The following 2 pages cover the basic technique that you need to know. Once you know the technique, the rest is easy! They do not cover product rule, quotient rules and implicit.

6 Basic Types

Type 1: Powers (year 1 recap)

Example 1

$y = x^3 + 5x^2 + 4x - 9$

Step 1: bring the power down to the front

Step 2: subtract one from the power

$y = 3x^{3-1} + 5(2)x^{2-1} + 4x^{1-1}$

Simplify

$\frac{dy}{dx} = 3x^2 + 10x + 4$

Example 2

$y = x(x+4)(x+2)$

Simplify first to get into the correct form to differentiate

$y = x(x^2 + 6x + 8)$

$y = x^3 + 6x^2 + 8x$

Now in the correct form to differentiate like in example 1

$\frac{dy}{dx} = 3x^2 + 12x + 8$

Example 3

$y = \frac{(x+4)(x-1)}{3\sqrt{x}}$

Simplify first to get into the correct form to differentiate

$y = \frac{x^2}{3x^{\frac{1}{2}}} + \frac{3x}{3x^{\frac{1}{2}}} + \frac{-4}{3x^{\frac{1}{2}}}$

$y = \frac{1}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}} - \frac{4}{3}x^{-\frac{1}{2}}$

Now in the correct form to differentiate like in example 1

$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} + \frac{2}{3}x^{-\frac{3}{2}}$

Type 2: Powers with a function inside (recognisable by a function inside a bracket raised to a power)

Example 4

$y = (3x-2)^5$

Let's colour code to explain

$y = (3x-2)^5$

Step 1: bring the power down to the front

Step 2: subtract one from the power, keep inside bracket the same

Step 3: multiply by the derivative of what is inside the bracket

Apply these 3 steps

Derivative of $3x - 2$

$\frac{dy}{dx} = 5(3x-2)^4(3)$

Re-order

$\frac{dy}{dx} = 5(3)(3x-2)^4$

Simplify

$= 15(3x-2)^4$

Example 5

$y = 3(5+x^2)^{\frac{3}{2}}$

Let's colour code to explain

$y = 3(5+x^2)^{\frac{3}{2}}$

Step 1: bring the power down to the front and multiply it with the number at the front (if a number exists which is does here)

Step 2: subtract one from the power, keep inside bracket the same

Step 3: multiply by the derivative of what is inside the bracket

Apply these 3 steps

Derivative of $5 + x^2$

$\frac{dy}{dx} = \left(\frac{3}{2}\right) \times 3\,(5+x^2)^{\frac{1}{2}}\,(2x)$

Re-order

$\frac{dy}{dx} = 3\left(\frac{3}{2}\right)(2)\,(x)(5+x^2)^{\frac{1}{2}}$

Simplify

$= 9x(5+x^2)^{\frac{1}{2}}$

Example 6

$y = \frac{5}{\sqrt{2-4x}}$

Firstly, we can bring the power up using indices rules and then it is just the harder powers type of differentiation shown in the two examples on the left

$\frac{dy}{dx} = 5(2-4x)^{-\frac{1}{2}}$

Now we can differentiate using our 3 steps on the left

$\frac{dy}{dx} = 5\left(-\frac{1}{2}\right)(2-4x)^{-\frac{3}{2}}(-4)$

$= 10(2-4x)^{-\frac{3}{2}}$

Note: we don't need quotient rule here since we just have a constant in the numerator

Type 3: Exponentials (recognisable by a base number raised to an algebraic power

Example 8

Type a: Base e to an unknown power

$y = 5e^{5x^2}$

Let's colour code to explain

$y = 5e^{5x^2}$

Step 1: copy the entire exponential (e to some unknown power)

Step 2: multiply by the derivative of the power

Note: don't worry about the 5 at the front, that is just hanging around at the front.

Apply these 2 steps

Derivative of $5x^2$

$\frac{dy}{dx} = 5e^{5x}(10x)$

Re-order

$\frac{dy}{dx} = 5(10x)e^{5x}$

Simplify

$\frac{dy}{dx} = 50xe^{5x}$

Example 11

Type b: Number other than base e to unknown power

$y = 2^{4x}$

Let's colour code to explain

$y = 2^{4x}$

Step 1: copy the entire exponential (number to some unknown power)

Step 2: multiply by the derivative of the power

Step 3: Multiply by ln of the base. Notice this extra purple part when we have an exponential which doesn't have a base of e. Here we have a base of 2.

Apply these 2 steps

Derivative of $4x$

$\frac{dy}{dx} = 2^{4x}(4)\ln 2$

Base is 2

Re-order

$\frac{dy}{dx} = 4\ln 2(2^{4x})$

Type 4: Natural log (recognised by ln)

Example 9

$y = \ln(3x+2)$

Let's colour code to explain

$y = \ln(3x+2)$

Step 1: This turn into a fraction

$\frac{?}{?}$

The ln completely disappears

Step 2: Fill in numerator and denominator

$\frac{derivative\ of\ argument}{copy\ of\ argument}$

Notice how the ln disappears

Apply these 2 steps

Derivative of $3x + 2$

$\frac{dy}{dx} = \frac{3}{3x+2}$

$y = 3\ln(x^2 + 3x + 5)$

Let's colour code to explain

$y = 3\ln(x^2 + 3x + 5)$

Step 1: This turns into a fraction

$\frac{?}{?}$

The ln completely disappears

Step 2: Fill in numerator and denominator

$\frac{derivative\ of\ argument}{copy\ of\ argument}$

Notice how the ln disappears

Note: don't worry about the 3 at the front, that is just hanging around at the front.

Apply these 2 steps

Derivative of $x^2 + 3x + 5$

$\frac{dy}{dx} = 3\left(\frac{2x+3}{x^2+3x+5}\right)$

$\frac{dy}{dx} = \frac{6x+9}{x^2+3x+5}$

Type 5 : Trigonometry

Example 10

$y = cos\,(3x)$

Let's colour code to explain

$y = cos\,(3x)$

Step 1: Change the trig function to what it is meant to become. Trig functions are a bit different. They change to different functions.

- $sin\,x \Rightarrow cos\,x$
- $cos\,x \Rightarrow -sin\,x$
- $tan\,x \Rightarrow sec^2\,x$
- $sec\,x \Rightarrow sec\,x\,tan\,x$
- $cosec\,x \Rightarrow -cosec\,x\,cot\,x$
- $cot\,x \Rightarrow -cosec^2x$

Watch out for the three functions that become negative

Notice how the angle stays the same

Step 2: multiply by the derivative of the angle

Apply these 2 steps

Here we have cos and $cos\,3x \Rightarrow -sin\,3x$:

Derivative of $3x$

$\frac{dy}{dx} = -\sin 3x\,(3)$

$= -3\sin 3x$

Type 5 : Trigonometry

Example 11

$y = sec\,(2x^3)$

Let's colour code to explain

$y = sec\,(2x^3)$

Step 1: Trig functions are a bit different. They change to different functions. Change the trig function to what it is meant to become:

- $sin\,x \Rightarrow cos\,x$
- $cos\,x \Rightarrow -sin\,x$
- $tan\,x \Rightarrow sec^2\,x$
- $sec\,x \Rightarrow sec\,x\,tan\,x$
- $cosec\,x \Rightarrow -cosec\,x\,cot\,x$
- $cot\,x \Rightarrow -cosec^2x$

Watch out for the three functions that become negative

Notice how the angle stays the same

Step 2: multiply by the derivative of the angle

Apply these 2 steps

Here we have sec and $sec\,2x^3 \Rightarrow -sec\,2x^3\,tan\,2x^3$:

Derivative of $2x^3$

$\frac{dy}{dx} = -\sec 2x^3 \tan 2x^3\,(6x^2)$

$= -6x^2 \sec 2x^3 \tan 2x^3$

Type 6: Inverse Trig (Further Maths Only)

Example 12

$y = \cos^{-1}(4x)$

Let's colour code to explain

$y = \sin^{-1}(4x)$

Step 1: These turn into a fraction, just like ln

$\frac{?}{?}$

The inverse trig completely disappears

Step 2 : Fill in numerator and denominator

Choose the first result since we have $\sin^{-1}$

- $sin^{-1}\,f(x) \Rightarrow \frac{derivative\ of\ angle}{\sqrt{1-(angle)^2}}$
- $cos^{-1}\,f(x) \Rightarrow \frac{-\,derivative\ of\ angle}{\sqrt{1-(angle)^2}}$
- $tan^{-1}\,f(x) \Rightarrow \frac{derivative\ of\ angle}{1+(angle)^2}$

Notice how the inverse trig disappears.

Derivative of $4x$

$\frac{dy}{dx} = \frac{4}{\sqrt{1-(4x)^2}}$

Combination of 2 types above (a type within a type)

Trig within a power

Example 1

$$y = (x + \sin 2x)^3$$

Here we have a mix of 2 types (harder power and trigonometry)

We deal with the harder power since that is the main function, but when we differentiate the angle which is part of the trig differentiation rule, we have to use our trig differentiation rule to do this

$$\frac{dy}{dx} = 3(x + \sin 2x)^2(1 + 2\sin 2x)$$

Exponential within a power

Example 2

$$y = (e^{4x} + 5)^6$$

Here we have a mix of 2 types (harder power and exponential)

We deal with the harder power since that is the main function, but when we differentiate inside the bracket which is part of the harder power differentiation rule, we have to use our exponential differentiation rule to do this

$$\frac{dy}{dx} = 6(e^{4x} + 5)^5(4e^{4x})$$

Exponential within a power

Example 3

$$y = \sqrt{e^{2x} + e^{-2x}}$$

Firstly, we need to write this as

$$y = (e^{2x} + e^{-2x})^{\frac{1}{2}}$$

Here we have a mix of 2 types (harder power and exponential)

We deal with the harder power since that is the main function, but when we differentiate inside the bracket which is part of the harder power differentiation rule, we have to use our exponential differentiation rule to do this

$$\frac{dy}{dx} = \frac{1}{2}(e^{2x} + e^{-2x})^{-\frac{1}{2}}(2e^{2x} + (-2)e^{-2x})$$

$$\frac{dy}{dx} = (e^{2x} - e^{-2x})(e^{2x} + e^{-2x})^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{e^{2x} - e^{-2x}}{\sqrt{e^{2x} + e^{-2x}}}$$

ln within a power

Example 4

$$y = (1 - \ln 2x)^3$$

Here we have a mix of 2 types (natural log and harder powers)

We deal with the harder power since that is the main function, but when we differentiate inside the bracket which is part of the harder power differentiation rule, we have to use our ln differentiation rule to do this

$$\frac{dy}{dx} = 3(1 - \ln 2x)^2\left(\frac{2}{2x}\right)$$

$$\frac{dy}{dx} = \frac{3}{x}(1 - \ln 2x)^2$$

Trig within a power

Example 5

$$y = \sin^3 4x$$

This is one that students so often **get wrong.** We have to first write the trig to a power in a more familiar way

$$y = \sin^3 4x = (\sin 4x)^3$$

Here we have a mix of 2 types (harder power and trig)

We deal with the harder power since that is the main function, but when we differentiate inside the bracket which is part of the harder power differentiation rule, we have to use our trig differentiation rule to do this

$$\frac{dy}{dx} = 3(\sin 4x)^2\,(4\cos 4x)$$

$$\frac{dy}{dx} = 12\cos 4x\,(\sin 4x)^2$$

Trig within an exponential

Example 6

$$y = e^{\cos x}$$

Here we have a mix of 2 types (exponential and trig)

We deal with the **exponential first** since that is the main function, but when we differentiate the power which is part of the exponential differentiation rule, we have to use our trig differentiation rule to do this

$$\frac{dy}{dx} = e^{\cos x}(-\sin x)$$

$$\frac{dy}{dx} = (-\sin x)e^{\cos x}$$

Trig of power

Example 7

$$y = \sin{(1 - 2x)}^3$$

Careful, this does not mean the whole trig function is squared, only the angle. $\sin^2 x = (\sin x)^2$

However, $\sin^2 x \neq \sin x^2$

Here we have a mix of 2 types (trig and harder power)

We deal with the trig since that is the main function, but when we differentiate the angle which is part of the trig differentiation rule, we have to use our harder power differentiation rule to do this

$$\frac{dy}{dx} = \cos(1 - 2x)^3\;(3)(1 - 2x)^2(-2)$$

$$\frac{dy}{dx} = -6(1 - 2x)^2\cos{(1 - 2x)^3}$$

Trig of ln

Example 8

$$y = \sin\,(\ln 2x)$$

Here we have a mix of 2 types (trig and harder power)

We deal with the trig since that is the main function, but when we differentiate the angle which is part of the trig differentiation rule, we have to use our trig differentiation rule to do this

$$\frac{dy}{dx} = \frac{\frac{2}{2x}}{\ln 2x}$$

$$\frac{dy}{dx} = \frac{\ln 2x}{x}$$

ln of trig

Example 9

$$y = \ln(sinx)$$

Here we have a mix of 2 types (log and trig)

We deal with the log first since that is the main function, but when we differentiate inside the argument part which is part of the log differentiation rule, we have to use our trig differentiation rules to do this

$$\frac{dy}{dx} = \frac{\cos x}{\sin x}$$

We can simplify this using a trig identity

$$\frac{dy}{dx} = \cot x$$

ln of a power

Example 10

$$y = \ln(1 - 2x)^3$$

Here we have a mix of 2 types (natural log and harder powers)

We deal with the log first since that is the main function, but when we differentiate inside the argument part which is part of the log differentiation rule, we have to use our harder power differentiation rules to do this

$$\frac{dy}{dx} = \frac{3(1 - 2x)^2(-2)}{(1 - 2x)^3}$$

$$\frac{dy}{dx} = \frac{-6(1 - 2x)^2}{(1 - 2x)^3}$$

ln of an exponential

Example 11

$$y = \ln(e^{2x} + 3x)$$

Here we have a mix of 2 types (natural log and harder powers)

We deal with the log first since that is the main function, but when we differentiate inside the argument part which is part of the log differentiation rule, we have to use our harder power differentiation rules to do this

$$\frac{dy}{dx} = \frac{2e^{2x} + 3}{e^{2x} + 3x}$$

ln of a fraction

Example 12

$$y = \ln\left(\frac{2x - 4}{x + 5}\right)$$

This can be written as two separate log terms

$$y = \ln(2x - 4) - \ln(x + 5)$$

Now the derivative of each log is

$$\frac{\textit{derivative of argument}}{\textit{copy of argument}} \text{ and } \frac{\textit{derivative of argument}}{\textit{copy of argument}}$$

$$\frac{dy}{dx} = \frac{2}{2x - 4} - \frac{1}{x + 5}$$

Combining the fractions,

$$\frac{dy}{dx} = \frac{2(x + 5) - 1(2x - 4)}{(2x - 4)(x + 5)}$$

Simplifying the numerator

$$\frac{dy}{dx} = \frac{14}{(2x - 4)(x + 5)}$$

Inverse Trig Of A Power (Further Maths only)

Example 13

$$y = 2arcsin\,\sqrt{1 - 2x}$$

We can write this as

$$y = 2\sin^{-1}(1 - 2x)^{\frac{1}{2}}$$

We deal with the inverse trig since that is the main function, but when we differentiate the angle which is part of the inverse trig differentiation rule, we have to use our harder power rule to do this

$$\frac{dy}{dx} = \frac{2\left(\frac{1}{2}\right)(1 - 2x)^{-\frac{1}{2}}(-2)}{\sqrt{1 - \left(\sqrt{1 - 2x}\right)^2}}$$

$$\frac{dy}{dx} = \frac{-\frac{2}{\sqrt{1 - 2x}}}{\sqrt{1 - (1 - 2x)}}$$

$$\frac{dy}{dx} = -\frac{2}{\sqrt{2x(1 - 2x)}}$$

Inverse Trig Within A Power (Further maths only)

Example 14

$$y = (arcsin\,x)^3$$

Here we have a mix of 2 types (harder power and inverse trigon)

We deal with the harder power since that is the main function, but when we differentiate the angle which is part of the inverse trig differentiation rule, we have to use our inverse trig differentiation rule to do this

$$\frac{dy}{dx} = 3(arcsin\,x)^2\left(\frac{1}{\sqrt{1 - x^2}}\right)$$

$$\frac{dy}{dx} = \left(\frac{3}{\sqrt{1 - x^2}}\right)(arcsin\,x)^2$$

Product rule - Combination of 6 basic types above multiplied

We use **product rule** when we have one of the 6 differentiation types (easy power, harder power, ln, exponential, trig, inverse trig) **MULTIPLIED together** (here we have an easy power and a harder power).
The formula for this is:

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

It is best to understand what the formula says in words:

Derivative = (copy 1st function)(differentiate 2nd function)+(differentiate 1st function)(copy 2nd function)

Example 1

$y = 2x(x^2-1)^5$

$y = 2x(x^2-1)^5$

Important: People often fall into the trap of not thinking that product rule is necessary here in this example, because they fail to realise that the easy power counts as a type so we have multiplication of 2 of the types (type 1 and type 2).

Way 1: Use the formula

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$y = 2x(x^2-1)^5$$

Let's call the first pink function u and the second blue function v

$$u = 2x, v = (x^2-1)^5$$

We differentiate each

$$\frac{du}{dx} = 2, \frac{dv}{dx} = 5(x^2-1)^4(2x)$$

Plug into the formula

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{dy}{dx} = 2x(5)(x^2-1)^4(2x) + (x^2-1)^5(2)$$

Way 2: Understand what formula is telling us

The formula basically says in English, differentiate one function at a time

(copy 1st function)(differentiate 2nd function)+(differentiate 1st function)(copy 2nd function)

$$y = 2x(x^2-1)^5$$

(copy 1st function)(differentiate 2nd function)+(differentiate 1st function)(copy 2nd function)

$$\frac{dy}{dx} = 2x(5)(x^2-1)^4(2x) + 2(x^2-1)^5$$

Simplify by re-ordering the constants from each term. Let's colour code to explain this.

$$\frac{dy}{dx} = 2x(5)(x^2-1)^4(2x) + (x^2-1)^5(2)$$

Pull the numbers and variables with less powers to the front of each term

$$\frac{dy}{dx} = 2x(5)(x^2-1)^4(2x) + (x^2-1)^5(2)$$

$$\frac{dy}{dx} = 20x^2(x^2-1)^4 + 2(x^2-1)^5$$

Example 2: Getting in a certain form

$y = (x+1)^4(2x-2)^5$. Show that $\frac{dy}{dx} = 2(9x+1)(x+1)^3(2x-2)^4$

$$\frac{dy}{dx} = (x+1)^4(5)(2x-2)^4(2) + (4)(x+1)^3(2x-2)^5$$

Simplify by multiplying constants and re-ordering.

$$\frac{dy}{dx} = (x+1)^4(5)(2x-2)^4(2) + (2x-2)^5(4)(x+1)^3$$

$$\frac{dy}{dx} = 10(x+1)^4(2x-2)^4 + 4(2x-2)^5(x+1)^3$$

To simplify further and get into the required form we must factorise by taking out what is common to both terms.

$$\frac{dy}{dx} = 10(x+1)^4(2x-2)^4 + 4(x+1)^3(2x-2)^5$$

Take out the HCF of the numbers
Take out the HCF of the pink terms (lowest power of each)
Take out the HCF of the blue terms (lowest power of each)

$$= 2(x+1)^3(2x-2)^4[5(x+1) + 2(2x-2)]$$

Notice how we subtracted the powers in order to get the powers of the terms inside the square bracket (or asked ourselves what power we need to add to the power we have outside the bracket to end up with the power we want)
Simplify what is inside the square bracket

$$= 2(x+1)^3(2x-2)^4[5x+5+4x-4]$$

Simplify again

$$= 2(x+1)^3(2x-2)^4(9x+1)$$

$$= 2(9x+1)(x+1)^3(2x-2)^4$$

Combination Of Any Of The 6 Basic Types Divided (Quotient Rule)

We use **quotient rule** when we have one of the 6 differentiation types (easy power, harder power, ln, exponential, trig, inverse trig) **DIVIDED together** (here we have an easy power and a harder power).

Just like for product rule, you're given a formula

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

In words:

$$\text{Derivative} = \frac{\text{copy denominator (differentiate numerator)} - \text{copy numerator (differentiate denominator)}}{\text{copy denominator}^2}$$

Tips:

- Once you have applied quotient rule work on the numerator. Do not expand first if you can factorise. Try to factorise out what is common first or if fractions in the numerator get a common denominator.
- Very unlikely we will need to expand the denominator unless it is trig and trying to get into a certain form
- We don't need quotient rule if the numerator is a constant. For example, $\frac{4}{(x-3)^2}$. Just bring the numerator up to give $4(x-3)^{-2}$.

  Even for things like $\frac{4x^2}{(x-3)^2}$ we can always bring the denominator up and use product rule (avoid quotient rule), but in order to get required forms it is better to work with quotient rule.

Example 1

$y = \frac{x^2-4x+12}{(x-3)^2}$.

Show that $\frac{dy}{dx} = -\frac{2(x+6)}{(x-3)^3}$

$$y = \frac{x^2-4x+12}{(x-3)^2}$$

Apply quotient rule

$$\frac{dy}{dx} = \frac{(x-3)^2(2x-4) - (x^2-4x+12)(2)(x-3)}{(x-3)^4}$$

Re-order: Bring the constant of 2 to the front of the second term in the numerator

$$\frac{dy}{dx} = \frac{(x-3)^2(2x-4) - 2(x-3)(x^2-4x+12)}{(x-3)^4}$$

Factorise

$$\frac{dy}{dx} = \frac{(x-3)^2(2x-4) - 2(x-3)(x^2-4x+12)}{(x-3)^4}$$

Take out the HCF of the pink terms in the numerator (lowest power)

$$\frac{dy}{dx} = \frac{(x-3)[(x-3)(2x-4) - 2(x^2-4x+12)]}{(x-3)^4}$$

Simplify what is inside the square bracket

$$\frac{dy}{dx} = \frac{(x-3)[2x^2-10x+12-2x^2+8x-24]}{(x-3)^4}$$

$$\frac{dy}{dx} = \frac{(x-3)[-2x-12]}{(x-3)^4}$$

$$\frac{dy}{dx} = \frac{-2x-12}{(x-3)^3} = -\frac{2(x+6)}{(x-3)^3}$$

Example 2

$y = \frac{5x^2+10x}{(x+1)^2}$. Show that $\frac{dy}{dx} = \frac{A}{(x+1)^n}$

where A and n are constants to be found

Apply quotient rule

$$\frac{dy}{dx} = \frac{(x+1)^2(10x+10) - (5x^2+10x)(2)(x+1)}{(x+1)^4}$$

Factorise

$$\frac{dy}{dx} = \frac{10(x+1)^3 - 2(x+1)(5x^2+10x)}{(x+1)^4}$$

We still need to get this into the form required. To do this, we factorise the numerator by taking out what is common
Take out the HCF of the constants which is 2
Take out the HCF of the pink terms which is $(x+1)$
(lowest power of each)
So we **factorise** $2(x+1)$ out from the numerator

$$\frac{dy}{dx} = \frac{2(x+1)[5(x+1)^2 - (5x^2+10x)]}{(x+1)^4}$$

Simplify what is inside the square bracket

$$\frac{dy}{dx} = \frac{2(x+1)[5x^2+10x+5-5x^2-10x]}{(x+1)^4}$$

$$\frac{dy}{dx} = \frac{10(x+1)}{(x+1)^4}$$

Cancel the common $(x+1)$ term

$$\frac{dy}{dx} = \frac{10}{(x+1)^3}$$

Example 3

$y = \frac{x-1}{\sqrt{x+1}}$

Show that $\frac{dy}{dx} = \frac{x+c}{k\sqrt{(x+1)^p}}$,where $c, k, p \in \mathbb{N}$

$$y = \frac{x-1}{(x+1)^{\frac{1}{2}}}$$

Apply quotient rule

$$\frac{dy}{dx} = \frac{(x+1)^{\frac{1}{2}}(1) - (x-1)\left(\frac{1}{2}\right)(x+1)^{-\frac{1}{2}}}{x+1}$$

Reorder the constants in the numerator

$$\frac{dy}{dx} = \frac{(x+1)^{\frac{1}{2}} - \frac{1}{2}(x-1)(x+1)^{-\frac{1}{2}}}{x+1}$$

Take out the lower power of $(x+1)^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{(x+1)^{-\frac{1}{2}}\left[(x+1) - \frac{1}{2}(x-1)\right]}{x+1}$$

Simplify what is inside the square brackets

$$\frac{dy}{dx} = \frac{(x+1)^{-\frac{1}{2}}\left[x+1-\frac{1}{2}x+\frac{1}{2}\right]}{x+1}$$

Simplify again

$$\frac{dy}{dx} = \frac{(x+1)^{-\frac{1}{2}}\left[\frac{1}{2}x+\frac{3}{2}\right]}{x+1}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}x+\frac{3}{2}}{(x+1)(x+1)^{-\frac{1}{2}}}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}x+\frac{3}{2}}{(x+1)^{1+\frac{1}{2}}} = \frac{\frac{1}{2}x+\frac{3}{2}}{(x+1)^{\frac{3}{2}}} = \frac{x+3}{2\sqrt{(x+1)^3}}$$

Example 4

$y = \frac{x-4}{2+\sqrt{x}}, x > 0.$

Show that $\frac{dy}{dx} = \frac{1}{A\sqrt{x}}, x > 0$

where A is a constant to be found

$$y = \frac{x-4}{2+\sqrt{x}}$$

Apply quotient rule

$$\frac{dy}{dx} = \frac{(2+\sqrt{x})(1) - (x-4)\left(\frac{1}{2}x^{-\frac{1}{2}}\right)}{(2+\sqrt{x})^2}$$

We can't factorise the numerator like we usually can.
Let's expand the numerator instead since all we can do.

$$= \frac{2+\sqrt{x}-\frac{1}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}}{(2+\sqrt{x})^2}$$

Group the common terms

$$= \frac{2+\frac{1}{2}x^{\frac{1}{2}}+2x^{-\frac{1}{2}}}{(2+\sqrt{x})^2}$$

Turn $x^{-\frac{1}{2}}$ into a fraction

$$= \frac{\frac{2}{1}+\frac{1}{2}x^{\frac{1}{2}}+\frac{2}{\sqrt{x}}}{(2+\sqrt{x})^2}$$

Get a **common denominator** in the numerator

$$= \frac{\frac{4\sqrt{x}}{2\sqrt{x}}+\frac{x}{2\sqrt{x}}+\frac{4}{2\sqrt{x}}}{(2+\sqrt{x})^2}$$

$$\frac{dy}{dx} = \frac{\frac{4\sqrt{x}+x+4}{2\sqrt{x}}}{(2+\sqrt{x})^2} = \frac{\frac{x+4\sqrt{x}+4}{2\sqrt{x}}}{(2+\sqrt{x})^2} = \frac{\frac{(\sqrt{x}+2)^2}{2\sqrt{x}}}{(2+\sqrt{x})^2}$$

Rewrite the fraction as numerator ÷ denominator

$$= \frac{(\sqrt{x}+2)^2}{2\sqrt{x}} \div \frac{(2+\sqrt{x})^2}{1}$$

$$= \frac{(\sqrt{x}+2)^2}{2\sqrt{x}} \times \frac{1}{(2+\sqrt{x})^2} = \frac{1}{2\sqrt{x}}$$

Example 5	Example 6 – less obvious to use quotient rule
Given that $y = \frac{3\sin x}{2\sin x + 2\cos x}$ show that $\frac{dy}{dx} = \frac{A}{1+\sin 2x}$ where A is a rational constant to be found.	Use the quotient rule to show that i. The derivative of $tan\ x$ is $\sec^2 x$ ii. The derivative of $sec\ x$ is $\sec x \tan x$

Example 5

Apply quotient rule

$$\frac{dy}{dx} = \frac{(2\sin x + 2\cos x)(3\cos x) - 3\sin x(2\cos x - 2\sin x)}{(2\sin x + 2\cos x)^2}$$

We can't factorise the numerator like we usually can. Let's expand the numerator instead since this is all we can do.

$$\frac{dy}{dx} = \frac{6\sin x\cos x + 6\cos^2 x - 6\sin x\cos x + 6\sin^2 x}{(2\sin x + 2\cos x)^2}$$

Collect like terms in the numerator

$$\frac{dy}{dx} = \frac{6\cos^2 x + 6\sin^2 x}{(2\sin x + 2\cos x)^2}$$

Factorise the numerator

$$\frac{dy}{dx} = \frac{6(\cos^2 x + \sin^2 x)}{(2\sin x + 2\cos x)^2}$$

Use the identity $\sin^2 x + \cos^2 x = 1$

$$\frac{dy}{dx} = \frac{6(1)}{(2\sin x + 2\cos x)^2}$$

Now we can see that out denominator still doesn't have the right form, so we need to expand it

$$\frac{dy}{dx} = \frac{6}{4\sin^2 x + 8\sin x\ cox + 4\cos^2 x}$$

Factorise the 4 out in the denominator

$$\frac{dy}{dx} = \frac{6}{4(\sin^2 x + \cos^2 x) + 8\sin x\ cosx}$$

Use the identity $\sin^2 x + \cos^2 x = 1$

$$\frac{dy}{dx} = \frac{6}{4(1) + 4(2\sin x\cos x)}$$

Use the identity $2sin\ x\cos x = \sin 2x$

$$\frac{dy}{dx} = \frac{6}{4 + 4\sin 2x}$$

Factorise the 4 out in the denominator

$$\frac{dy}{dx} = \frac{6}{4(1 + \sin 2x)}$$

$$A = \frac{6}{4} = \frac{3}{2}$$

Example 6

i.

We can use a trig identity for $tan\ x$ to write it as a fraction in order to use quotient rule.

$$y = \tan x = \frac{\sin x}{\cos x}$$

Now that we have expression $y = \tan x$ as a fraction we can use the quotient rule.

$$\frac{(copy\ denominator)(\boldsymbol{differentiate}\ numerator) - (copy\ numerator)(\boldsymbol{differentiate}\ denominator)}{(denominator)^2}$$

$$\frac{dy}{dx} = \frac{\cos x(\cos x) - \sin x(-\sin x)}{(\cos x)^2}$$

Simplifying the numerator

$$\frac{dy}{dx} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$\frac{dy}{dx} = \frac{1}{\cos^2 x}$$

$$\frac{dy}{dx} = \sec^2 x$$

ii. The expression $\frac{d}{dx}(\sec x)$ means "the derivative of $\sec x$ with respect to x", so if

$$y = \sec x = \frac{1}{\cos x}$$

Using quotient rule

$$\frac{(copy\ denominator)(\boldsymbol{differentiate}\ numerator) - (copy\ numerator)(\boldsymbol{differentiate}\ denominator)}{(denominator)^2}$$

$$\frac{dy}{dx} = \frac{\cos x(0) - 1(-\sin x)}{(\cos x)^2}$$

Simplifying the numerator

$$\frac{dy}{dx} = \frac{\sin x}{\cos^2 x}$$

Rewriting as the product of two fractions

$$\frac{dy}{dx} = \frac{\sin x}{\cos x}\left(\frac{1}{\cos x}\right)$$

Using trig identities

$$\frac{dy}{dx} = \tan x \sec x$$