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GCSE/iGCSE Maths Formulae Sheet

2D Shapes

Area of Triangle	$\frac{1}{2}$ x base x height
Area of Parallelogram	base x height
Area of Rectangle	$l \times w$
Area of Trapezoid	$\frac{1}{2} \times$ (sum of parallel sides) x height
Circumference & Area: Circle	$c = 2\pi r,\ A = \pi r^2$
Length of an arc	$\frac{\theta}{360} \times 2\pi r$
Area of a Sector	$\frac{\theta}{360} \times \pi r^2$

3D Shapes

Cuboid Surface area	$SA = 2xy + 2xz + 2yz$ where x, y , z are side lengths
Cuboid Volume	$V = xyz$ where x, y , z are side lengths
Cylinder Surface Area	$SA = 2\pi rh + 2\pi r^2$ Note: Curved part: $2\pi rh$
Cylinder Volume	$V = \pi r^2 h$
Cone Surface Area	$SA = \pi rl + \pi r^2$ Note: Curved part: πrl, l is slant length
Cone Volume	$V = \frac{1}{3}\pi r^2 h$
Sphere Surface Area	$SA = 4\pi r^2$ Note: Hemisphere $3\pi r^2$
Sphere Volume	$v = \frac{4}{3}\pi r^3$ Note: Hemisphere= $\frac{2}{3}\pi r^3$
Prism Volume	$V =$Area of cross section x height
Pyramid Volume	$V = \frac{1}{3} \times base\ area \times h$

Indices

Multiplication	$x^a \times x^b = x^{a+b}$ $(x^a)^b = x^{ab}$ $(cx^a y^b)^d = c^d x^{ad} y^{bd}$
Division	$x^a \div x^b = \frac{x^a}{x^b} = x^{a-b}$
Negative Powers	$x^{-n} = \frac{1}{x^n}$
Fractions	$\left(\frac{x}{y}\right)^{-n} = \frac{y^n}{x^n}$ and $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$
Fractional Powers	$a^{\frac{n}{m}} = \left(\sqrt[m]{a}\right)^n$

Percentages

One amount as a % of the other amount (wants answer as a %)	a as a percentage of b $\frac{a}{b} \times 100$ Look for the words as a percent of
Percentage gain/loss (wants answer as a %)	$\frac{\text{difference}}{\text{original}} \times 100$ Look for the words percentage gain/loss/increase/decrease
Find percentage of an amount	$\frac{\%}{100} \times$ amount
Given % of an amount, find the full amount	$\frac{\text{given amount}}{\frac{\%}{100}}$
Increasing/decreasing by a %	$\text{amount}\left(1 \pm \frac{\%}{100}\right)$ + if increase − if decrease
Given % of an amount after amount has been added or subtracted, find the full amount	$\frac{\text{Amount}}{1 \pm \frac{\%}{100}}$ + if increase − if decrease Look for the words originally, at the beginning, before...

Pyramid Method For Percentages:

Pink horizontal line means divide (÷)
blue line means multiply (×)

Cross off what you want and do the resulting operation (multiplication or division)
Note: we use the pyramid on the right if increasing/decreasing by an amount

Simple Interest (interest on initial amount)	amount + $\left(\text{amount} \times \frac{\%}{100} \times \text{time}\right)$ Interest = amount × $\frac{\%}{100}$ × time Note: Make sure t and % are same unit of time
Compound Interest (interest added also earns interest)	FV = PV$\left(1 + \frac{r}{100}\right)^t$ FV=future value, PV=present value t=time, r= interest rate

Quadratics

Quadratic Function: Solutions to $ax^2 + bx + c = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},\ a \neq 0$
Completing The Square $ax^2 \pm bx + c = 0$	$a\left(x \pm \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$
Max/Min Value	$c - \frac{b^2}{4a}$

Compound Measures

Speed	speed= $\frac{\text{distance}}{\text{time}}$
Density	density= $\frac{\text{mass}}{\text{volume}}$
Pressure	pressure= $\frac{\text{force}}{\text{area}}$

Pink horizontal line means divide (÷)
Green vertical line means multiply (×)

cross off what you want and do the resulting operation (multiplication or division)

Congruent Shapes

SSS (side side side)	Three sides of each triangle equal
SAS (side angle side)	Two sides and included angle equal
AAS (angle angle side)	Two angles and corresponding side equal
RHS (right hypotenuse leg)	Contains right angle and hypotenuse and another side equal

Direct/Indirect Proportion

y is ... proportional to x

Directly: $y = kx$, Inversely: $y = \frac{k}{x}$

Statistics

Frequency Density	 Frequency density = $\frac{\text{frequency}}{\text{class width}}$
Pie chart	Angle= $\frac{\text{category frequency}}{\text{total}} \times 360$
Cumulative frequency	This is a running total of the frequencies
Box Plot	

Fractions/Decimals/Percentages

Simplifying Fractions	Step 1: Find a factor of both numbers i.e. a number that fits in both the numerator AND denominator Step 2: Say how many times for each Step 3: Check whether you can do steps 1 and 2 again.
Fraction Of Amount	$\frac{a}{b}$ of amount Step 1: Divide amount by b Step 2: Multiply answer found by a
Improper to Mixed	Step 1: Divide the numerator by the denominator Step 2: Write down the whole number answer to step 1 Step 3: Put the remainder in the numerator. The new denominator remains the same as that of the original improper fraction.
Mixed to Improper	Step 1: Multiply the whole number by the fraction's denominator Step 2: Add the numerator to step 1 and this is the new numerator Step 3: write the result the top of the original denominator
$+$ *and* $-$ Fractions	Need a common denominator (the smallest number that that both the numerator and denominator fit into)
× Fractions	Don't need common denominator. Can cancel diagonally or vertically, not horizontally.
÷ Fractions	Don't need a common denominator. "Keep change flip"
Decimal to Fraction	Write over 10,100,1000 etc depending on how many places after the decimal and simplify.
Decimal to Percent	Multiply by 100
Fraction to Decimal	Write as an equivalent fraction over 10,100,1000 etc and then easy to divide by this number OR Use short division if can't write as an equivalent fraction
Fraction to Percent	Turn into a decimal and then just a decimal to percent question i.e. multiply decimal found by 100
Percent to Decimal	Divide by 100
Percent to Fraction	Write over 100 and simplify

Geometry

Straight Line Equation	• Slope intercept $y = mx + c$ • General $ax + by + d = 0$ To get this form we put all the terms from form 1 on one side and multiply all terms by the denominators to get rid of the fractions (if we have them)
Straight Line Gradient/Slope Between 2 Points $(x_1, y_1), (x_2, y_2)$	slope = $\frac{y_2 - y_1}{x_2 - x_1}$ OR $\frac{y_1 - y_2}{x_1 - x_2}$ In English this formula just says: subtract the y coordinates and divide by the answer we get by subtracting the x coordinates. It doesn't matter which way round we subtract, just so long as we keep the same direction
Coordinates of midpoint of 2 points $(x_1, y_1), (x_2, y_2)$	midpoint = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ In English this formula just says: Add the x coordinates and divide by 2 (i.e. find the average) and add the y coordinates and divide by 2 (i.e. find the average)
Distance Between 2 Points $(x_1, y_1), (x_2, y_2)$	distance = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

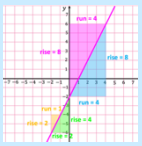
Methods to find the equation of a straight line

$y = mx + c$
gradient/slope, y intercept

We use the letter m to represent slope and c to represent y intercept. If we can find the gradient m and y intercept c then we are done.

Step 1: Find the **gradient** m using one of the following 5 ways

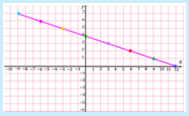
- **Way 1:** If given a graph ⟹ pick any 2 points on the graph and use $\frac{rise}{run}$



⟹

Using the pink triangle : $m = \frac{rise}{run} = \frac{8}{4} = 2$
Using the blue triangle. : $m = \frac{rise}{run} = \frac{8}{4} = 2$
Using the orange triangle. : $m = \frac{rise}{run} = \frac{2}{1} = 2$
Using the green triangle. : $m = \frac{rise}{run} = \frac{4}{2} = 2$

- **Way 2:** If given a graph ⟹ pick any 2 points on the graph and use the slope formula



⟹ The graph on the left has the following coordinates

(−9,7) (−6,6) (−3,5) (0,4) (3,3) (6,2) (9,1) (12,0)

Pick any pair of coordinates. Let's choose (−3,5) and (3,3)

Way 1: slope = $\frac{5-3}{-3-3} = \frac{2}{-6} = -\frac{1}{3}$

Way 2: slope = $\frac{3-5}{3-(-3)} = \frac{-2}{6} = -\frac{1}{3}$

- **Way 3:** If given another line parallel to ⟹ locate m for this line and use same slope

$y = 2x - 1$ We can spot this straight away gradient = 2	$y = x + 2$ This means the same as $y = 1x + 2$ gradient = 1	$y = -x + 4$ This means the same as $y = -1x + 4$ gradient = −1	$y = -2 + 3x$ Need to re-order this first since it is not in the form $y = mx + c$ $y = 3x - 2$ gradient = 3	$y = 2 - 4x$ Need to re-order this first $y = -4x + 2$ gradient = −4
$y + x = 4$ We need to use algebra to re-arrange $y = -x + 4$ This means the same as $y = -1x + 4$ gradient = −1	$y - 2x = 5$ We need to use algebra to re-arrange $y = 2x + 5$ gradient = 2	$2x + 4y = 5$ We need to use algebra to re-arrange $4y = -2x + 5$ $y = \frac{-2x+5}{4}$ We can split this up in order to separate the gradient and y intercept $y = -\frac{2}{4}x + \frac{5}{4}$ We can simplify the fraction $y = -\frac{1}{2}x + \frac{5}{4}$ gradient = $-\frac{1}{2}$	$5x - 2y = 7$ Need to re-order this first since it is not in the form $y = mx + c$ $-2y = -5x + 7$ $y = \frac{-5x+7}{-2}$ We can split this up in order to separate the gradient and y intercept $y = \frac{5}{2}x - \frac{7}{2}$ gradient = $\frac{5}{2}$	$x + 2y + 5 = 0$ We need to use algebra to re-arrange $2y = -x - 5$ $y = \frac{-x-5}{2}$ We can split this up in order to separate the gradient and y intercept $y = -\frac{1}{2}x - \frac{5}{2}$ gradient = $-\frac{1}{2}$

- **Way 4:** If given another line old perpendicular to ⟹ locate m for this line and don't use the same slope but instead "flip the fraction and change the sign" (this I just the fact that perpendicular slopes multiply to make −1.
  - If a line has slope 2 (**note: this means the same as** $\frac{2}{1}$) then a perpendicular slope is $-\frac{1}{2}$
  - If a line has slope $-\frac{2}{5}$ then a perpendicular slope is $\frac{5}{2}$
  - If a line has slope $\frac{1}{3}$ then a perpendicular slope is −3
- **Way 5:** If given 2 points ⟹ use formula $\frac{y_2 - y_1}{x_2 - x_1}$

Step 2: Find the **y intercept c** using one of the following 2 ways

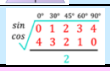
- **Way 1:** read it off the graph (if given graph this is where the graph crosses the y axis
- **Way 2:** plug the point given (x, y) into the equation (replace x with the x value and y with the y value).

$y = mx + c$

Make sure the slope m from step 1 is plugged in and solve/re-arrange for c using algebra. Make sure you plug in the point that the line passes through, not just any point.

Circles	$(x - a)^2 + (y - b)^2 = r^2$ centre (a, b), radius r

Right Angled Trigonometry

Pythagoras	$a^2 + b^2 = c^2$ if given hyp⟹subtract, if finding hyp⟹add
SOHCAHTOA	$\sin x° = \frac{opp}{hyp}$, $\cos x° = \frac{adj}{hyp}$, $\tan x° = \frac{opp}{adj}$
	
Exact Trig Values	

Non Right-Angled Trigonometry

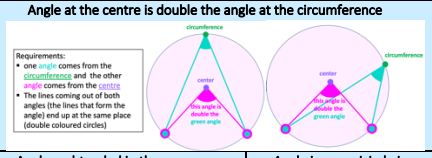
Sine Rule	Finding a side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Finding an angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Cosine Rule	Finding a side: $a^2 = b^2 + c^2 - 2bc\cos A$ Finding an angle: $A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$
Area of Triangle	$\frac{1}{2}absinC$
Sine Rule	Finding a side: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ Finding an angle: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Functions

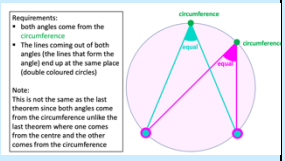
Inverse	Replace $f(x)$ with y, swap x & y, solve for y
Composite	$fg(x)$ means plug $g(x)$ into $f(x)$
Transformations $af(bx + c) + d$ "anything in a bracket affects x and does the opposite"	a=vertical stretch sf a, b=horizontal stretch sf $\frac{1}{b}$ c=translation c units x direction d=translation d units in y direction $f(-x)$=reflcn in y axis $-f(x)$=reflcn in x axis

Circle Theorems

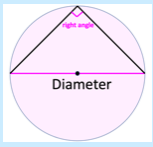
Angle at the centre is double the angle at the circumference



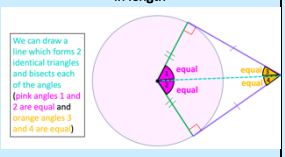
Angles subtended in the same segment by a chord are equal



Angle in a semicircle is a right angle



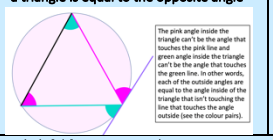
Tangents which meet at a point are equal in length



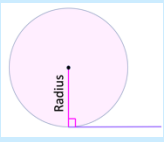
Opposite angles of a cyclic quadrilateral add to 180°

Watch out: All points need to be on the circumference!

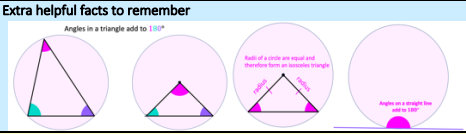
Alternate segment theorem: The angle between a tangent and a side of a triangle is equal to the opposite angle



A tangent meets a radius at 90°

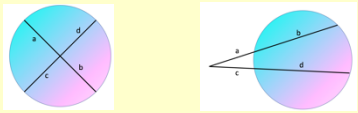


Extra helpful facts to remember



For two intersecting chords, the products of their diagonals are equal

$ab = cd$ $\quad a(a + b) = c(c + d)$



Series (iGCSE only)

Arithmetic sequence:	nth term: $u_n = a + (n - 1)d$ sum of n terms $S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$ a =first term, d= common diff, l=last term
Geometric sequence:	$u_n = ar^{n-1}$ $S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}$, $r \neq 1$ where a =first term, r= common ratio

Differentiation (iGCSE only)

Rule	$x^n \Rightarrow nx^{n-1}$ Remember: Constants go to 0
Turning/Stationary Points (Max/Min)	Solve $\frac{dy}{dx} = 0$
Proving whether Max/Min	Use knowledge of shape of graph $+x^2$ happy face min $-x^2$ sad face max $+x^3$ max on left, min on right $-x^3$ min on left, max on right