Venn Diagrams With Set Notation

Set Notation Definitions:

Don't worry too much if you don't fully understand the definitions below. Just memorise what the symbols mean/represent at first. You will understand how to use them when you get to the examples on page 2 onwards.

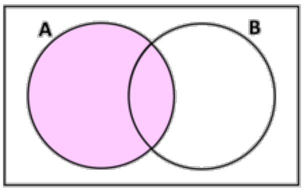
- ξ = the whole entire set
- $\{\ldots\}$ = this is used to display the elements which are in a set. We put the elements inside this curly bracket.
  For example A= $\{2,3,5\}$ means set A contains the elements 2, 3 and 5
- $\in$ =element of (means is inside of/a member of)
  For example A= $\{2,3,5\}$. We can say $2 \in$ A since 2 is inside the set A
- $\notin$ =not an element of (means is not inside of/not a member of)
  For example A= $\{2,3,5\}$. We can say $7 \notin$ A since 7 is not inside the set A
- $\emptyset$ =empty set (means no elements inside the set, it is empty)
- $\cap$ = intersection (this means '**and**')
- $\cup$ =union (this means '**or**')
- $'$ = complement (this means '**not**')
- $\supset$ =subset/contained in. Think of subset as an inclusion and meaning contained completely inside.
  For example $B \subset A$. Every element in B is inside A, but A might have more elements than B
- $\not\subset$ =not a subset of/not contained in
- $n(\ldots)$ = number of elements in the set (this does not mean what elements, it just means how many)

Venn diagram - 9 basic representations

Firstly, you **MUST** know and understand the following 9 basic Venn diagram representations of the following

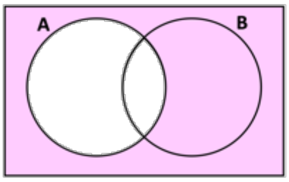
$P(A)$ $P(A')$ $P(B)$ $P(B')$ $P(A \cap B)$ $P(A \cup B)$ $P(A \cap B')$ $P(A' \cap B)$ $P(A' \cap B')$

$P(A)$



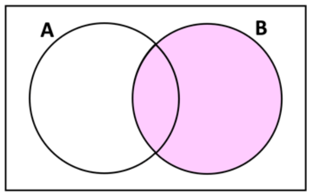
In English: This says all of A. Notice how we include the central part since it is also part of A. The ENTIRE circle on the left is A.

$P(A')$



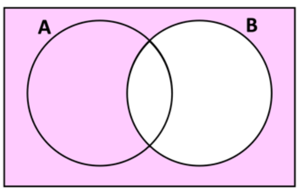
In English: This says everything that is NOT in A, hence the entire circle of A is not included

$P(B)$



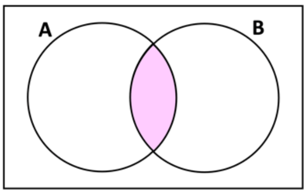
In English: This says all of B. Notice how we include the central part since it is part of B. The ENTIRE circle on the right is B.

$P(B')$



In English: This says everything that is NOT in B, hence the entire circle of A is not included

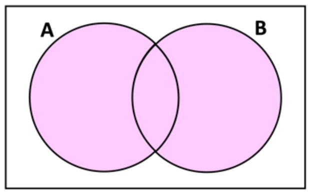
$P(A \cap B)$



In English: This says everything that is in A AND at the same time must be in B.

Remember: $\cap$ means AND which means we have to be in A AND B hence both have to be true at the same time.

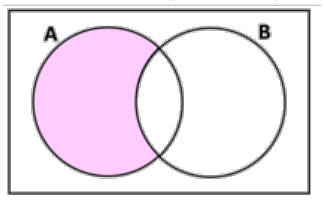
$P(A \cup B)$



In English: This says everything that is EITHER in A or B.

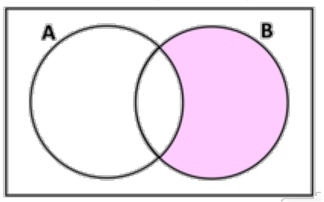
Remember: $\cup$ means OR which means either in A OR B and hence we merge all of A and B together.

$P(A \cap B')$



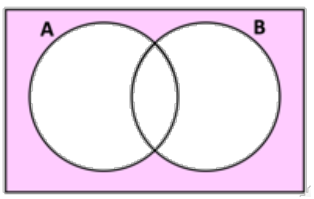
In English: This says everything that is A AND at the same time must not be in B, so we delete the part of a that touches B.

$P(A' \cap B)$



In English: This says everything that is not in A AND at the same time must not be in A so we delete the part of B which touches A.

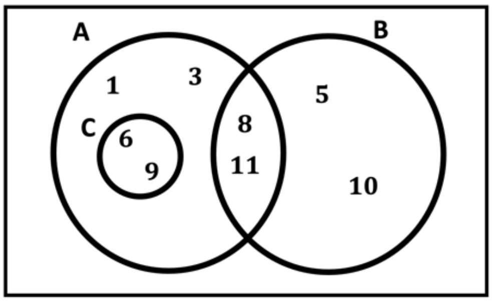
$P(A' \cap B')$



In English: This says everything not in A AND at the same time also not in B. This is everything on the outside.

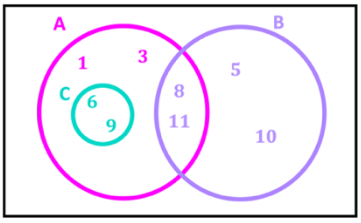
Type 1: Given a Venn diagram, work out the sets

Given the Venn diagram on the right, find all of the following sets

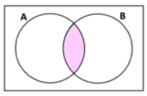


i. A
ii. B
iii. C
iv. $A \cap B$
v. $A \cup B$
vi. $n(A)$
vii. $n(B)$
viii. $n(A' \cap B)$
ix. Are the following five statements true or false
$6 \in A$
$3 \in B$
$6 \in C$
$5 \notin A$
$C \supset A$
$A \supset C$

Let's colour code for ease of explanation

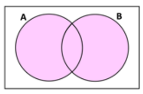


i.
A is the full circle of A. We consider everything that is in this circle.
A= $\{1,3,6,8,9,11\}$
ii.
B is the full circle of B. We consider everything that is in this circle.
B= $\{5,8,10,11\}$
iii.
C is the full circle of C.
C= $\{6,9\}$
iv.
$\cap$ means and, so $A \cap B$ means the members in A and B hence in both sets at the same time. This is the middle part



If you still don't understand this go back and read page 2 above.
$A \cap B = \{8, 11\}$

v.
$\cup$ means or, so $A \cup B$ means either A or B hence everything that inside of the circles.



If you still don't understand this go back and read page 2 above.
$A \cup B = \{1,3,5,6,8,9,10,11\}$

vi.
This means the number of elements in A (not what elements, but instead <u>how many</u> elements). We have 6 elements since we have 1,3, 6, 8, 9, 11 in the full circle of A

$$n(A) = 6$$

vii.
This means the number of elements in B (not what elements, but instead <u>how many</u> elements). We have 4 elements since we have 5, 8, 10, 11 in the full circle of B

$$n(B) = 4$$

viii.
This means not touching any part of A, but at the same time has to be touching K: {5, 13}

ix.

$6 \in A$ true since 6 is inside A
$3 \in B$ is not true since 3 is not inside B
$6 \in C$ true since 6 is inside c
$5 \notin A$ is true since 5 is not inside A
$C \supset A$ is true since C is completely contained in A. A has all the elements of C and no mor than C.
$A \supset C$ is false since A is not completely contained in C. A has more elements than C.

Type 2: Given the sets, fill in a Venn diagram

Example 1:

ξ = odd numbers less than 35
$A = \{1, 5, 9, 13, 17\}$
$B = \{1, 9, 17, 25, 33\}$

List the members of the following the sets

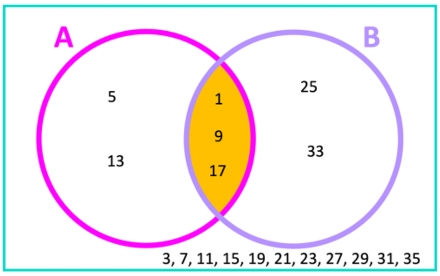
i. $A \cap B$
ii. $A \cup B$
iii. $A \cap B'$

Find

iv. $n(A' \cap B')$

ξ = odd numbers less than 35
$A = \{1, 5, 9, 13, 17\}$
$B = \{1, 9, 17, 25, 33\}$

Let's draw a Venn diagram for A and B and colour code for ease of explanation



How did we fill in the diagram above?

Firstly, we always fill in the centre part (orange part):
This is the numbers common to both sets (the same numbers that appears in both sets) which are 1, 9, 17

Now we can fill in the crescent/half-moon parts of A and B:
A has 1, 5, 9, 13 and 17 in total and we have already filled in 1, 9, 17 in the centre which counts as part of A so we just have to put 5, 13 in the left (moon shape) part of A
B has 1, 9, 17, 25 and 33 and we have already filled in 1, 9, 17 in the centre which counts as part of B so we just have to put 25 and 33 in the right part (moon shape) of B

Next we fill in the outside part:
The entire set is odd numbers less than 35, so outside we have to put the odd numbers less than 35 which haven't been considered yet which are 3, 7, 11, 15, 19, 21, 23, 27, 29, 31, 35

Make sure you understand page 2 before proceeding.

i. This means the members in both sets (and) hence the middle region: {1, 9, 17}
ii. This means the region in either set (or) hence both circles: {1, 5, 9, 13, 17, 25, 33}
iii. This means A but not touching any part of B hence the left crescent/moon shape: {5, 13}
iv. This means the number of elements
$A' \cap B'$ means neither in A nor in B hence the outside region: {3, 7, 11, 15, 19, 21, 23, 27, 29, 31, 35}
The number of elements is 11

Example 2: With 3 events

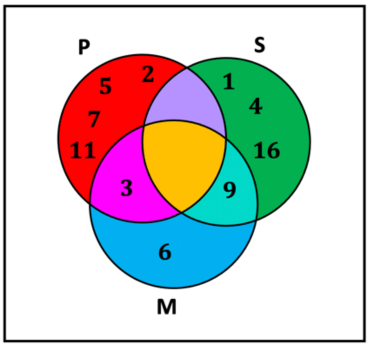
$$\xi = \{1,2,3,4,5,6,7,9,11,16\}$$
$$P = \{2,3,5,7\ 11\}$$
$$S = \{1,4,9,16\}$$
$$M = \{3,6,9\}$$

i. List the members of the set $P \cap M$

ii. Write down the value of $n(M' \cap P)$

iii. Write down the value of $n(P' \cap S)$

We need to draw a Venn diagram to show this information first which will help us answer the question



How did we fill in this Venn diagram?

Again, we start by filling in the centre

Any elements common to all 3 sets P, S and M? No, so leave this empty.

Any elements common to P and S? No so leave this empty

Any elements common to M and S? Yes 9

Any elements common to P and M? Yes 3

What is left over for P? 2, 5, 7 and 11 are left over since 3 is already accounted for

What is left over for S? 1, 4 and 16 since 9 is already accounted for

What is left over for M? 6 only since 3 and 9 is already accounted for

i.

This is the members in P and M hence the pink and orange region

The orange region is empty

The pink region has one element which is 3

So, we only have $\{3\}$

ii.

Remember n means the **number** of elements, not what elements

This means the elements not in the full circle of M, but must be in P

This is the red and purple region.

The are no elements in the purple region

There are only elements in the red region hence this is 4 since 4 numbers

ii.

Remember n means the **number** of elements, not what elements

This means the elements not in the full circle of P, but must be in S

This is the dark green and turquoise region

$$3+1 = 4$$

Example 3: With Words

$$\xi = \{\text{students in year 12}\}$$
$$G = \{\text{students who study German}\}$$
$$F = \{\text{students who study French}\}$$
$$M = \{\text{students who study Maths}\}$$

i. Describe $G \cap M$

ii. $G \cap M = \emptyset$
Use this information to write a statement about the students who study German in year 12

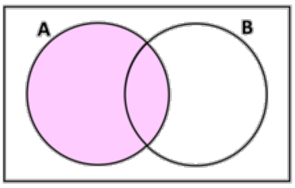
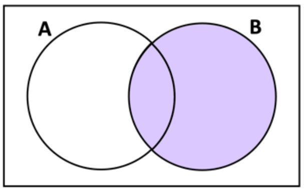
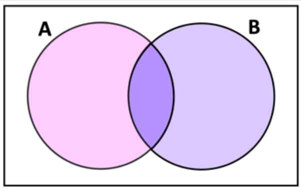
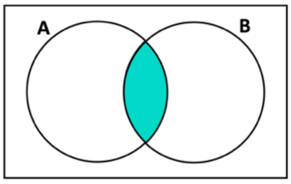
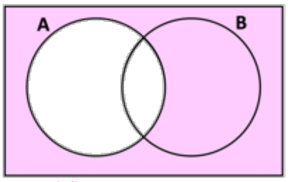
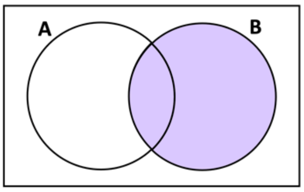
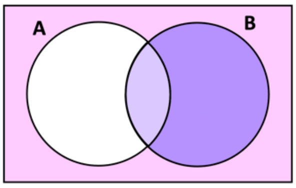
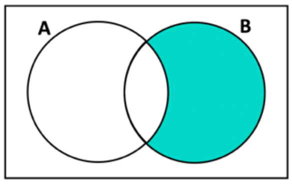
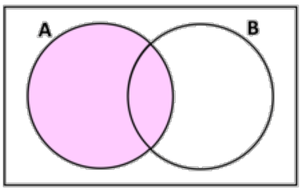
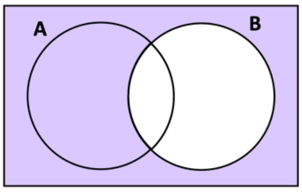
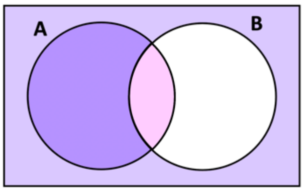
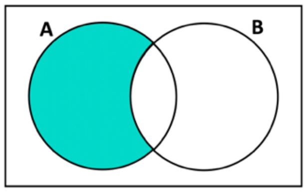
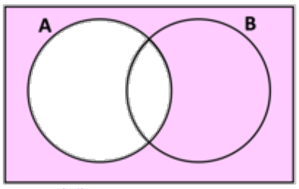
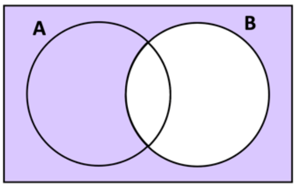
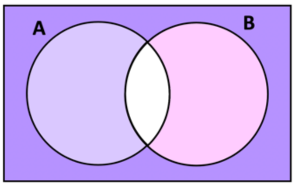
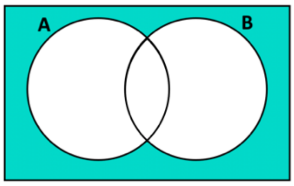
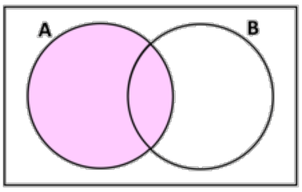
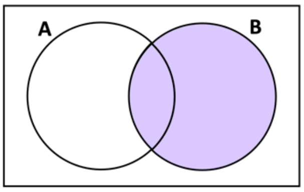
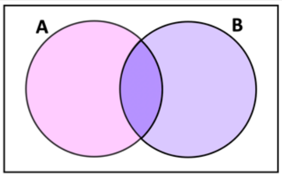
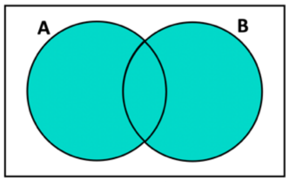
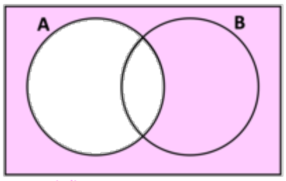
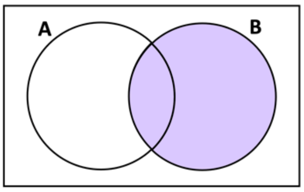
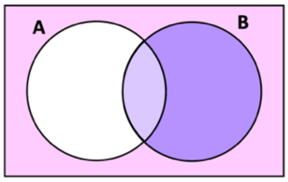
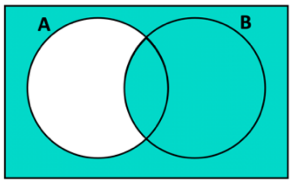
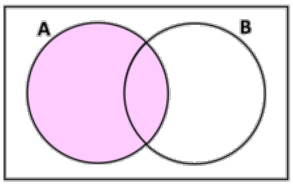
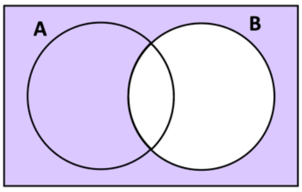
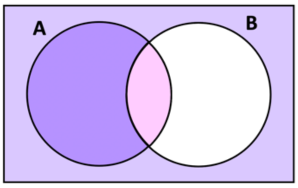
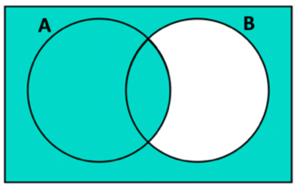
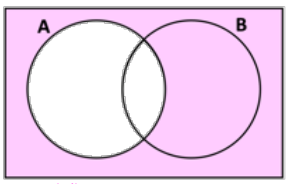
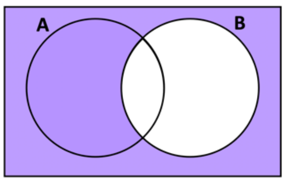
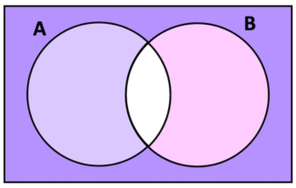
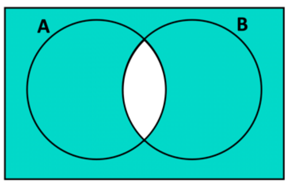
iii. Preety is a student in year 12.
Preety $\notin F$
Use this information to write a statement about Preety.

i. Students who student both German and Maths ii. There are no students who study both German and Maths iii. Pretty does not study French

For harder questions, we need a more in depth understanding of $\cap$ and $\cup$. The following pages will explain this.

Each row in the table on the next page is a type you have to know and the columns show the 4 steps for each.

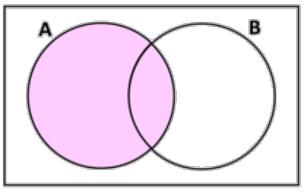
How to deal with an ∪ (**union which means or**) versus an ∩ (**intersection which means and**)

	Step 1: (Shade the LHS)	Step 2: (Shade the RHS)	Step 3: Put Steps 1 and 2 Together	Step 4: Deal with the Symbol (This is the answer)
Type 1: $P(A \cap B)$	 P(A) = Everything in A	 P(B) = Everything in B	 The darker region indicates the overlap where the regions were shaded twice	 ∩ (means and) so we take what was shaded in BOTH diagrams i.e. the double shading common to both diagrams (overlap)
Type 2: $P(A' \cap B)$ Note: ′ means not	 $P(A')$ = Everything NOT in A	 $P(B)$ = Everything in B	 The darker region indicates the overlap where the regions were shaded twice	 ∩ (means and) so we take what was shaded in BOTH diagrams i.e. the double shading common to both diagrams (overlap)
Type 3: $P(A \cap B')$ Note: ′ means not	 P(A) = Everything in A	 $P(B')$ = Everything NOT in B	 The darker region indicates the overlap where the regions were shaded twice	 ∩ (means and) so we take what was shaded in BOTH diagrams i.e. the double shading common to both diagrams (overlap)
Type 4: $P(A' \cap B')$ Note: ′ means not	 $P(A')$ = Everything NOT in A	 $P(B')$ = Everything NOT in B	 The darker region indicates the overlap where the regions were shaded twice	 ∩ (means and) so we take what was shaded in BOTH diagrams i.e. the double shading common to both diagrams (overlap)
Type 5: $P(A \cup B)$	 $P(A)$ =Everything in A	 $P(B)$ =Everything in B	 The darker region indicates the overlap where the regions were shaded twice	 ∪ (means or) so we **merge/combine** everything that is shaded
Type 6: $P(A' \cup B)$ Note: ′ means not	 $P(A')$ = Everything NOT in A	 $P(B)$ =Everything in B	 The darker region indicates the overlap where the regions were shaded twice	 ∪ (means or) so we **merge/combine** everything that is shaded
Type 7: $P(A \cup B')$ Note: ′ means not	 P(A) = Everything in A	 $P(B')$ = Everything NOT in B	 The darker region indicates the overlap where the regions were shaded twice	 ∪ (means or) so we **merge/combine** everything that is shaded
Type 8: $P(A' \cup B')$ Note: ′ means not	 $P(A')$ = Everything NOT in A	 $P(B')$ = Everything NOT in B	 The darker region indicates the overlap where the regions were shaded twice	 ∪ (means or) so we **merge/combine** everything that is shaded

Results

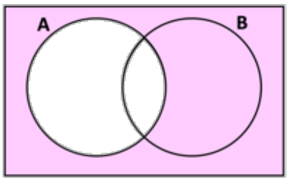
From the above you should now be comfortable all the following (you can either memorise them or use the method mentioned above to find them).

$P(A)$



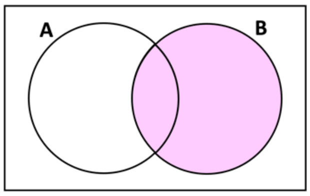
In English: This says all of A. Notice how we include the central part since it is part of A.

$P(A')$



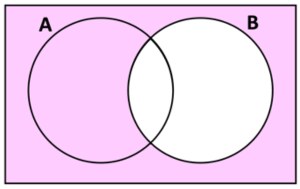
In English: This says everything that isn't in A

$P(B)$



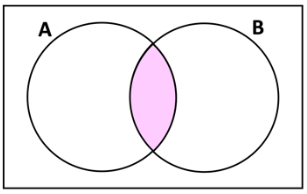
In English: This says all of B. . Notice how we include the central part since it is part of B.

$P(B')$



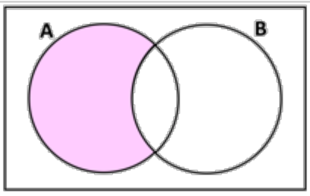
In English: This says everything that isn't in B

$P(A \cap B)$



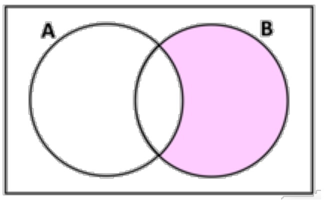
In English: This says everything that is in AND at the same time must be in B

$P(A \cap B')$



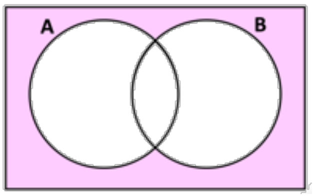
In English: This says everything that is A AND at the same time must not be in B.

$P(A' \cap B)$



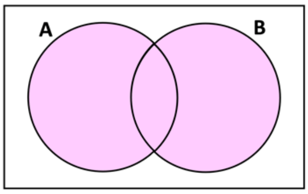
In English: This says everything that is not in A AND at the same time must be in B.

$P(A' \cap B')$



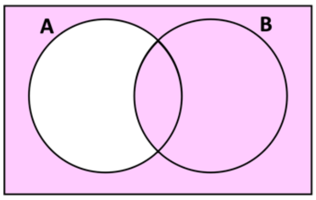
In English: This says everything in not in A AND at the same time not in B.

$P(A \cup B)$



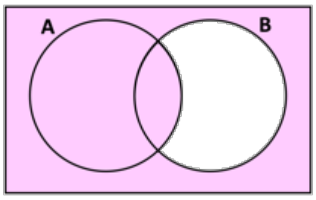
In English: This says everything that is EITHER in A or B.

$P(A' \cup B)$



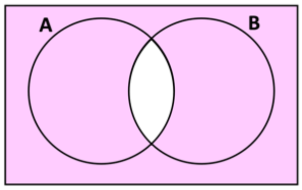
In English: This says everything that is EITHER not in A or in B. Don't let the fact that it says not in A confuse you that we have shaded part of A. The shaded part of A comes from when it could be in B. Remember ∪ means in either. They don't have to be simultaneously like for ∪.

$P(A \cup B')$



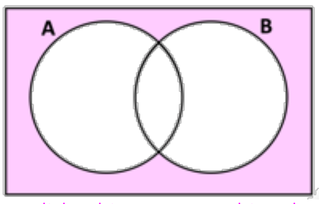
In English: This says everything that is EITHER in A or not in B. Don't let the fact that it says not in B confuse you that we have shaded part of B. The shaded part of B comes from when it could be in A. Remember ∪ means in either. They don't have to be simultaneously like for ∪.

$P(A' \cup B')$



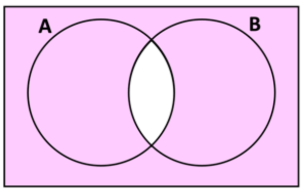
In English: This says everything that is EITHER not in A or not in B. . Don't let the fact that it says not in A and B confuse you that we have shaded part of A and B. The shaded part of B comes from when in not A and the shaded part of A comes when in not B.

$P(A \cup B)'$



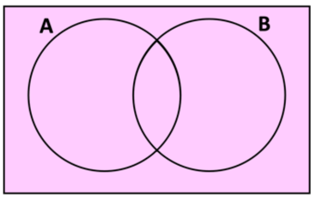
In English: This says everything that is EITHER in A or not in B

$P(A \cap B)'$



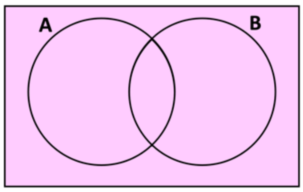
In English: This says consider everything that is in a and B and then shade the reverse

$P(A \cup A')$



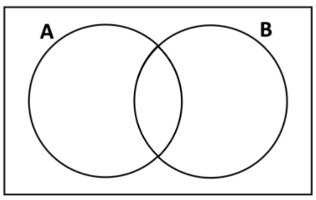
In English: This says everything that is EITHER in A or not in A which is everything

$P(B \cup B')$



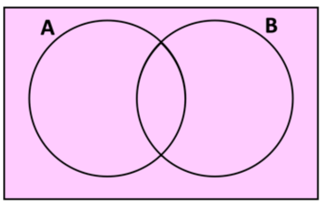
In English: This says everything that is EITHER in B or not in B which is everything

$P(\emptyset)$



In English: This says the empty set.

$P(\xi)$



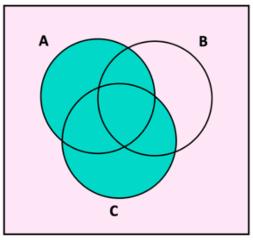
In English: This just says the **entire** set

What about if we have 3 events and have to shade?

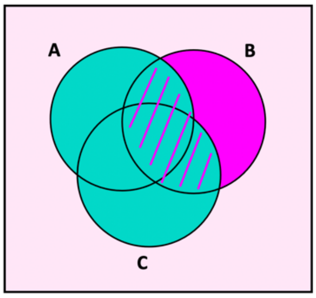
Example 1: P ($(A \cup C) \cap B$)

$P((A \cup C) \cap B)$

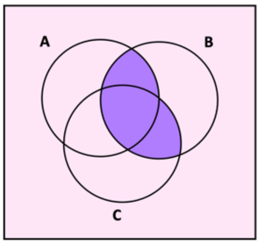
We deal with what is in the bracket first (think of BIDMAS). This means we shade $P(A \cup C)$ first



Now we deal with the $\cap B$ part. This means we choose the parts that are also in B i.e. choose the green parts which are within the pink circle



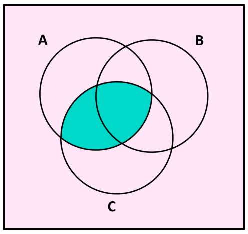
This gives $P((A \cup C) \cap B)$



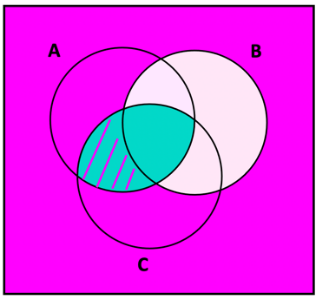
Example 2: P ($B' \cap (A \cap C)$)

$P(B' \cap (A \cap C))$

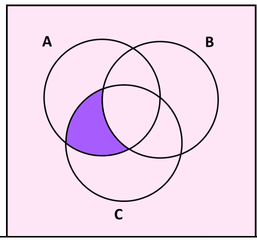
We deal with what is in the bracket first (think of BIDMAS). This means we shade $P(A \cap C)$ first



Now we deal with the $B' \cap$ part
This means we are choosing the parts of $A \cap C$ that are also NOT in B, i.e. choose the green parts which are within the pink shaded region



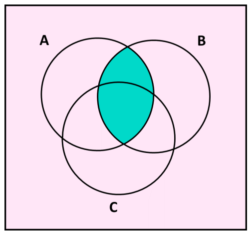
This gives $P(B' \cap (A \cap C))$



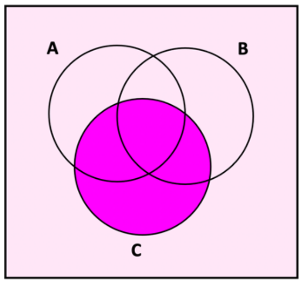
Example 3: P ($(A \cap B) \cup C$)

$P((A \cap B) \cup C)$

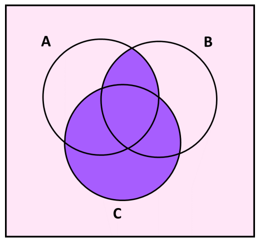
Consider the bracket first; shade $A \cap B$



Now deal with $\cup C$ so also shade everything in C



Merge both our answers together. This gives $P((A \cap B) \cup C)$



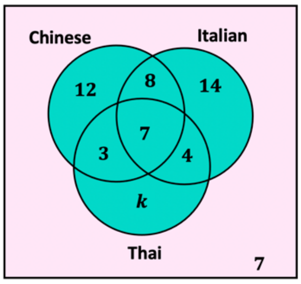
To try:
Shade the following
$P((A \cup B) \cap C)$
$P(A \cup (B \cap C'))$
$P(A \cup (B \cap C'))$
$P((A \cup B') \cap C)$
$P(A \cap (B' \cup C))$

Venn Diagrams Without Set Notation

Example 1: Already drawn and filled in for us

There are 60 members of a club. The members indicate their liking for Chinese, Italian and Thai takeaway food in the Venn diagram below. If a member is selected at random, what is the probability that they like:

i. Italian
ii. Only Italian
iii. None of these choices
iv. All of the choices
v. Only two types of these choice
vi. Thai or Italian
vii. Italian & Thai
viii. At least one of these choices
ix. Chinese or Italian, but not Thai
x. Chinese and Italian, but not Thai
xi. Exactly one of the three choices of takeaway

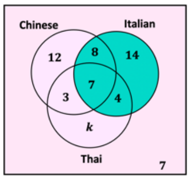


Total must add to 60

So $60 = 12 + 8 + 14 + 3 + 7 + 4 + k$

So $k = 5$

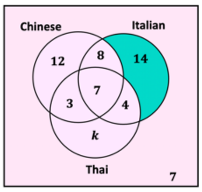
i.



Probability they like Italian
This is the full circle of Italian

$$\frac{8+14+7+4}{60} = \frac{33}{60} = \frac{11}{20}$$

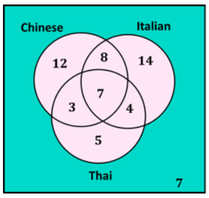
ii.



Probability they like only Italian
This is Italian and nothing else

$$\frac{14}{60} = \frac{7}{30}$$

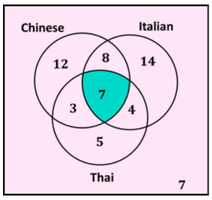
iii.



Probability they like none of these choices

$$\frac{7}{60}$$

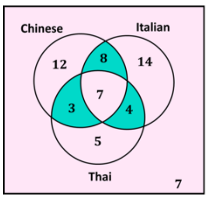
iv.



Probability they like all of these choices
This is the common part ot all 3

$$\frac{7}{60}$$

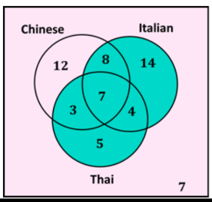
v.



Probability they like only two out of three

$$\frac{8+3+4}{60} = \frac{15}{60} = \frac{1}{4}$$

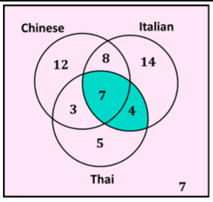
vi.



Probability they like Thai or Italian

$$\frac{8+14+7+3+4+5}{60} = \frac{41}{60}$$

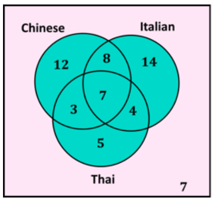
vii.



Probability they like Italian and Thai
This is the middle part of Italian and Thai

$$\frac{7+4}{60}=\frac{11}{60}$$

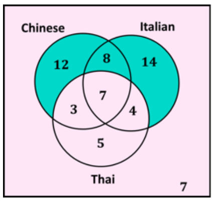
viii.



Probability they like at least one of these choices

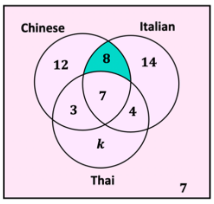
$$\frac{12+8+14+7+3+4+5}{60}=\frac{53}{60}$$

ix.



Probability they like Chinese or Italian, but not Thai

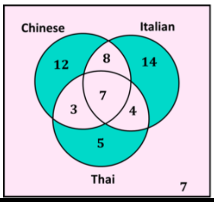
$$\frac{12+8+14+7+3+4+5}{60}=\frac{53}{60}$$



Probability they like Chinese **and** Italian, but not Thai
This is the middle part of Chinese and Italian, but then take away anything in Thai

$$\frac{8}{60}=\frac{53}{60}$$

x.



Probability they like exactly one of the three

$$\frac{12+5+14}{60}=\frac{31}{60}$$

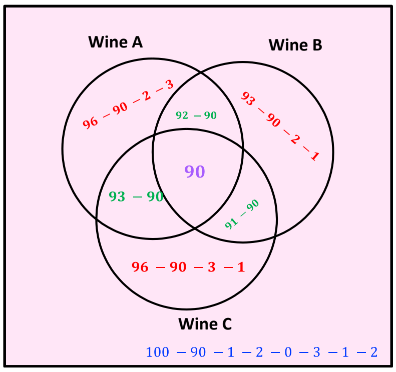
Example 2: To be drawn and filled in

96 people like wine A
93 people like wine B
96 people like wine C
92 people like wine A and B
91 people like wine B and C
93 people like wine A and C
90 like all three
There are 100 people in total
Find the probability that a person

i. Doesn't like any wine
ii. Likes wine A, but not wine B
iii. Likes any wine except C
iv. Likes exactly one of the wines
v. Likes exactly two kinds of wine
vi. Likes at least one wine
vii. Likes wine A but not B or C
viii. Likes A and B but not C
ix. Likes wine A or B
x. Likes wine A or B or both
xi. Given that the person likes wine A, what is the probability that they like wine
xii. Given that the person likes wine A or B or both, what is the probability that they do not like A

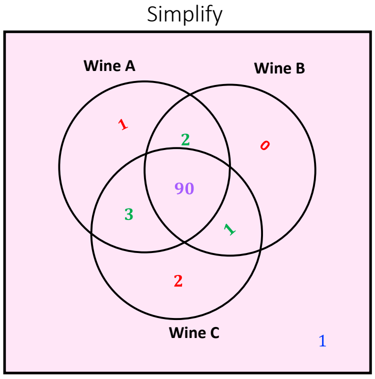
We always fil in the middle part first

90 like all 3



92 like A and B, 91 like B and C, 93 like A and C
96 like A, 93 like B, 96 like C
100 in total

Simplify



Now we can answer the questions

i. $\frac{1}{100}$

ii. $\frac{1+3}{100} = \frac{4}{100}$

iii. $\frac{1+2+0}{100} = \frac{3}{100}$

iv. $\frac{1+2+0}{100} = \frac{3}{100}$

v. $\frac{2+3+1}{100} = \frac{6}{100}$

vi. $\frac{1+2+0+90+3+1+2}{100} = \frac{99}{100}$

vii. $\frac{1}{100}$

viii. $\frac{2}{100}$

ix. $\frac{1+2+90+3+1}{100} = \frac{97}{100}$

x. $\frac{1+2+0}{100} = \frac{3}{100}$

xi. $\frac{\text{like A and C}}{\text{total that like A}} = \frac{3+90}{90+1+2+3} = \frac{93}{96}$

xii. $\frac{\text{likes A or B or both AND doesn't like A}}{\text{likes A or B or both}} = \frac{\text{likes B and doesn't like A}}{\text{likes A or B or both}} = \frac{0+1}{1+2+0+90+3+1} = \frac{1}{97}$