebra

25)



We have $m_2 > \mu m_1$, meaning that the pull of B is larger than friction at A, so the system is in motion and B is going down.

Consider object A:

(we have to look at 2 directions since we have forces in the horizontal and vertical direction)

Consider object B:

(we only look at the vertical direction since only have forces in this direction)

Vertical:	Horizontal	Vertical:
Take ↑ as positive	Take → as positive	Take ↓ as positive
There is no acceleration in this direction since the motion is horizontal	→: $T - F = m_1 a$	↓: $m_2 g - T = m_2 a$
↑ : $R - m_1 g = m_1(0)$	We take $F = \mu R = \mu m_1 g$	$T = m_2 g - m_2 a$ ③
$R = m_1 g$ ①	$T = m_1 a + \mu m_1 g$ ②	

We had the following equations

$R = m_1 g$ ①

$T = m_1 a + \mu m_1 g$ ②

$T = m_2 g - m_2 a$ ③

Let's set ② and ③ equal

$$m_1 a + \mu m_1 g = m_2 g - m_2 a$$

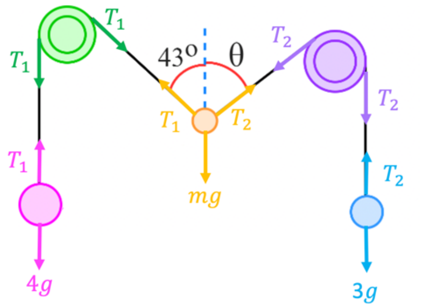
$$m_1 a + m_2 a = m_2 g - \mu m_1 g$$

$$a(m_1 + m_2) = m_2 g - \mu m_1 g$$

$$a = \frac{g(m_2 - \mu m_1)}{m_1 + m_2} ms^{-2}$$

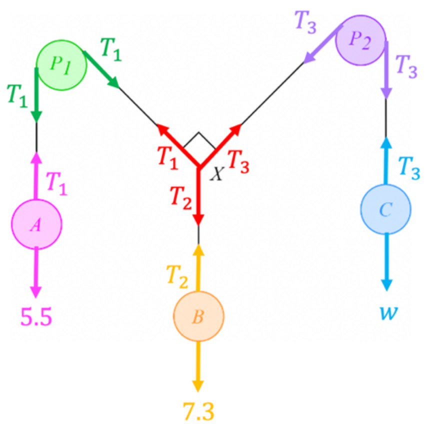
4.3 Vertical - 2 Pulleys and Diagonal Forces (year 2 only)

26)



Consider pink	Consider blue	Consider orange
$\uparrow: T_1 - 4g = 4(0)$ $T_1 = 39.2$	$\uparrow: T_2 - 3g = 3(0)$ $T_2 = 3g = 29.4$	$\rightarrow: -T_1 \sin 43 + T_2 \sin\theta = m(0)$ $-39.2 \sin 43 + 29.4 \sin\theta = m(0)$ $\sin\theta = 0.90933$ $\theta = 65.4°$ $\uparrow: T_1 \cos 43 + T_2 \cos\theta - mg = m(0)$ $39.2 \cos 43 + 29.4 \cos 65.4 - mg = m(0)$ $mg = 40.908$ $m = 4.17\ kg$

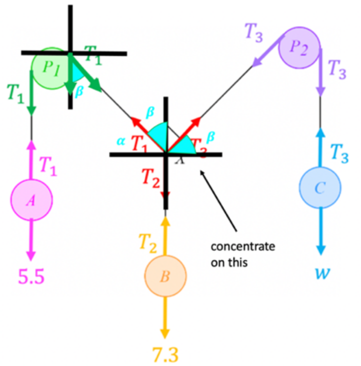
27)



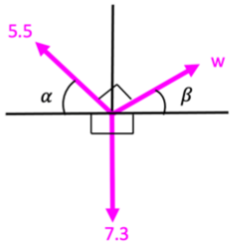
First of all, we need to find the tensions first

Consider A	Consider B	Consider C
$T_1 - 5.5 = 5.5(0)$ $T_1 = 5.5$	$T_2 - 7.3 = 7.3(0)$ $T_2 = 7.3$	$T_3 - w = w(0)$ $T_3 = w$

Let's concentrate on the red forces below



Way 1: Resolving (best method)



R(→): $w\cos\beta - 5.5\cos\alpha = 0$

$w\cos\beta = 5.5\cos\alpha$ ①

R(↑): $w\sin\beta + 5.5\sin\alpha - 7.3 = 0$

$w\sin\beta = 7.3 - 5.5\sin\alpha$ ②

2 equations, 3 unknowns

We also know that all the angles add to 360°

$90 + \alpha + 90 + \beta + 90 = 360$

$\alpha + \beta = 90$

$\alpha = 90 - \beta$

① becomes $w\cos\beta = 5.5\cos(90 - \beta)$

$w\cos\beta = 5.5\sin\beta$ ③

② becomes

$w\sin\beta = 7.3 - 5.5\sin(90 - \beta)$

$w\sin\beta = 7.3 - 5.5\cos\beta$ ④

Solve simultaneously ③ and ④

④ ÷ ③ :

$\frac{w\sin\beta}{w\cos\beta} = \frac{7.3 - 5.5\cos\beta}{5.5\sin\beta}$

$\frac{\sin\beta}{\cos\beta} = \frac{7.3 - 5.5\cos\beta}{5.5\sin\beta}$

$5.5\sin\beta\frac{\sin\beta}{\cos\beta} = 7.3 - 5.5\cos\beta$

$5.5\sin^2\beta = \cos\beta(7.3 - 5.5\cos\beta)$

$5.5\sin^2\beta = 7.3\cos\beta - 5.5\cos^2\beta$

$5.5(\sin^2\beta - \cos^2\beta) = 7.3\cos\beta$

$5.5(1) = 7.3\cos\beta$

$\cos\beta = \frac{5.5}{7.3}$

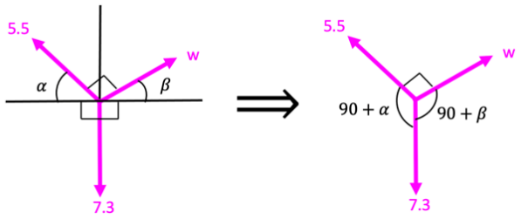
$\beta = \cos^{-1}\left(\frac{5.5}{7.3}\right) = 41.1°$

$③^2 + ④^2$:

$(w\cos\beta)^2 + (w\sin\beta)^2$

$= (5.5\sin\beta)^2 + (7.3 - 5.5\cos\beta)^2$

Way 2: Lami's Method



$$\frac{5.5}{\sin(90+\beta)} = \frac{7.3}{\sin 90} = \frac{w}{\sin(90+\alpha)}$$

Let's use $\frac{7.3}{\sin 90}$ in both equations since no unknown in here

$\frac{5.5}{\sin(90+\beta)} = \frac{7.3}{\sin 90}$

$5.5\sin 90 = 7.3\sin(90 + \beta)$

$5.5(1) = \sin 90 cos\beta + \cos 90\sin\beta$

$5.5 = 7.3\cos\beta$

$\cos\beta = \frac{5.5}{7.3}$

$\beta = 41.1°$

$\frac{w}{\sin(90+\alpha)} = \frac{7.3}{\sin 90}$

$w\sin 90 = 7.3\sin(90 + \alpha)$

$w(1) = 7.5(\sin 90 cos\beta + \cos 90\sin\beta)$

$w = 7.3\, cos\alpha$ ①

Note: we know the sum of the angle is 360°

$90 + \alpha + 90 + \beta + 90 = 360$

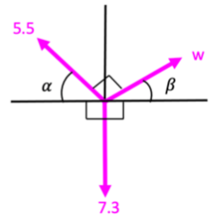
$\alpha + \beta = 90$

$\alpha + 41.4 = 90$

$\alpha = 48.6$

① becomes $w = 7.3\cos(48.6) = 4.8$

Way 3: Vector Triangle



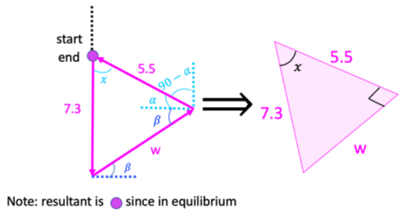
All angles add to 360°

$90 + \alpha + 90 + \beta + 90 = 360$

$\alpha + \beta = 90$

(so we have a right angled triangle)

we can build a vector triangle



We can use SOHCAHTOA

$cos\, x = \frac{5.5}{7.3}$

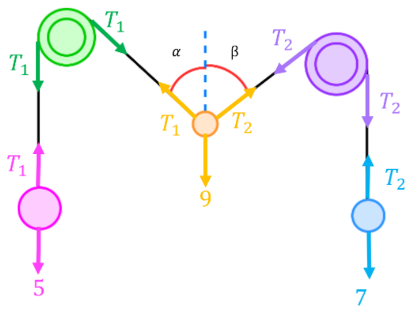
$x = 41.1$

$sin\, 41.1 = \frac{w}{7.3}$

$w = 4.82$

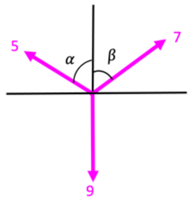
We simplify both sides $w^2\cos^2\beta + w^2\sin^2\beta =$ $30.25\sin^2\beta + 53.29 -$ $80.3\cos\beta + 30.25\cos^2\beta$ $w^2 = 30.25(1) + 53.29 -$ $80.3\cos\beta$ $w^2 = 83.59 - 80.3\cos\beta$ $w^2 = 83.59 - 80.3\cos 41.1$ $w^2 = 23.3$ $w = 4.8$ angle $AP_1X = \beta = 41.1°$		

28)



Consider pink	Consider blue
$\uparrow: T_1 - 5 = \frac{5}{g}(0)$ $T_1 = 5$	$\uparrow: T_2 - 7 = \frac{7}{g}(0)$ $T_2 = 7$

Way 1: Resolving (best method)	Way 2: Lami's Method	Way 3: Vector Triangle



R(→): $7\sin\beta - 5\sin\alpha = 0$ ①

R(↑): $7\cos\beta + 5\cos\alpha - 9 = 0$ ②

2 equations, 2 unknowns

① becomes $7\sin\beta = 5\sin\alpha$ ③

② becomes $7\cos\beta = 9 - 5\cos\alpha$ ④

③^2 + ④^2 :

$$(7\sin\beta)^2 + (7\cos\beta)^2 = (5\sin\alpha)^2 + (9 - 5\cos\alpha)^2$$

$$49\sin^2\beta + 49\cos^2\beta = 25\sin^2\alpha + 81 - 90\cos^2\alpha + 25\cos^2\alpha$$

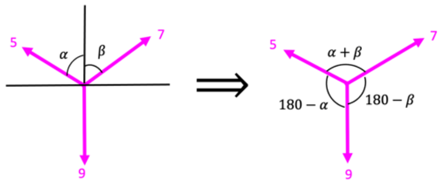
$$49(\cos^2\beta + \sin^2\beta) = 25(\sin^2\alpha + \cos^2\alpha) + 81 - 90\cos\alpha$$

$$49(1) = 25(1) + 81 - 90\cos\alpha$$

$$\cos\alpha = \frac{57}{90}$$
$$\alpha = 50.7 \approx 51°$$

We can plug into ② now:

$$7\cos\beta + 5\cos\alpha - 9 = 0$$
$$7\cos\beta + 5\left(\frac{57}{90}\right) - 9 = 0$$
$$\cos\beta = \frac{5}{6}$$
$$\beta = 33.6$$
$$\beta \approx 34°$$



$$\frac{5}{\sin(180-\beta)} = \frac{9}{\sin(\alpha+\beta)} = \frac{7}{\sin(180-\alpha)}$$

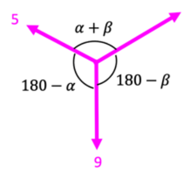
This is harder than the example above since we have less information about the angles and can't form an equation based on the sum of the angles (the sum of the angles is already given so we're missing a whole equation)

We have $\sin(180 - x) = \sin x$ (think about the graph)

$$\frac{5}{\sin\beta} = \frac{9}{\sin(\alpha+\beta)} = \frac{7}{\sin\alpha}$$

Fundamentally this doesn't have enough equations. Way 1 had 2 equations and 2 unknowns

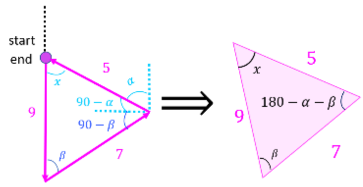
$$\frac{5}{\sin\beta} = \frac{9}{\sin\alpha\cos\beta - \cos\alpha\sin\beta} = \frac{7}{\sin\alpha}$$



All angles add to 360°

$$x + \beta + 180 - \alpha - \beta = 180$$
$$180 - \alpha + x = 180$$
$$\alpha = x$$

we can build a vector triangle



Note: resultant is ● since in equilibrium

By the cosine rule,

$$5^2 = 9^2 + 7^2 - 2(9)(7)\cos\beta$$
$$126\cos\beta = 105$$
$$\cos\beta = \frac{105}{126}$$
$$\beta \approx 34°$$

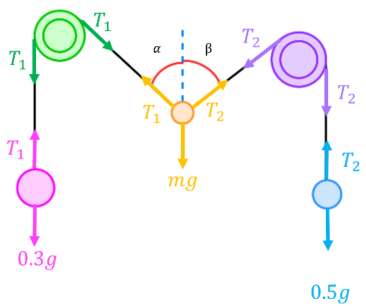
By the cosine rule again,

$$7^2 = 9^2 + 5^2 - 2(5)(9)\cos x$$
$$90\cos x = 57$$
$$\cos x = \frac{57}{90}$$
$$x \approx 51°$$

We know $\alpha = x$,

$$\alpha \approx 51°$$

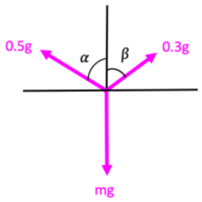
29)



Consider pink	Consider blue
↑: $T_1 - 0.3g = 0.3(0)$ $T_1 = 0.3g$	↑: $T_2 - 0.5g = 0.5(0)$ $T_2 = 0.5g$

i.

Way 1: Resolving (best method)



R(→): $0.3g\sin\beta - 0.5g\sin\alpha = 0$ ①

R(↑): $0.3g\cos\beta + 0.5g\cos\alpha - mg = 0$ ②

2 equations, 2 unknowns

① becomes $0.3g\sin\beta = 0.5g\sin\alpha$ ③

② becomes $0.3g\cos\beta = -0.5g\cos\alpha + mg$ ④

③^2 + ④^2 :

$(0.3g\sin\beta)^2 + (0.3g\cos\beta)^2 = (0.5g\sin\alpha)^2 + (-0.5g\cos\alpha + mg)^2$

$0.09g^2\sin^2\beta + 0.09g^2\cos^2\beta = 0.25g^2\sin^2\alpha + 0.25g^2\cos^2\alpha - mg^2\cos\alpha + m^2g^2$

$0.09g^2(\cos^2\beta + \sin^2\beta) = 0.25g^2(\sin^2\alpha + \cos^2\alpha) - mg^2\cos\alpha + m^2g^2$

$0.09g^2(1) = 0.25g^2(1) - mg^2\cos\alpha + m^2g^2$

$0.09 = 0.25 - m\cos\alpha + m^2$

We are told $m = 0.7$

$$0.09 = 0.25 - 0.7\cos\alpha + 0.7^2$$
$$\cos\alpha = \frac{13}{14}$$
$$\alpha = 21.8°$$

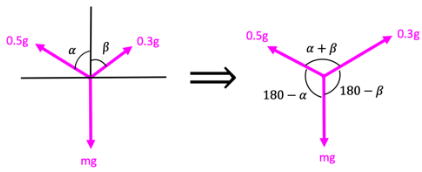
We can plug into ② now:

$$0.3g\cos\beta + 0.5g\cos\alpha - mg = 0$$

$$0.3g\cos\beta + 0.5g\left(\frac{13}{14}\right) - 0.7g = 0$$

$$\cos\beta = \frac{11}{14}$$
$$\beta \approx 38.2°$$

Way 2: Lami's Method



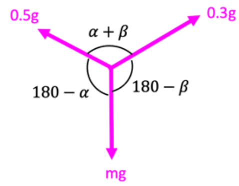
$$\frac{5}{\sin(180-\beta)} = \frac{9}{\sin(\alpha+\beta)} = \frac{7}{\sin(180-\alpha)}$$

We have $\sin(180 - x) = \sin x$ (think about the graph).

$$\frac{5}{\sin\beta} = \frac{9}{\sin(\alpha+\beta)} = \frac{7}{\sin\alpha}$$

Can't solve this

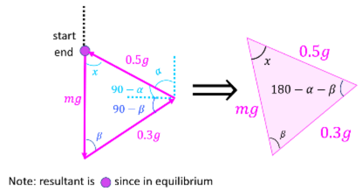
Way 3: Vector Triangle



All angles add to 360°

$$x + \beta + 180 - \alpha - \beta = 180$$
$$180 - \alpha + x = 180$$
$$\alpha = x$$

Now we can build the vector triangle



By the cosine rule,

$$(0.5g)^2 = (mg)^2 + (0.3g)^2 - 2mg0.3g\cos\beta$$
$$0.6mg^2\cos\beta = (m^2 - 0.16)g^2$$
$$\cos\beta = \frac{m^2 - 0.16}{0.6m} = \frac{11}{14}$$
$$\beta \approx 38.2°$$

Repeating the same thing for $\alpha = x$,

$$(0.3g)^2 = (mg)^2 + (0.5g)^2 - 2mg0.5g\cos\alpha$$
$$mg^2\cos\alpha = (m^2 + 0.16)g^2$$
$$\cos\alpha = \frac{m^2 + 0.16}{m} = \frac{13}{14}$$
$$\alpha \approx 21.8°$$

ii.

If using way 1:

We had previously that

$$0.09 = 0.25 - m\cos\alpha + m^2$$

This can be re-arranged

$$m^2 - \cos\alpha m + 0.16 = 0$$

$$b^2 - 4ac \geq 0 \text{ since } m \text{ is real}$$

$$(-cos\ \alpha)^2 - 4(1)(0.16) \geq 0$$

$$\cos^2 \alpha \geq 0.64$$

$$-0.8 \leq cos\ \alpha \leq 0.8$$

$$cos\ \alpha < 0.8$$

If using way 3:

the length of any one side of the triangle of forces cannot exceed the sum of the length of the other two sides.

The case $m = 0.8$ is excluded because the pulleys are not in the same vertical line

iii.

The easiest method is to use our vector triangle formula from above.

$$\cos \beta = \frac{m^2 - 0.16}{0.6m} = \frac{11}{14}$$

If we substitute $m = 0.4$, cos$\beta = 0$,$\beta = 90$, so the string at the right is horizontal.

iv. K cannot be above the level of the pulleys